GCE AS/A Level – LEGACY

0976/01

S19-0976-01

MATHEMATICS – C4
Pure Mathematics

FRIDAY, 14 JUNE 2019 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{2+17x+9x^2}{(x+1)^2(x+3)} .$$

(a) Express $f(x)$ in terms of partial fractions. [4]

(b) **Using your result in part *(a)***,

(i) find an expression for $f'(x)$,

(ii) verify that $f(x)$ has a stationary value when $x = 1$. [3]

2. The curve *C* has equation

$$x^6 + 6x^3y + 16y^2 = 28.$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-\left(3x^5+9x^2y\right)}{16y+3x^3}$. [3]

(b) Find the coordinates of each of the points on *C* where the **normal** is parallel to the x-axis. [4]

3. *(a)* Given that $\theta \neq 90°$, find all values of θ in the range $0° \leqslant \theta \leqslant 180°$ satisfying

$$5\tan 2\theta = 8\cot\theta.$$

Give your answers in degrees, correct to two decimal places. [4]

(b) (i) Express $\sqrt{33}\sin\phi + 4\cos\phi$ in the form $R\cos(\phi - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$\sqrt{33}\sin\phi + 4\cos\phi = 6 .$$

(iii) Find the least possible value for k so that

$$\sqrt{33}\sin\phi + 4\cos\phi = k$$

has solutions. Give a reason for your answer. [7]

4. *(a)* (i) Expand $\left(1+\frac{x}{8}\right)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2.

Express each of the coefficients in its simplest form.

(ii) State the range of values of x for which your expansion is valid. [3]

(b) By writing $x = -1$ in your expansion in part *(a)*, find an approximate value for $\sqrt[3]{7}$ in the form $\frac{a}{b}$, where a, b are integers whose values are to be found. [2]

5. The region *R* is bounded by the curve $y = 4 + 3\cos x$, the x-axis and the lines $x = \frac{\pi}{3}$, $x = \frac{\pi}{2}$.

Find the volume of the solid generated when *R* is rotated through four right angles about the x-axis. Give your answer correct to one decimal place. [6]

6. The curve *C* has the parametric equations $x = \frac{9t^2}{2}$, $y = 6t$. The point *P* lies on *C* and has parameter p.

(a) Show that the **normal** to *C* at the point *P* has equation

$$4y + 6px = 27p^3 + 24p.$$ [5]

(b) The normal to *C* at the point *P* intersects *C* again at the point with parameter 2.

(i) Show that $9p^3 - 28p - 16 = 0$.

(ii) Hence show that *P* can be one of two points. Find the coordinates of each of these two points. [6]

7. *(a)* Find $\int (5x-1)\sin 3x \, \mathrm{d}x$. [4]

(b) Use the substitution $u = 3x + 1$ to evaluate

$$\int_0^1 \frac{x}{(3x+1)^4} \, \mathrm{d}x.$$

Express your answer in the form $\frac{1}{n}$, where n is an integer whose value is to be found. [5]

8. Part of the surface of a small lake is covered by algae. Initially, the area of the lake covered by algae is A m^2. After t years, the area of the lake covered by algae is X m^2. The rate of increase of X is directly proportional to X^3.

(a) Write down a differential equation satisfied by X. [1]

(b) After 8 years, the area of the lake covered by algae has increased to twice the initial area. What multiple of the initial area will be covered by algae after 10 years? [7]

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9. The position vectors of the points *A* and *B* are given by

$$\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 7\mathbf{k},$$
$$\mathbf{b} = 11\mathbf{i} - 3\mathbf{j} + 8\mathbf{k},$$

respectively.

(a) Write down the vector **AB**. [1]

(b) The point *C* lies on the line *AB* and is such that *AC* : *CB* = 2 : 3.
Find the position vector of *C*. [2]

(c) The vector equation of the line *L* is given by

$$\mathbf{r} = -4\mathbf{i} + 3\mathbf{j} + 20\mathbf{k} + \lambda(-5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}).$$

Show that,

(i) *B* lies on *L*,

(ii) **AB** is perpendicular to *L*. [5]

10. Prove by contradiction the following proposition.

When x is real and $x \neq 0$,

$$\left|4x + \frac{1}{x}\right| \geqslant 4.$$

The first two lines of the proof are given below.

Assume that there is a real value of x *such that*

$$\left|4x + \frac{1}{x}\right| < 4.$$

Then squaring both sides, we have: [3]

END OF PAPER