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1) $A = \begin{pmatrix} 3 & 7 \\ -2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}$

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Nawr $A^{-1} = \dfrac{1}{3\times 0 - (-2\times 7)} \begin{pmatrix} 0 & -7 \\ 2 & 3 \end{pmatrix}$

$$A^{-1} = \frac{1}{14}\begin{pmatrix} 0 & -7 \\ 2 & 3 \end{pmatrix}$$

Felly $X = \dfrac{1}{14}\begin{pmatrix} 0 & -7 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}$

$$X = \frac{1}{14}\begin{pmatrix} 0 & -28 \\ 10 & 14 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & -2 \\ \frac{10}{14} & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & -2 \\ \frac{5}{7} & 1 \end{pmatrix}$$

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2) $\underline{a} = 2\underline{i} + 3\underline{j} - \underline{k}$, $\underline{b} = 4\underline{j} + 5\underline{k}$,
$\underline{c} = 7\underline{i} - 3\underline{k}$, $\underline{d} = -3\underline{i} - \underline{j} - 5\underline{k}$

a) $\underline{AB} = -\underline{a} + \underline{b}$
$= -2\underline{i} - 3\underline{j} + \underline{k} + 4\underline{j} + 5\underline{k}$
$= -2\underline{i} + \underline{j} + 6\underline{k}$

Hafaliad fector AB: $\underline{r} = \underline{a} + \lambda(\underline{AB})$
$\underline{r} = 2\underline{i} + 3\underline{j} - \underline{k} + \lambda(-2\underline{i} + \underline{j} + 6\underline{k})$

$\underline{CD} = -\underline{c} + \underline{d}$
$= -7\underline{i} + 3\underline{k} - 3\underline{i} - \underline{j} - 5\underline{k}$
$= -10\underline{i} - \underline{j} - 2\underline{k}$

Hafaliad fector CD: $\underline{r} = \underline{c} + \mu(\underline{CD})$
$\underline{r} = 7\underline{i} - 3\underline{k} + \mu(-10\underline{i} - \underline{j} - 2\underline{k})$

b) Er mwyn i AB ag CD fod yn berpendicwlar, rhaid i
1) y llinellau groesdorri mewn un pwynt;
2) yr ongl rhwng y llinellau fod yn $90°$.

Edrychwn ar amod 2) yn gyntaf.
Os yw'r ongl rhwng y llinellau'n $90°$, yna mae'r lluoswm sgalar o gyfeiriadau'r llinellau'n sero.

$(-2\underline{i} + \underline{j} + 6\underline{k}) \cdot (-10\underline{i} - \underline{j} - 2\underline{k})$
$= (-2 \times -10) + (1 \times -1) + (6 \times -2)$
$= 20 - 1 - 18$
$= 1.$ Nid yw hwn yn sero felly nid yw'r llinellau'n berpendicwlar. (Nid oes raid felly ystyried amod 1).)

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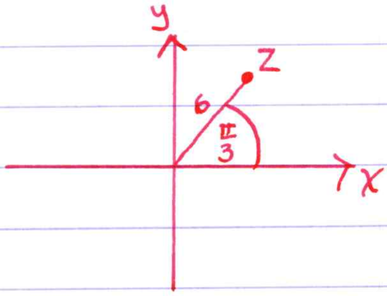
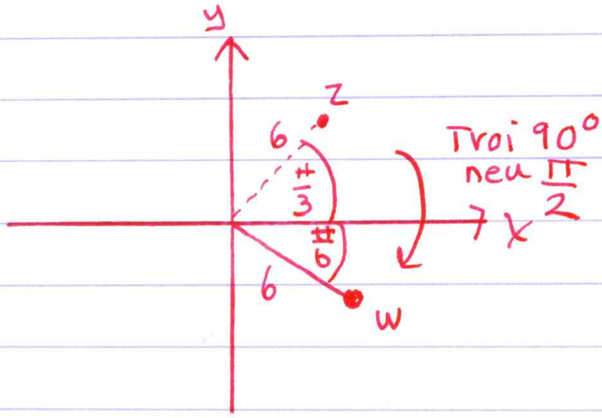
3) $\cancel{z = x + iy}$
$\cancel{w = u + iv}$



a) $|z| = 6$, $\arg z = \frac{\pi}{3}$

$z = 6\cos\left(\frac{\pi}{3}\right) + 6\sin\left(\frac{\pi}{3}\right)i$
$z = 6\left(\frac{1}{2}\right) + 6\left(\frac{\sqrt{3}}{2}\right)i$
$z = 3 + 3\sqrt{3}\,i$



$w = 6\cos\left(\frac{\pi}{6}\right) - 6\sin\left(\frac{\pi}{6}\right)i$
$w = 6\left(\frac{\sqrt{3}}{2}\right) - 6\left(\frac{1}{2}\right)i$
$w = 3\sqrt{3} - 3i$

$\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

b) $\frac{z}{w} = \frac{3 + 3\sqrt{3}\,i}{3\sqrt{3} - 3i}$

$= \left(\frac{3 + 3\sqrt{3}\,i}{3\sqrt{3} - 3i}\right) \times \left(\frac{3\sqrt{3} + 3i}{3\sqrt{3} + 3i}\right)$

$= \frac{9\sqrt{3} + 9i + 9 \times 3i + 9\sqrt{3}\,i^2}{9 \times 3 + \cancel{9\sqrt{3}\,i} - \cancel{9\sqrt{3}\,i} - 9i^2}$

$= \frac{9\sqrt{3} + 9i + 27i - 9\sqrt{3}}{27 + 9}$

$= \frac{36i}{36i}$

$\frac{z}{w} = i$

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4) Achos $n=1$: Mae $9^n+15 = 9^1+15$

$= 9+15$

$= 24.$

Mae 24 yn lluosrif 8 $(24 = 3\times 8)$ felly mae'r gosodiad yn wir ar gyfer $n=1$.

Cymerwch bod y gosodiad yn wir ar gyfer $n=k$, felly mae 9^k+15 yn lluosrif 8.

Edrychwn ar yr achos $n=k+1$.

$$9^{k+1}+15 = 9\times 9^k + 15$$
$$= 8\times 9^k + 9^k + 15$$

Yn ôl yr hypothesis anwythol, mae 9^k+15 yn lluosrif 8, fel bod $9^k+15 = 8p$ ar gyfer rhyw rif cyfan p.

Felly $8\times 9^k + 9^k + 15 = 8\times 9^k + 8p$
$$= 8(9^k+p)$$

Mae $9^{k+1}+15$ felly yn lluosrif o 8.
Mae hyn yn profi'r achos $n=k+1$, felly trwy anwythiad mathemategol mae 9^n+15 yn lluosrif 8 ar gyfer pob cyfanrif positif n.

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5) Mae $x = -\frac{1}{2}$ a $x = -3$ yn wreiddiau i'r hafaliad

$$2x^4 - x^3 - 15x^2 + 23x + 15 = 0$$

Felly mae $(x+\frac{1}{2})$ a $(x+3)$ yn ffactorau o ochr chwith yr hafaliad.

Felly mae $(x+\frac{1}{2})(x+3)$ yn ffactor hefyd

$$= x^2 + 3x + \frac{1}{2}x + \frac{3}{2}$$

$$= x^2 + \frac{7}{2}x + \frac{3}{2}.$$

Rhannu allan:

$$\begin{array}{r|l} & \quad\quad\quad\quad\quad\quad 2x^2 - 8x + 10 \\ \hline x^2+\frac{7}{2}x+\frac{3}{2} & 2x^4 - x^3 - 15x^2 + 23x + 15 \\ & \underline{2x^4 + 7x^3 + 3x^2} \\ & \quad\; -8x^3 - 18x^2 + 23x + 15 \\ & \quad\; \underline{-8x^3 - 28x^2 - 12x} \\ & \quad\quad\quad\quad 10x^2 + 35x + 15 \\ & \quad\quad\quad\quad \underline{10x^2 + 35x + 15} \end{array}$$

Felly $2x^4 - x^3 - 15x^2 + 23x + 15$

$$= (x+\frac{1}{2})(x+3)(2x^2 - 8x + 10)$$

$$= 2(x+\frac{1}{2})(x+3)(x^2 - 4x + 5)$$

$$= (2x+1)(x+3)(x^2 - 4x + 5)$$

Datrys $x^2 - 4x + 5 = 0$:

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2}$$

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

$$x = \frac{4 \pm 2\sqrt{-1}}{2}$$

$$x = 2 \pm \sqrt{-1}$$

$$x = 2 \pm i$$

Y ddau wreiddyn arall yw $\underline{2+i}$ a $\underline{2-i}$.

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6) $z = x + iy$

$$|z - 1| = |z - 2i|$$
$$|(x+iy) - 1| = |(x+iy) - 2i|$$
$$|(x-1) + iy| = |x + (y-2)i|$$
$$\sqrt{(x-1)^2 + y^2} = \sqrt{x^2 + (y-2)^2}$$
$$(x-1)^2 + y^2 = x^2 + (y-2)^2$$
$$(x-1)(x-1) + y^2 = x^2 + (y-2)(y-2)$$
$$\cancel{x^2} - 2x + 1 + \cancel{y^2} = \cancel{x^2} + \cancel{y^2} - 4y + 4$$
$$-2x + 1 = -4y + 4$$
$$4y = 2x + 4 - 1$$
$$4y = 2x + 3$$
$$y = \frac{2}{4}x + \frac{3}{4}$$
$$y = \frac{1}{2}x + \frac{3}{4}$$

Mae hwn yn llinell syth o'r ffurf $y = mx + c$.
Y graddiant yw $\frac{1}{2}$ a'r rhyngdoriad-y yw $\frac{3}{4}$.

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7) a) $\sum_{r=1}^{2m} (r+2)^2$

$= \sum_{r=1}^{2m} (r+2)(r+2)$

$= \sum_{r=1}^{2m} r^2+4r+4$

$= \left(\sum_{r=1}^{2m} r^2\right) + 4\left(\sum_{r=1}^{2m} r\right) + 4\left(\sum_{r=1}^{2m} 1\right)$

$= \frac{1}{6}(2m)(2m+1)(2(2m)+1) + \cancel{4}^{2}\left(\frac{2m(2m+1)}{\cancel{2}}\right) + 4(2m)$

$= \frac{1}{6}(2m)(2m+1)(4m+1) + 2(2m(2m+1)) + 8m$

$= \frac{m}{3}(2m+1)(4m+1) + 4m(2m+1) + 8m$

$= \frac{m}{3}(8m^2+2m+4m+1) + 8m^2+4m+8m$

$= \frac{m}{3}(8m^2+6m+1) + 8m^2 + 12m$

$= \frac{m}{3}(8m^2+6m+1) + \frac{24}{3}m^2 + \frac{36}{3}m$

$= \frac{m}{3}(8m^2+6m+1+24m+36)$

$= \frac{m}{3}(8m^2+30m+37)$

fel bod $a=8$, $b=30$, $c=37$.

b) $\sum_{r=11}^{20} (r+2)^2 = \left(\sum_{r=1}^{20} (r+2)^2\right) - \left(\sum_{r=1}^{10} (r+2)^2\right)$

$2m=20,\ m=10$ $\qquad$ $2m=10,\ m=5$

$= \frac{10}{3}(8\times10^2+30\times10+37) - \frac{5}{3}(8\times5^2+30\times5+37)$

$= \frac{10}{3}\times 1137 - \frac{5}{3}\times 387$

$= 3790 - 645$

$= \underline{\underline{3145}}$

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8) $A = (3, 5, 6)$, $B = (5, -1, 7)$, $C = (-1, 7, 0)$

DULL 1

Mae'r fectorau $\underline{AB}$ a $\underline{BC}$ yn gorwedd yn y plân.
Mae'r normal $\underline{n}$ yn berpendicwlar i $\underline{AB}$ ag i $\underline{BC}$.

$$\begin{aligned}\underline{AB} &= -\underline{a} + \underline{b} \\ &= -3\underline{i} - 5\underline{j} - 6\underline{k} + 5\underline{i} - \underline{j} + 7\underline{k} \\ &= 2\underline{i} - 6\underline{j} + \underline{k}\end{aligned}$$

$$\begin{aligned}\underline{BC} &= -\underline{b} + \underline{c} \\ &= -5\underline{i} + \underline{j} - 7\underline{k} - \underline{i} + 7\underline{j} \\ &= -6\underline{i} + 8\underline{j} - 7\underline{k}\end{aligned}$$

Gadewch i $\underline{n} = p\underline{i} + q\underline{j} + r\underline{k}$ fod y normal cyffredin.

Mae $\underline{AB}$ ag $\underline{n}$ yn berpendicwlar felly

$$\underline{AB} \cdot \underline{n} = 0$$
$$(2\underline{i} - 6\underline{j} + \underline{k}) \cdot (p\underline{i} + q\underline{j} + r\underline{k}) = 0$$
$$2p - 6q + r = 0 \quad \text{—— } 1$$

Mae $\underline{BC}$ ag $\underline{n}$ yn berpendicwlar felly

$$\underline{BC} \cdot \underline{n} = 0$$
$$(-6\underline{i} + 8\underline{j} - 7\underline{k}) \cdot (p\underline{i} + q\underline{j} + r\underline{k}) = 0$$
$$-6p + 8q - 7r = 0 \quad \text{—— } 2$$

Gadewch i ni ffeindio'r normal ble mae $r = 1$.

① $\Rightarrow$ $2p - 6q + 1 = 0$ ② $\Rightarrow$ $-6p + 8q - 7 = 0$

$2p - 6q = -1$ $-6p + 8q = 7$

[×3] $6p - 18q = -3$

$$\begin{aligned} 6p - 18q &= -3 \\ -6p + 8q &= 7 \\ \hline -10q &= 4 \\ \hline q &= -\tfrac{2}{5} \end{aligned}$$

ADIO

Yn ôl yn ①: $2p - 6q + 1 = 0$

$$2p - 6\left(-\tfrac{2}{5}\right) + 1 = 0$$

$$2p = -\tfrac{17}{5}$$

$$p = -\tfrac{17}{10}$$

Y normal cyffredin yw $\underline{n} = -\frac{17}{10}\underline{i} - \frac{2}{5}\underline{j} + \underline{k}$

Hafaliad fector y plân Π yw $\underline{r} \cdot \underline{n} = d$

$$\underline{r} \cdot \left(-\tfrac{17}{10}\underline{i} - \tfrac{2}{5}\underline{j} + \underline{k}\right) = d$$

ble mae $d = \underline{a} \cdot \underline{n}$

$$= (3\underline{i} + 5\underline{j} + 6\underline{k}) \cdot \left(-\tfrac{17}{10}\underline{i} - \tfrac{2}{5}\underline{j} + \underline{k}\right)$$

$$= \left(3 \times -\tfrac{17}{10}\right) + \left(5 \times -\tfrac{2}{5}\right) + (6 \times 1)$$

$$= -\tfrac{11}{10}$$

Felly hafaliad fector y plân Π yw $\underline{r} \cdot \left(-\frac{17}{10}\underline{i} - \frac{2}{5}\underline{j} + \underline{k}\right) = -\frac{11}{10}$

Yn y ffurf Cartesaidd: $(x\underline{i} + y\underline{j} + z\underline{k}) \cdot \left(-\frac{17}{10}\underline{i} - \frac{2}{5}\underline{j} + \underline{k}\right) = -\frac{11}{10}$

$$-\tfrac{17}{10}x - \tfrac{2}{5}y + z = -\tfrac{11}{10}$$

$$-17x - 4y + 10z = -11$$

$$\underline{17x + 4y - 10z = 11}$$

DULL 2

Gadewch i ni ffeindio'r hafaliad $\underline{r} \cdot \underline{n} = d$ ble mae $d = 1$.

Gadewch i'r normal i'r plân fod o'r ffurf $\underline{n} = p\underline{i} + q\underline{j} + r\underline{k}$.

Ar gyfer y pwynt A, $\underline{a} \cdot \underline{n} = 1$

$$(3\underline{i} + 5\underline{j} + 6\underline{k}) \cdot (p\underline{i} + q\underline{j} + r\underline{k}) = 1$$

$$3p + 5q + 6r = 1 \quad \text{—— } (1)$$

Ar gyfer y pwynt B, $\underline{b} \cdot \underline{n} = 1$

$$(5\underline{i} - \underline{j} + 7\underline{k}) \cdot (p\underline{i} + q\underline{j} + r\underline{k}) = 1$$

$$5p - q + 7r = 1 \quad \text{—— } (2)$$

Ar gyfer y pwynt C, $\underline{c} \cdot \underline{n} = 1$

$$(-\underline{i} + 7\underline{j}) \cdot (p\underline{i} + q\underline{j} + r\underline{k}) = 1$$

$$-p + 7q = 1 \quad \text{—— } (3)$$

(3) $\Rightarrow$ $7q - 1 = p$ —— (4)

Yn amnewid am p yn (2):

$$5(7q - 1) - q + 7r = 1$$

$$35q - 5 - q + 7r = 1$$

$$34q + 7r = 6$$

[×6] $204q + 42r = 36$

Yn amnewid am p yn (1):

$$3(7q - 1) + 5q + 6r = 1$$

$$21q - 3 + 5q + 6r = 1$$

$$26q + 6r = 4$$

[×7] $182q + 42r = 28$

$$\begin{array}{rrcl} & 204q + 42r & = & 36 \\ - & 182q + 42r & = & 28 \\ \hline & 22q & = & 8 \end{array}$$

$$q = \frac{8}{22}$$

$$q = \frac{4}{11}$$

Yn amnewid am q yn (4):

$$7q - 1 = p$$
$$7\left(\frac{4}{11}\right) - 1 = p$$
$$p = \frac{17}{11}$$

Yn amnewid am p a q yn (1):

$$3p + 5q + 6r = 1$$
$$3\left(\frac{17}{11}\right) + 5\left(\frac{4}{11}\right) + 6r = 1$$
$$\frac{71}{11} + 6r = 1$$
$$6r = -\frac{60}{11}$$
$$r = -\frac{10}{11}$$

Felly y normal yw $\underline{n} = \frac{17}{11}\underline{i} + \frac{4}{11}\underline{j} - \frac{10}{11}\underline{k}$

Hafaliad y plân Π yw $\underline{r} \cdot \underline{n} = 1$

$$\underline{r} \cdot \left(\frac{17}{11}\underline{i} + \frac{4}{11}\underline{j} - \frac{10}{11}\underline{k}\right) = 1$$

Yn y ffurf Cartesaidd: $(x\underline{i} + y\underline{j} + z\underline{k}) \cdot \left(\frac{17}{11}\underline{i} + \frac{4}{11}\underline{j} - \frac{10}{11}\underline{k}\right) = 1$

$$\frac{17}{11}x + \frac{4}{11}y - \frac{10}{11}z = 1$$

$$\underline{17x + 4y - 10z = 11}$$

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9) $z = x + iy$ $\qquad w = z^2 - 1$

$w = u + iv$

a) $w = z^2 - 1$

$u + iv = (x + iy)^2 - 1$

$u + iv = (x + iy)(x + iy) - 1$

$u + iv = x^2 + 2xyi + i^2y^2 - 1$

$u + iv = x^2 + 2xyi - y^2 - 1$

$u + iv = x^2 - y^2 - 1 + 2xyi$

Yn cymharu darnau real a dychmygol

$u = x^2 - y^2 - 1, \quad v = 2xy$ ✓

b) $y = 3x$

Yn amnewid i mewn i'r ddau hafaliad o ran (a)

$u = x^2 - y^2 - 1$ $\qquad v = 2xy$

$u = x^2 - (3x)^2 - 1$ $\qquad v = 2x(3x)$

$u = x^2 - 9x^2 - 1$ $\qquad v = 6x^2$

$u = -8x^2 - 1$ $\qquad \frac{v}{6} = x^2$

(1) $\qquad$ (2)

Yn amnewid am x^2 o (2) i (1)

$u = -8\left(\frac{v}{6}\right) - 1$

$u = -\frac{4}{3}v - 1$

$3u = -4v - 3$

$4v = -3u - 3$

$v = \frac{-3}{4}u - \frac{3}{4}$ $\qquad$ neu $\qquad 3u + 4v + 3 = 0$

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10) $px^2 + qx + r = 0$ Gwreiddiau α, β

Mae $\alpha + \beta = \frac{-q}{p}$ a $\alpha\beta = \frac{r}{p}$

$\quad$ ① $\quad$ ②

a) Yr hafaliad ciwbig efo gwreiddiau $\alpha, \beta, \alpha+\beta$ yw

$$(x-\alpha)(x-\beta)(x-(\alpha+\beta)) = 0$$
$$(x^2 - \beta x - \alpha x + \alpha\beta)(x-(\alpha+\beta)) = 0$$
$$(x^2 - x(\beta+\alpha) + \alpha\beta)(x-(\alpha+\beta)) = 0$$
$$(x^2 - x(\alpha+\beta) + \alpha\beta)(x-(\alpha+\beta)) = 0$$

Yn amnewid o ① a ②:

$$(x^2 - x(\tfrac{-q}{p}) + \tfrac{r}{p})(x - (\tfrac{-q}{p})) = 0$$
$$(x^2 + \tfrac{q}{p}x + \tfrac{r}{p})(x + \tfrac{q}{p}) = 0$$
$$x^3 + \frac{q}{p}x^2 + \frac{r}{p}x + \frac{q}{p}x^2 + \frac{q^2}{p^2}x + \frac{qr}{p^2} = 0$$
$$p^2x^3 + pqx^2 + prx + pqx^2 + q^2x + qr = 0$$
$$p^2x^3 + 2pqx^2 + (pr+q^2)x + qr = 0$$

b) $px^2 - qx - r = 0$ Gwreiddiau $2\alpha, \gamma$

Mae $2\alpha + \gamma = \frac{-(-q)}{p}$ a $2\alpha\gamma = \frac{-r}{p}$

$\quad$ ③ $\quad$ ④

② + ④ $\Rightarrow$ $\alpha\beta + 2\alpha\gamma = \frac{r}{p} + \frac{-r}{p}$

$$\alpha\beta + 2\alpha\gamma = 0$$
$$\cancel{\alpha}\beta = -2\cancel{\alpha}\gamma$$
$$\underline{\beta = -2\gamma}\ \checkmark$$