GCE AS/A Level – LEGACY

0982/01

S19-0982-01

MATHEMATICS – M3
Mechanics

THURSDAY, 20 JUNE 2019 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle *P*, of mass 2 kg, moves along a horizontal x-axis. At time t seconds, its velocity is v ms^{-1} and its displacement from the origin *O* is x metres. The particle moves under the action of a tractive force $(148 - 26x)$ N and a resistive force $(8v + 26t)$ N.

(a) Show that x satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 13x = 74 - 13t.$$ [2]

(b) Given that the particle *P* is at the origin *O* when $t = 0$ and is moving with velocity 5 ms^{-1}, find an expression for x in terms of t and determine the value of x when $t = 0{\cdot}5$. [13]

2. A particle is moving in a straight line with Simple Harmonic Motion with centre *O*. When the particle is 5 m from *O* its speed is 24 ms^{-1} and when the particle is 12 m from *O* its speed is 10 ms^{-1}.

(a) Show that the amplitude of the motion is 13 m and find the period of the motion. [7]

(b) Determine the maximum magnitude of the acceleration. [2]

(c) Find, correct to two decimal places, the distance of the particle from *O*, 3 seconds after it has passed through *O*. [3]

(d) How long after passing through *O* is the speed of the particle a half of its maximum speed? Give your answer correct to two decimal places. [5]

3. A car of mass 900 kg is initially at rest on a straight horizontal road. It is then towed by a truck with a horizontal force of 432 N. The resistance to motion of the car has magnitude $12v^2$ N, where v ms^{-1} is the speed of the car at time t seconds.

(a) (i) Show that the motion of the car satisfies the differential equation

$$75\frac{\mathrm{d}v}{\mathrm{d}t} = 36 - v^2.$$

(ii) Calculate, correct to 2 decimal places, the time taken for the speed of the car to reach 3 ms^{-1}.

(iii) Find an expression for v at time t seconds. [9]

(b) By forming another differential equation, determine the distance the car has travelled by the time its speed has reached 3 ms^{-1}. [8]

4. Particle *A*, of mass 2 kg, and particle *B*, of mass 3 kg, are connected by a light inextensible string of length l metres. Initially, both particles are lying at rest on a smooth horizontal surface a distance l metres apart, with the string just slack. Particle *B* is given a blow of impulse 50 Ns in a direction away from *A* at an angle 60° to the line joining the initial positions of *A* and *B* as shown in the diagram.

Immediately after the blow, the speed of particle *A* is v ms^{-1}.

(a) Determine the value of v and the impulsive tension in the string. [6]

(b) Calculate the magnitude and direction of the velocity of *B* immediately after the impulse. [6]

5. A uniform ladder, of mass 12 kg and length 6 m, rests with its top against a smooth vertical wall and its bottom end on rough horizontal ground. The ladder is inclined at an angle of 60° to the ground. The coefficient of friction between the ladder and the ground is μ. A person of mass 78 kg climbs the ladder.

(a) Given that $\mu = 0{\cdot}4$, determine how far the person can climb up the ladder before the ladder slips. [10]

(b) Given instead that the person can climb to the top of the ladder, determine the minimum value of μ. [3]

(c) State one modelling assumption you have made in your solution. [1]

END OF PAPER

