wjec cbac

GCE AS/A Level – LEGACY

0978/01

S19-0978-01

MATHEMATICS – FP2

Further Pure Mathematics

MONDAY, 24 JUNE 2019 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Using the substitution $x = \sin^2\theta$, evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{3+\sin^4\theta}\,\mathrm{d}\theta\,.$$

Give your answer in the form $\dfrac{\pi}{M\sqrt{N}}$, where M and N are positive integers. [5]

2. The function f is given by

$$f(x) = x^2 - 2x + 2.$$

(a) Sketch a graph of $y = f(x)$, indicating the coordinates of the minimum point. [2]

(b) The set S = [2, 5]. Determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [7]

3. *(a)* By putting $t = \tan\left(\dfrac{x}{2}\right)$, show that the equation

$$3\sin x + \cos x = 2$$

can be written in the form

$$3t^2 - 6t + 1 = 0.$$ [2]

(b) Hence find the general solution, correct to the nearest degree, of the equation

$$3\sin x + \cos x = 2.$$ [6]

4. *(a)* Using mathematical induction, prove de Moivre's Theorem, namely that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta,$$

where n is a positive integer. [7]

(b) Using de Moivre's Theorem, show that

$$\cos 4\theta = a\cos^4\theta + b\cos^2\theta + c,$$

where a, b, c are integers to be determined. [5]

5. The equation of the hyperbola *H* is

$$\frac{x^2}{4} - \frac{y^2}{16} = 1.$$

(a) Show that the point $P(2\sec\theta, 4\tan\theta)$ lies on *H*, where θ is not an odd multiple of $\frac{\pi}{2}$. [1]

(b) Find the eccentricity and the coordinates of the foci of *H*. [3]

(c) The point (2, 0) is denoted by *V*. Show that the locus of the midpoint of *PV* as θ varies is a hyperbola having the same eccentricity as *H*. [6]

6. The complex number w is equal to $\frac{1}{2}\left(-1+\sqrt{3}\mathrm{i}\right)$.

(a) Show that $w^3 = 1$. [2]

(b) Hence show that if u is a cube root of the complex number z then uw is also a cube root of z. [2]

(c) (i) Verify that $1 + \mathrm{i}$ is a cube root of $-2 + 2\mathrm{i}$.

(ii) Using the result in part *(b)*, find in Cartesian form the cube root of $-2 + 2\mathrm{i}$ lying in the 2nd quadrant of the Argand diagram.

(iii) State the argument of $1 + \mathrm{i}$ and deduce the argument of the cube root of $-2 + 2\mathrm{i}$ lying in the 2nd quadrant.

(iv) Hence show that $\tan 15° = 2 - \sqrt{3}$. [10]

7. The function f is given by

$$f(x) = \frac{7x^2 + 4x + 2}{(2x+1)(3x+2)(x-3)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence evaluate the integral

$$\int_0^1 f(x)\,\mathrm{d}x.$$

Give your answer correct to three significant figures. [6]

(c) (i) State the equations of all the asymptotes on the graph of f.

(ii) Show that there are no real values of x for which $f(x) = 0$.

(iii) Write down the range of f.

(iv) Sketch the graph of f. [6]

END OF PAPER