GCE AS/A Level – LEGACY

0979/01

S19-0979-01

MATHEMATICS – FP3
Further Pure Mathematics

TUESDAY, 25 JUNE 2019 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Determine the two roots of the equation

$$\cosh 2x = 5\sinh x + 4,$$

giving your answers correct to two decimal places. [6]

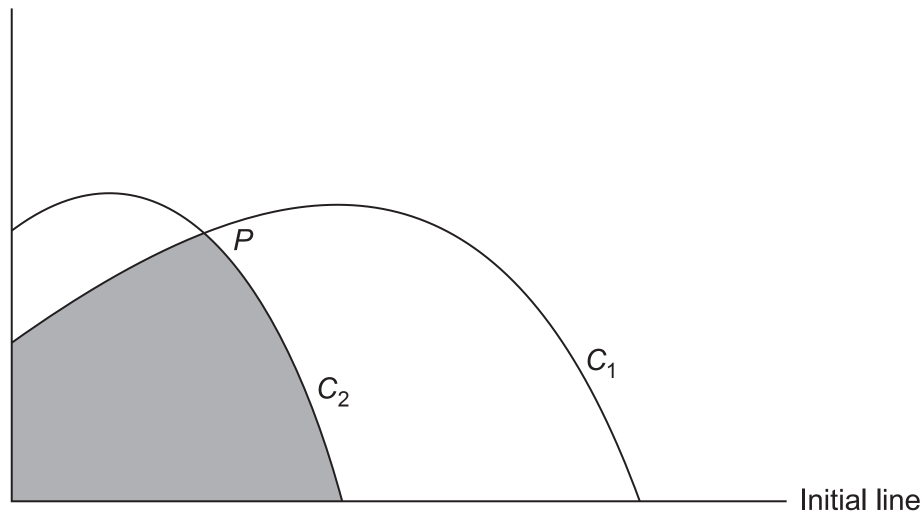
2. The curve *C* has equation $y = \ln(\sec x)$. Show that the length of the arc joining the points (0, 0) and $\left(\frac{\pi}{3}, \ln 2\right)$ on *C* is equal to $\ln\left(a + \sqrt{b}\right)$, where a, b are positive integers to be determined. [5]

3. Using the substitution $x = \sinh\theta$, evaluate the integral

$$\int_0^1 \frac{x^2}{\left(x^2+1\right)^{\frac{3}{2}}}\,\mathrm{d}x\,.$$

Give your answer correct to three significant figures. [8]

4.



The diagram shows the initial line, the line $\theta = \frac{\pi}{2}$ and sketches of the curves C_1 and C_2 with polar equations

$$C_1 : r = 2\mathrm{e}^{-2\theta} \quad \left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right),$$

$$C_2 : r = \mathrm{e}^{-\theta} \quad \left(0 \leqslant \theta \leqslant \frac{\pi}{2}\right).$$

(a) Find the polar coordinates of the point on C_1 at which the tangent is parallel to the initial line. [6]

(b) (i) Find the polar coordinates of *P*, the point of intersection of C_1 and C_2.

(ii) Find the area of the shaded region. [8]

5. Determine the value of the integral

$$\int_0^{\frac{\pi}{2}} \frac{1}{2+\sin x}\,\mathrm{d}x\ ,$$

giving your answer in the form $\dfrac{\pi\sqrt{a}}{b}$ where a, b are positive integers to be determined. [8]

6. The integral I_n is defined for $n \geqslant 0$ by

$$I_n = \int_0^1 \cosh^n\theta\,\mathrm{d}\theta\ .$$

(a) Show that, for $n \geqslant 2$,

$$nI_n = (n-1)I_{n-2} + \sinh 1(\cosh 1)^{n-1}.$$ [5]

(b) Evaluate I_4 correct to three significant figures. [5]

7. Consider the quartic equation $x^4 - 8x^2 - 16x - 12 = 0$.
You are given that the equation has a root α lying between 3 and 4.

(a) Show that the sequence obtained using the Newton-Raphson method to determine the value of α can be written in the form

$$x_{n+1} = \frac{ax_n^4 + bx_n^2 + c}{4x_n^3 - 16x_n - 16}\ ,$$

where a, b, c are integers to be determined. [4]

(b) Using this sequence, with $x_0 = 3$,

(i) write down in full the decimal value of x_1 as given by your calculator,

(ii) determine the value of α correct to three decimal places. [3]

(c) It is suggested that an alternative method for determining the value of α could be obtained using the sequence given by

$$x_{n+1} = (8x_n^2 + 16x_n + 12)^{\frac{1}{4}} \quad \text{with } x_0 = 3{\cdot}5.$$

(i) By differentiating an appropriate function, show that this sequence is convergent.

(ii) Using this sequence, write down in full the decimal value of x_1 as given by your calculator.

(iii) Determine the value of α correct to six decimal places. It is not necessary to record the values of x_2, x_3, x_4....., etc. [8]

TURN OVER

8. *(a)* Write down the Taylor series of $f(x)$ about $x = a$ as far as the quadratic term. Hence show that

$$\int_{a-h}^{a+h} f(x)\,\mathrm{d}x \approx 2hf(a) + \frac{h^3}{3}f''(a)\,,$$

where h is small. [4]

(b) Use this result to estimate the value of

$$\int_{0}^{0\cdot 5} \ln(1+\sin x)\,\mathrm{d}x\ .$$ [5]

END OF PAPER