GCE AS/A Level – LEGACY

0977/01

S19-0977-01

MATHEMATICS – FP1
Further Pure Mathematics

MONDAY, 13 MAY 2019 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Differentiate $\frac{1}{x^3}$ from first principles. [6]

2. Given that

$$z = (4-\mathrm{i})^2 + \frac{7+\mathrm{i}}{3-\mathrm{i}} + 7,$$

(a) express z in the form $a + \mathrm{i}b$, where a, b are real, [6]

(b) find the modulus and the argument of z. [2]

3. Given that

$$S_n = 5^2 + 11^2 + 17^2 + \ldots + (6n-1)^2,$$

obtain an expression for S_n giving your answer in the form $an^3 + bn^2 + cn$, where a, b, c are positive integers. [6]

4. The transformation *T* in the plane consists of an anticlockwise rotation through 45° about the origin, followed by a reflection in the line $y = x$, followed by a clockwise rotation through 45° about the origin.

(a) Find the 2×2 matrix representing *T*. [6]

(b) Identify *T* as a single transformation. [1]

5. Consider the quadratic equation $ax^2 + bx + c = 0$, where a, b, c are positive constants.

(a) Given that one of the roots of this equation is double the other root, show that $b^2 = kac$, where k is a constant whose value is to be determined. [4]

(b) Given instead that the roots of this equation differ by 1, show that

$$b^2 = 4ac + a^2.$$ [4]

(c) In each of the cases considered in parts *(a)* and *(b)*, show that the roots cannot be complex. [2]

6. The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix}.$$

(a) Show that $\mathbf{A}$ is non-singular. [3]

(b) Show that

$$\mathbf{A}^3 = 2\mathbf{A}^2 + 5\mathbf{A} + 2\mathbf{I}.$$ [5]

(c) Hence obtain a quadratic expression in $\mathbf{A}$ for $\mathbf{A}^4$. [4]

7. *(a)* Using logarithmic differentiation, show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(2^x\right) = 2^x \ln 2.$$ [2]

The function f is defined for $x > 0$ by

$$f(x) = 2^x - 2x.$$

(b) The graph of $y = f(x)$ has a single stationary point.

(i) Find its x-coordinate, giving your answer correct to three significant figures.

(ii) Determine whether it is a minimum or a maximum. [6]

(c) Evaluate

$$\int_1^2 f(x)\mathrm{d}x,$$

giving your answer correct to three significant figures. [4]

8. Use mathematical induction to prove that $5^n - (-1)^n$ is divisible by 6 for all positive integers n. [6]

TURN OVER

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z^2 - z - \mathrm{i}.$$

(a) Show that

$$u = x^2 - y^2 - x$$

and find an expression for v in terms of x and y. [4]

(b) The point P moves along the line $y = x - 1$. Determine the equation of the locus of Q. [4]

END OF PAPER

