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1) a) $$\frac{9}{(x-1)(x+2)^2} \equiv \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$$\frac{9}{(x-1)(x+2)^2} \equiv \frac{A(x+2)^2}{(x-1)(x+2)^2} + \frac{B(x-1)(x+2)}{(x-1)(x+2)^2} + \frac{C(x-1)}{(x-1)(x+2)^2}$$

$$9 \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

Yn amnewid $x=1$:

$$9 \equiv A(1+2)^2 + B(0)(3) + C(0)$$
$$9 \equiv 9A$$
$$A \equiv 1$$

Yn amnewid $x=-2$:

$$9 \equiv A(0)^2 + B(-3)(0) + C(-3)$$
$$9 \equiv -3C$$
$$C \equiv -3$$

Yn cymharu cyfernodau x^2:

$$0 \equiv A + B$$
$$0 \equiv 1 + B$$
$$B \equiv -1$$

Felly $$\frac{9}{(x-1)(x+2)^2} = \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$$

b) $$\int \frac{9}{(x-1)(x+2)^2} = \int \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}\,dx$$

$$= \ln|x-1| - \ln|x+2| - 3\int (x+2)^{-2}\,dx$$

$$= \ln|x-1| - \ln|x+2| - 3\frac{(x+2)^{-1}}{-1} + K$$

$$= \ln\left|\frac{x-1}{x+2}\right| + \frac{3}{x+2} + K$$

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2) $\dfrac{4-x}{\sqrt{1+2x}} = (4-x)(1+2x)^{-\frac{1}{2}}$

Nawr $(1+2x)^{-\frac{1}{2}} = 1 + (-\frac{1}{2})(2x) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2\times1}(2x)^2$

$+ \dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times2\times1}(2x)^3 + \cdots$

$= 1 - x + \frac{3}{8}(4x^2) - \frac{5}{16}(8x^3) + \cdots$

$= 1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \cdots$

Yr ateb yw $(4-x)\left(1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \cdots\right)$

$= 4 - 4x + \frac{12}{2}x^2 - \frac{20}{2}x^3 + \cdots$

$-x + x^2 - \frac{3}{2}x^3 + \frac{5}{2}x^4 + \cdots$

$= 4 - 5x + 7x^2 - \frac{23}{2}x^3 + \cdots$

Mae'r ehangiad yn ddilys ar gyfer $|2x| < 1$

$|2| \times |x| < 1$

$2|x| < 1$

$|x| < \frac{1}{2}$

Felly $-\frac{1}{2} < x < \frac{1}{2}$.

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3) $x_{n+1} = 4x_n - 3$ $\quad x_3 = 113$

a)
$$x_3 = 4x_2 - 3$$
$$113 = 4x_2 - 3$$
$$113 + 3 = 4x_2$$
$$4x_2 = 116$$
$$x_2 = \frac{116}{4}$$
$$\underline{x_2 = 29.}$$

$$x_2 = 4x_1 - 3$$
$$29 = 4x_1 - 3$$
$$29 + 3 = 4x_1$$
$$4x_1 = 32$$
$$x_1 = \frac{32}{4}$$
$$\underline{x_1 = 8}$$

b) Y gyfres yw 8, 29, 113, ...

Cyfres rifyddol? 8, 29, 113, ...

+21 +84

Dim gwahaniaeth cyffredin felly ddim yn gyfres rifyddol.

Cyfres geometrig? 8, 29, 113

× 3.625 × 3.89655....

Dim cymhareb cyffredin felly ddim yn gyfres geometrig.

Casgliad: Nid yw'r gyfres yn gyfres rifyddol na chwaith yn gyfres geometrig.

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4) a) $5\sin x - 12\cos x$

$R\sin(x-\alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$

Felly $R\cos\alpha = 5$, $R\sin\alpha = 12$

[R] $\dfrac{5}{\cos\alpha} = \dfrac{12}{\sin\alpha}$

$\tan\alpha = \dfrac{12}{5}$

$\alpha = \tan^{-1}\left(\dfrac{12}{5}\right)$

$\alpha = 67.38°$

i 2 le degol

[$\cos^2\alpha + \sin^2\alpha = 1$]

$\left(\dfrac{5}{R}\right)^2 + \left(\dfrac{12}{R}\right)^2 = 1$

$\dfrac{25}{R^2} + \dfrac{144}{R^2} = 1$

$25 + 144 = R^2$

$R^2 = 169$

$R = 13$

Felly $5\sin x - 12\cos x = 13\sin(x - 67.38°)$

i 2 le degol

b) Gwerth minimwm $\dfrac{4}{5\sin x - 12\cos x + 15}$

= Gwerth minimwm $\dfrac{4}{13\sin(x-67.38°)+15}$

Mae $\sin(x-67.38°)$ yn amrywio rhwng -1 ac 1

Mae $13\sin(x-67.38°)$ yn amrywio rhwng -13 ac 13

Mae $13\sin(x-67.38°)+15$ yn amrywio rhwng 2 a 28

Mae $\dfrac{4}{13\sin(x-67.38°)+15}$ yn amrywio rhwng $\dfrac{4}{2}$ a $\dfrac{4}{28}$

Felly'r gwerth minimwm yw $\dfrac{4}{28} = \dfrac{1}{7}$

c) $5\sin x - 12\cos x + 3 = 0$

$13\sin(x - 67.38°) + 3 = 0$

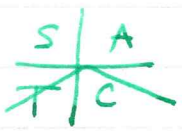
$13\sin(x - 67.38°) = -3$

$\sin(x - 67.38°) = -\frac{3}{13}$

$x - 67.38° = \sin^{-1}\left(\frac{-3}{13}\right)$

$x - 67.38° = \ldots, -13.34°, 193.34°, 346.66°, \ldots$

$x = \ldots, 54.0°, 260.7°, 414.0°, \ldots$



Atebion rhwng 0° a 360°: $x = 54.0°, 260.7°$
(i un lle degol).

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5) a) $|1-3x| > 7$

Naill ai $1-3x > 7$

$-3x > 6$

$x < -2$

neu $1-3x < -7$

$-3x < -8$

$x > \frac{-8}{-3}$

$x > \frac{8}{3}$

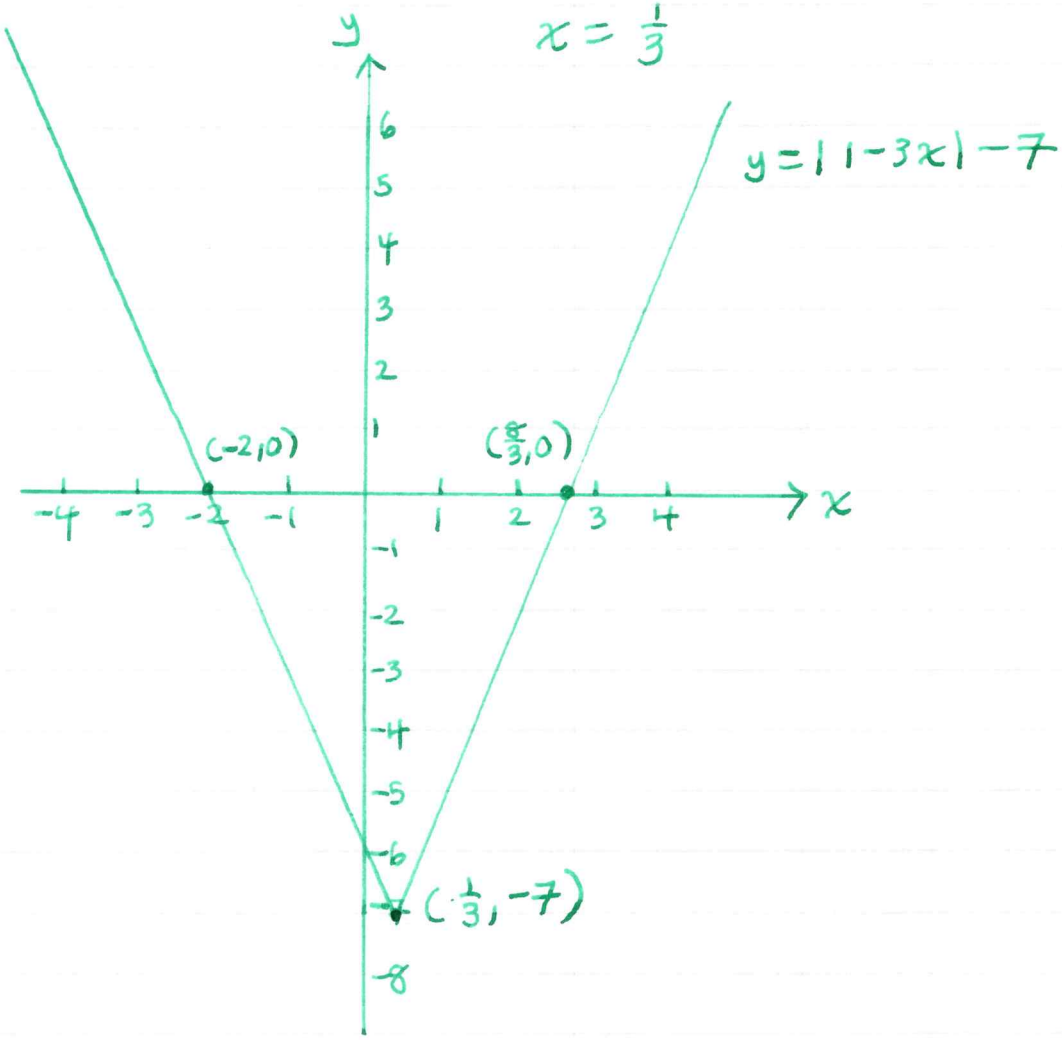
Ateb: naill ai $x < -2$ neu $x > \frac{8}{3}$

b) $y = |1-3x| - 7$

Y pwynt minimwm yw ble mae $1-3x = 0$

$1 = 3x$

$x = \frac{1}{3}$



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6) C: $x = \sin\theta$, $y = \cos 2\theta$

a) $\frac{dx}{d\theta} = \cos\theta$ $\quad$ $\frac{dy}{d\theta} = -2\sin 2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\frac{dy}{dx} = -2\sin 2\theta \times \frac{1}{\cos\theta}$

$\frac{dy}{dx} = \frac{-2\sin 2\theta}{\cos\theta}$

Os yw $\theta = \frac{\pi}{4}$,

$x = \sin\left(\frac{\pi}{4}\right)$, $y = \cos\left(2 \times \frac{\pi}{4}\right)$, $\frac{dy}{dx} = \frac{-2\sin\left(2 \times \frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}$

$x = \frac{1}{\sqrt{2}}$ $\quad$ $y = \cos\left(\frac{\pi}{2}\right)$ $\quad$ $= \frac{-2\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{4}\right)}$

$y = 0$ $\quad$ $= \frac{-2 \times 1}{\frac{1}{\sqrt{2}}}$

$= -2\sqrt{2}$

Hafaliad y tangiad: $y - y_1 = m(x - x_1)$

$y - 0 = -2\sqrt{2}\left(x - \frac{1}{\sqrt{2}}\right)$

$y = -2\sqrt{2}x + \frac{2\sqrt{2}}{\sqrt{2}}$

$y = -2\sqrt{2}x + 2$

(Felly $m = -2\sqrt{2}$, $c = 2$.)

b) $x + y = 1$

$\sin\theta + \cos 2\theta = 1$

$\sin\theta + 1 - 2\sin^2\theta = 1$

$0 = 2\sin^2\theta - \sin\theta$

$0 = \sin\theta(2\sin\theta - 1)$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

Naill ai $\sin\theta = 0$ neu $2\sin\theta - 1 = 0$

$\theta = \sin^{-1}(0)$ $\quad$ $\sin\theta = \frac{1}{2}$

S	A
T	C

$\theta = 0, \pi, \ldots$ $\quad$ $\theta = \sin^{-1}(\frac{1}{2})$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$

S	A
T	C

→ Ar gyfer y datrysiadau yma
mae $x = \sin\theta$
$x = 0$
hefyd $y = \cos 2\theta$
$y = \cos(0)$ neu $\cos(2\pi)$ neu ..
$y = 1$.
Felly un pwynt croestoriad yw $(0, 1)$

→ Ar gyfer y datrysiadau yma
mae $x = \sin\theta$
$x = \frac{1}{2}$
hefyd $y = \cos 2\theta$
$y = \cos\left(\frac{2\pi}{6}\right)$ neu $\cos\left(\frac{10\pi}{6}\right)$ neu ...
$y = \cos\left(\frac{\pi}{3}\right)$ neu $\cos\left(\frac{5\pi}{3}\right)$ neu ...
$y = \frac{1}{2}$
Felly ail bwynt croestoriad yw $\left(\frac{1}{2}, \frac{1}{2}\right)$

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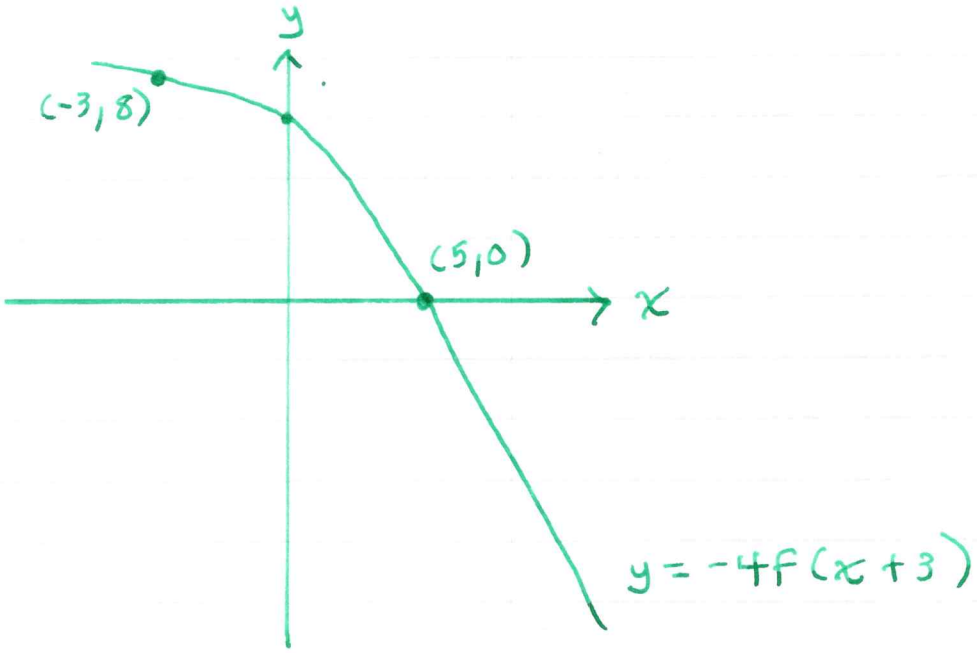
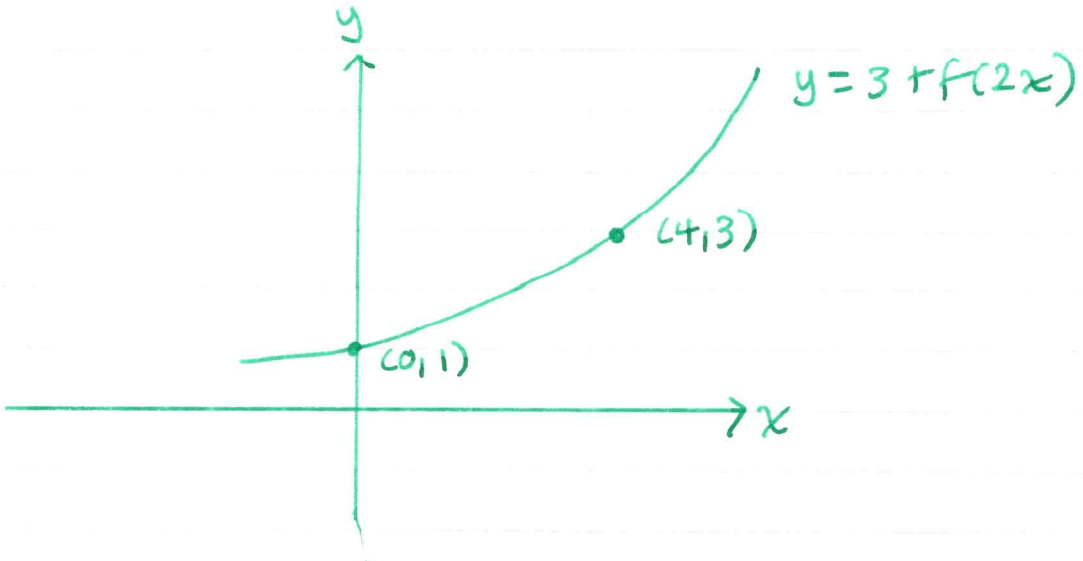
7) a) Symud 3 uned i'r chwith yna lluosi efo -4.



b) Cywasgu at yr echelin y $\rightarrow\leftarrow$ yna 3 uned i fyny.



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8) a) Cyfres rifyddol: $a = 3$

3ydd term: $a + 2d = 3 + 2d$

Term 19: $a + 18d = 3 + 18d$

Term 67: $a + 66d = 3 + 66d$

Maer 3 term uchod yn ffurfio cyfres geometrig

a, ar, ar^2, ...

$3+2d$, $3+18d$, $3+66d$, ...

$\times r$ $\times r$

Mae $r = \frac{3+18d}{3+2d}$ ac hefyd $r = \frac{3+66d}{3+18d}$

Felly $\frac{3+18d}{3+2d} = \frac{3+66d}{3+18d}$

$(3+18d)(3+18d) = (3+66d)(3+2d)$

$9 + 54d + 54d + 324d^2 = 9 + 6d + 198d + 132d^2$

$192d^2 - 96d = 0$

$96d(2d-1) = 0$

Naill ai $96d = 0$ neu $2d - 1 = 0$

$d = 0$ $\quad$ $d = \frac{1}{2}$

Nid oes cyfres rifyddol os yw $d = 0$

b) i)

Diwrnod	Llun	Mawrth	Mercher	Iau	Gwener
Term	1	2	3	4	5
Gweithwyr	100	112	124	136	148
		+12	+12	+12	+12

Mae hwn yn gyfres rifyddol efo $a = 100$, $d = 12$

Ar ddiwedd yr wythfed wythnos, mae term $8 \times 5 = 40$ yn ymddangos.

$$\begin{aligned} \text{Term } 40 &= a + 39d \\ &= 100 + 39 \times 12 \\ &= \underline{568} \end{aligned}$$

ii) Mae pawb yn derbyn £55 y diwrnod.

Mae angen ffeindio swm y 40 term cyntaf ac yna lluosi efo 55.

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{40} = \frac{40}{2}\left(2 \times 100 + (40-1) \times 12\right)$$

$$S_{40} = 20(200 + 39 \times 12)$$

$$S_{40} = 13,360.$$

Cyfanswm y cyflogau yw 13360×55

$$= \underline{£734,800}$$

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9) a) Mae $\tan\alpha = 2\cot\beta$

$$\tan\alpha = \frac{2}{\tan\beta} \quad \text{——} (1)$$

Nawr $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ yr llyfryn fformiwlâu

$= \dfrac{\tan\alpha + \tan\beta}{1 - \left(\frac{2}{\tan\beta}\right)\tan\beta}$ yn ammewid o (1)

$= \dfrac{\tan\alpha + \tan\beta}{1 - 2}$

$= \dfrac{\tan\alpha + \tan\beta}{-1}$

$= -1(\tan\alpha + \tan\beta)$

$= -(\tan\alpha + \tan\beta).$

CED.

b) $4\tan\theta = 3\sec^2\theta - 7$ $\qquad \sec^2\theta = 1 + \tan^2\theta$

$4\tan\theta = 3(1 + \tan^2\theta) - 7$

$4\tan\theta = 3 + 3\tan^2\theta - 7$

$0 = 3\tan^2\theta - 4\tan\theta - 4$

$0 = (3\tan\theta + 2)(\tan\theta - 2)$

Naill ai $3\tan\theta + 2 = 0$ neu $\tan\theta - 2 = 0$

$\tan\theta = -\frac{2}{3}$ $\qquad \tan\theta = 2$

$\theta = \tan^{-1}\left(-\frac{2}{3}\right)$ $\qquad \theta = \tan^{-1}(2)$

$\theta = -33.69^\circ, 146.31^\circ,$ $\qquad \theta = 63.43^\circ, 243.43^\circ$

326.31°

S | A
T | C

S | A
T | C

Atebion rhwng 0° a 360°, i 2 le degol:

$\theta = 63.43^\circ, 146.31^\circ, 243.43^\circ, 326.31^\circ$

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10) a) i) $y = x^5 \ln x$

$$\frac{dy}{dx} = x^5\left(\frac{1}{x}\right) + 5x^4 \ln x$$

$$\frac{dy}{dx} = x^4 + 5x^4 \ln x$$

$$\frac{dy}{dx} = x^4(1 + 5\ln x)$$

ii) $y = \dfrac{e^{3x}}{x^3 - 1}$

$$\frac{dy}{dx} = \frac{(x^3-1)(3e^{3x}) - e^{3x}(3x^2)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{3e^{3x}(x^3 - 1 - x^2)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{3e^{3x}(x^3 - x^2 - 1)}{(x^3-1)^2}$$

iii) $y = (\tan x + 7x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(\tan x + 7x)^{-\frac{1}{2}}(\sec^2 x + 7)$$

$$\frac{dy}{dx} = \frac{\sec^2 x + 7}{2\sqrt{\tan x + 7x}}$$

b) $3y + 4xy^2 - 5x^3 = 8$

$$3\left(\frac{dy}{dx}\right) + 4x\left(2y \times \frac{dy}{dx}\right) + 4(y^2) - 15x^2 = 0$$

$$\frac{dy}{dx}(3 + 8xy) = 15x^2 - 4y^2$$

$$\frac{dy}{dx} = \frac{15x^2 - 4y^2}{3 + 8xy}$$

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11) $f(x) = \dfrac{\sqrt{x^2-1}}{x}$ ar gyfer $x \geq 1$.

a) Gadewch i $y = \dfrac{\sqrt{x^2-1}}{x}$

$$yx = \sqrt{x^2-1}$$
$$(yx)^2 = x^2-1$$
$$y^2x^2 = x^2-1$$
$$y^2x^2 - x^2 = -1$$
$$x^2(y^2-1) = -1$$
$$x^2 = \frac{-1}{y^2-1}$$
$$x = \pm\sqrt{\frac{-1}{y^2-1}}$$

Rhaid dewis y gwreiddyn positif gan fod $x \geq 1$.

Felly $x = \sqrt{\dfrac{-1}{y^2-1}}$

Felly $f^{-1}(x) = \sqrt{\dfrac{-1}{x^2-1}}$ $\left(\text{neu } \sqrt{\dfrac{1}{1-x^2}}\right)$

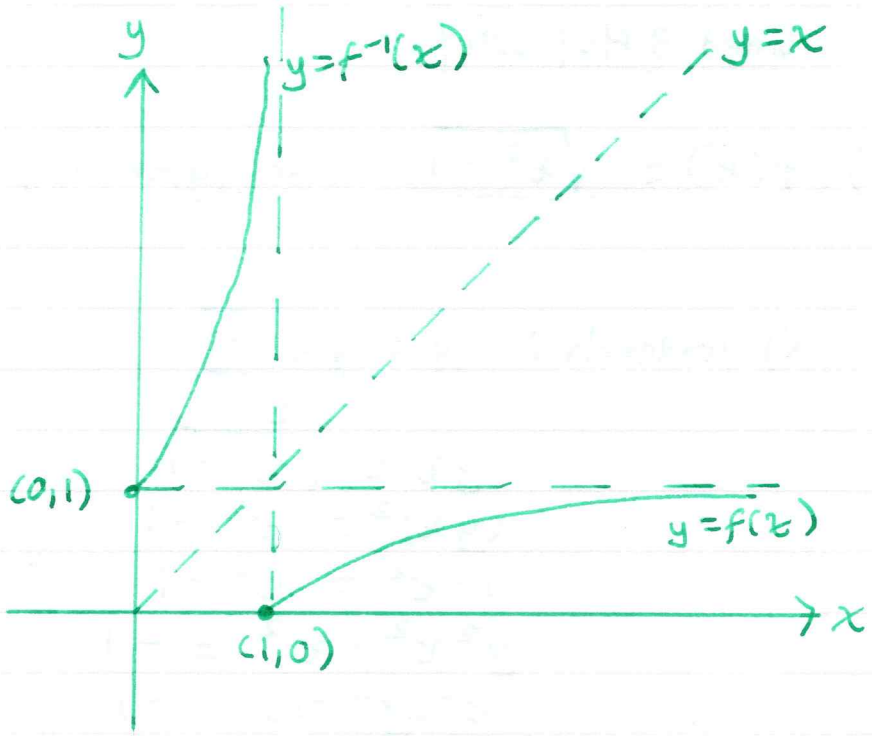
Parth f^{-1} yw amrediad f.

Mae $f(1) = \dfrac{\sqrt{1^2-1}}{1}$

$$= \frac{0}{1}$$
$$= 0$$

Fel mae x yn cynyddu mae'r effaith o dynnu 1 oddi wrth x^2 cyn cymryd yr ail isradd yn lleihau.
Fel mae x yn tueddu at ∞ mae rhifiadur y ffracsiwn mwy neu'n lai'n hafal i'r enwadur. Felly amrediad $f(x)$ yw $[0, 1)$. Felly parth $f^{-1}(x)$ yw $\underline{[0, 1)}$.

Brasluniad:



b) Ni ellir ffurfio $ff(x)$ oherwydd nid oes rhifau yn amrediad $f(x)$ sydd yn ymddangos ym mharth $f(x)$.

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12) a) Arwynebedd y sector $OAB = \frac{1}{2}r^2\theta$

Arwynebedd y triongl $OAB = \frac{1}{2}r^2\sin\theta$

Arwynebedd y segment $OAB = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$.

Hwn yw'r segment leiaf yn y diagram.

Arwynebedd y segment mwyaf
= Arwynebedd y cylch − Arwynebedd y segment leiaf
$= \pi r^2 - (\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta)$
$= \pi r^2 - \frac{1}{2}r^2\theta + \frac{1}{2}r^2\sin\theta$.

Gan fod arwynebedd y ddau segment yn y gymhareb 1:2 mae
arwynebedd y segment leiaf × 2 = ar. y segment mwyaf

$2(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta) = \pi r^2 - \frac{1}{2}r^2\theta + \frac{1}{2}r^2\sin\theta$

$r^2\theta - r^2\sin\theta = \pi r^2 - \frac{1}{2}r^2\theta + \frac{1}{2}r^2\sin\theta$

$\frac{3}{2}r^2\theta - \frac{3}{2}r^2\sin\theta = \pi r^2$

$\frac{3}{2}\theta - \frac{3}{2}\sin\theta = \pi$

$3\theta - 3\sin\theta = 2\pi$

$3\theta - 2\pi = 3\sin\theta$

$\sin\theta = \frac{3\theta - 2\pi}{3}$

$\sin\theta = \theta - \frac{2\pi}{3}$.

QED

b) i) $\sin\theta = \theta - \frac{2\pi}{3}$

$\sin\theta - \theta + \frac{2\pi}{3} = 0$

Gadewch i $f(\theta) = \sin\theta - \theta + \frac{2\pi}{3}$.

Mae $f(2.6) = \sin(2.6) - 2.6 + \frac{2\pi}{3}$
$= 0.0098964742 1\ldots$

Mae $f(2.7) = \sin(2.7) - 2.7 + \frac{2\pi}{3}$
$= -0.1782250174\ldots$

Mae'r newid arwydd yn dangos bod θ yn gorwedd rhwng 2.6 a 2.7.

ii) Dull Newton-Raphson:

$$\theta_{n+1} = \theta_n - \frac{f(\theta_n)}{f'(\theta_n)}$$ (cofiwch y fformiwla).

Nawr $f'(\theta) = \cos\theta - 1$

Felly $\theta_{n+1} = \theta_n - \dfrac{\sin\theta_n - \theta_n + \frac{2\pi}{3}}{\cos\theta_n - 1}$

Cychwyn efo $\theta_0 = 2.6$

$\theta_1 = 2.6053296$

$\theta_2 = 2.6053256 75$

$\theta_3 = 2.6053256 75$

$\vdots$

Felly, i 3 lle degol, $\underline{\underline{\theta = 2.605}}$

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13) a) $\frac{dA}{dt} \propto A$

$\frac{dA}{dt} = kA$

b) $\frac{dA}{dt} = kA$

$\frac{dA}{A} = k\,dt$

$\int \frac{1}{A}\,dA = \int k\,dt$

$\ln|A| = kt + c$

Os yw $t = 0$, mae $A = 0.2$

felly $\ln(0.2) = k(0) + c$

$\ln(0.2) = c.$

Felly $\ln|A| = kt + \ln(0.2)$

Os yw $t = 1$, mae $A = 1.48$

felly $\ln(1.48) = k(1) + \ln(0.2)$

$\ln(1.48) - \ln(0.2) = k$

$k = \ln\left(\frac{1.48}{0.2}\right)$

$k = \ln(7.4)$

Felly $\ln|A| = \ln(7.4)t + \ln(0.2)$

$A = e^{\ln(7.4)t + \ln(0.2)}$

$A = e^{t\ln(7.4)} \times e^{\ln(0.2)}$

$A = e^{\ln(7.4^t)} \times e^{\ln(0.2)}$

$A = 7.4^t \times 0.2$

$A = 0.2 \times 7.4^t$

Felly $p = 0.2$, $q = 7.4$

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14) a) $\int (e^{2x} + 6\sin 3x)\,dx$

$= \frac{1}{2}e^{2x} - 2\cos 3x + K$

b) $\int 7(x^2+\sin x)^6(2x+\cos x)\,dx$

$= (x^2+\sin x)^7 + K$

c) $\int \frac{1}{x^2}\ln x\,dx$

Gadewch i: $u = \ln x$ $\quad \frac{dv}{dx} = x^{-2}$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = -x^{-1}$

$$\int \frac{1}{x^2}\ln x\,dx = (\ln x)(-x^{-1}) - \int \frac{1}{x}(-x^{-1})\,dx$$

$$= -\frac{\ln x}{x} - \int \frac{1}{x}\left(-\frac{1}{x}\right)dx$$

$$= -\frac{\ln x}{x} + \int x^{-2}\,dx$$

$$= -\frac{\ln x}{x} - x^{-1} + K$$

$$= -\frac{1}{x}(\ln x + 1) + K$$

ch) $\int_0^{\frac{\pi}{3}} \frac{\sin x}{(2\cos x+1)^2}\,dx$

$u = 2\cos x + 1$

$\frac{du}{dx} = -2\sin x$

$\frac{du}{-2\sin x} = dx$

Terfannau:

[0] $u = 2\cos(0) + 1$

$u = 3$

$[\frac{\pi}{3}]$ $u = 2\cos(\frac{\pi}{3}) + 1$

$u = 2$

Felly $\int_0^{\frac{\pi}{3}} \frac{\sin x}{(2\cos x + 1)^2}\, dx$

$= \int_3^2 \frac{\cancel{\sin x}}{u^2} \left(\frac{du}{-2\cancel{\sin x}} \right)$

$= \int_3^2 \frac{1}{-2u^2}\, du$

$= -\frac{1}{2} \int_3^2 u^{-2}\, du$

$= -\frac{1}{2} \left[-u^{-1} \right]_3^2$

$= \frac{1}{2} \left[u^{-1} \right]_3^2$

$= \frac{1}{2} \left(2^{-1} - 3^{-1} \right)$

$= \frac{1}{2} \left(\frac{1}{2} - \frac{1}{3} \right)$

$= \frac{1}{12}$

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15) Tybiwch fod $\sqrt{6}$ yn gymarebol. Yna mae'n bosibl ysgrifennu $\sqrt{6}$ yn y ffurf $\frac{a}{b}$, lle mae a, b yn gyfanrifau sydd heb ffactorau cyffredin.

$\therefore \quad \frac{a}{b} = \sqrt{6}$

$\therefore \quad a = \sqrt{6}\, b$

$\therefore \quad a^2 = 6b^2$

$\therefore$ mae 6 yn ffactor o a^2

$\therefore$ mae 6 yn ffactor o a ac felly mae $a = 6K$, lle mae K yn gyfanrif.

Yn amnewid $a = 6K$ i mewn i $a^2 = 6b^2$, cawn

$(6K)^2 = 6b^2$

$36K^2 = 6b^2$

$6K^2 = b^2$

$\therefore$ mae 6 yn ffactor o b^2

$\therefore$ mae 6 yn ffactor o b

Ond os yw 6 yn ffactor o a a b, yna mae hyn yn gwrth-ddweud y dybiaeth gynt bod a, b yn gyfanrifau sydd heb ffactor cyffredin. (Rydym wedi darganfod ffactor cyffredin, sef 6.)

Felly nid yw $\sqrt{6}$ yn gymarebol.