GCE AS/A Level – LEGACY

0983/01

S19-0983-01

MATHEMATICS – S1
Statistics

WEDNESDAY, 12 JUNE 2019 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The random variable X has a Poisson distribution with mean 4.
Calculate

(a) $E(X^2)$, [3]

(b) $P(X \geqslant 2 \mid X \geqslant 1)$. [3]

2. A bag contains 12 balls of which 5 are red, 4 are blue and 3 are white. A random selection of 3 of these balls is made, without replacement. Calculate the probability that the selection contains 2 balls of the same colour and another ball of a different colour. Give your answer in the form $\frac{N}{44}$, where N is an integer. [6]

3. The events *A* and *B* are such that

$$P(A) = 0{\cdot}3, \qquad P(B) = 0{\cdot}2.$$

(a) Given that $P(A \cup B) = 0{\cdot}44$, show that *A* and *B* are independent. [4]

(b) Given instead that $P(A \cup B) = 0{\cdot}4$, calculate

(i) $P(A \mid B)$,

(ii) $P(A \mid B')$. [7]

4. Bethan plays the following game. She throws a fair six-sided dice. If she obtains a ‘1’, she tosses two fair coins. If she obtains a ‘2’ or a ‘3’, she tosses three fair coins. If she obtains a ‘4’, ‘5’ or ‘6’, she tosses four fair coins.

(a) Calculate the probability that all the coins tossed fall ‘heads’. [3]

(b) Given that all the coins tossed fall ‘heads’, calculate the probability that she obtained a ‘1’ when she threw the dice. [3]

5. *(a)* The number of letters, X, arriving each morning at a house may be modelled by a Poisson distribution with mean 5. Determine the probability that, on a randomly chosen day, the number of letters arriving is

(i) equal to 2,

(ii) more than 3. [4]

(b) The number of letters, Y, arriving each morning at another house may be modelled by a Poisson distribution with mean μ. Given that

$$P(Y = 2) = P(Y = 0 \text{ or } Y = 1),$$

determine the value of μ. [5]

6. The discrete random variable X has the following probability distribution,

$$P(X = x) = kx \qquad \text{for } x = 1, 2, 3, 4, 5,$$
$$P(X = x) = 0 \qquad \text{otherwise,}$$

where k is a constant.

(a) Show that $k = \frac{1}{15}$. [2]

(b) Calculate

(i) $E(X)$,

(ii) $E\left(\frac{1}{X}\right)$. [6]

(c) Two independent observations X_1, X_2 are taken from the distribution of X. Determine the value of $P(X_1 + X_2 = 4)$. [3]

7. Jim decides to try his luck at a rifle range at a fairground so he pays £6 for 5 shots at a target. You may assume that the probability of each shot hitting the target is 0·3 and that successive shots are independent. Let X denote the number of shots that hit the target.

(a) State the distribution of X, including its parameters. [2]

(b) (i) Determine the mean and the variance of X.

(ii) Without the use of tables, calculate $P(X = 2)$.

(iii) Find $P(2 \leqslant X \leqslant 4)$. [6]

(c) Jim is given a reward of £2.50 for every shot that hits the target. Given that £P denotes his overall profit from firing the 5 shots,

(i) write down an expression for P in terms of X,

(ii) calculate the mean and the variance of P. [5]

TURN OVER

8. The continuous random variable X has probability density function f given by

$$f(x) = \frac{3}{4}x^2(2-x) \qquad \text{for } 0 \leqslant x \leqslant 2,$$
$$f(x) = 0 \qquad \text{otherwise.}$$

(a) (i) Determine the mean of X.

(ii) The mode of X is defined as the value of x which maximises $f(x)$.
Determine the mode of X. [7]

(b) (i) Find an expression for $F(x)$, valid for $0 \leqslant x \leqslant 2$, where F denotes the cumulative distribution function of X.

(ii) Show that the median of X lies between the mean and the mode. [6]

END OF PAPER