# GCE AS/A LEVEL – NEW

2305U10-1

S19-2305U10-1

**MONDAY, 13 MAY 2019 – AFTERNOON**

# FURTHER MATHEMATICS – AS unit 1
## FURTHER PURE MATHEMATICS A

1 hour 30 minutes

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

**INFORMATION FOR CANDIDATES**

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**Reminder**: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

**1.** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 7 \\ -2 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}$.

The matrix **X** is such that **AX = B**. Showing all your working, find the matrix **X**. [6]

**2.** The position vectors of the points *A*, *B*, *C*, *D* are given by

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \qquad \mathbf{b} = 4\mathbf{j} + 5\mathbf{k}, \qquad \mathbf{c} = 7\mathbf{i} - 3\mathbf{k}, \qquad \mathbf{d} = -3\mathbf{i} - \mathbf{j} - 5\mathbf{k},$$

respectively.

*(a)* Find the vector equations of the lines *AB* and *CD*. [3]

*(b)* Determine whether or not the lines *AB* and *CD* are perpendicular. [4]

**3.** The complex numbers $z$ and $w$ are represented by the points *Z* and *W* in an Argand diagram.

The complex number $z$ is such that $|z| = 6$ and $\arg z = \frac{\pi}{3}$.

The point *W* is a 90° clockwise rotation, about the origin, of the point *Z* in the Argand diagram.

*(a)* Express $z$ and $w$ in the form $x + \mathrm{i}y$. [3]

*(b)* Find the complex number $\frac{z}{w}$. [4]

**4.** Prove, by mathematical induction, that $9^n + 15$ is a multiple of 8 for all positive integers $n$. [7]

**5.** Given that $x = -\frac{1}{2}$ and $x = -3$ are two roots of the equation

$$2x^4 - x^3 - 15x^2 + 23x + 15 = 0,$$

find the remaining roots. [6]

**6.** The complex number $z$ is represented by the point $P(x, y)$ in an Argand diagram.
Given that

$$|z - 1| = |z - 2\mathrm{i}|,$$

show that the locus of *P* is a straight line. [3]

 

**7.** *(a)* Find an expression for $\sum_{r=1}^{2m}(r+2)^2$ in the form $\frac{1}{3}m(am^2+bm+c)$, where $a$, $b$, $c$ are integers whose values are to be determined. [4]

*(b)* Hence, calculate $\sum_{r=11}^{20}(r+2)^2$. [4]

**8.** The plane $\Pi$ contains the three points $A(3, 5, 6)$, $B(5, -1, 7)$ and $C(-1, 7, 0)$.

Find the vector equation of the plane $\Pi$ in the form $\mathbf{r.n} = d$.
Express this equation in Cartesian form. [9]

**9.** The complex numbers $z$ and $w$ are represented by the points $P(x, y)$ and $Q(u, v)$ respectively in Argand diagrams and

$$w = z^2 - 1.$$

*(a)* Show that $v = 2xy$ and obtain an expression for $u$ in terms of $x$ and $y$. [4]

*(b)* The point $P$ moves along the line $y = 3x$. Find the equation of the locus of $Q$. [4]

**10.** The quadratic equation $px^2 + qx + r = 0$ has roots $\alpha$ and $\beta$, where $p, q, r$ are non-zero constants.

*(a)* A cubic equation is formed with roots $\alpha$, $\beta$, $\alpha + \beta$.

Find the cubic equation with coefficients expressed in terms of $p, q, r$. [6]

*(b)* Another quadratic equation $px^2 - qx - r = 0$ has roots $2\alpha$ and $\gamma$.

Show that $\beta = -2\gamma$. [3]

**END OF PAPER**