Deillio'r Hafaliadau Mudiant

Deriving the Equations of Motion

@mathemateg

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Deillio'r Hafaliadau Mudiant / *Deriving the Equations of Motion*

Tybiaethau / *Assumptions*:

$$\text{Cyflymder Cyfartalog} = \frac{\text{Dadleoliad}}{\text{Amser}} \qquad \text{Average Velocity} = \frac{\text{Displacement}}{\text{Time}} \quad \text{①}$$

$$\text{Cyflymiad} = \frac{\text{Newid mewn cyflymder}}{\text{Amser}} \qquad \text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} \quad \text{②}$$

Diffinio newidynnau / *Defining variables:*

Newidyn / *Variable*	Enw / *Name*	Uned / *Unit*
S	Dadleoliad / *Displacement*	m
u	Cyflymder cychwynnol / *Initial velocity*	ms^{-1}
v	Cyflymder terfynol / *Final velocity*	ms^{-1} ③
a	Cyflymiad / *Acceleration*	ms^{-2}
t	Amser / *Time*	s

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$v = u + at$

Yn defnyddio newidynnau ③ i ysgrifennu hafaliad ②:
Using the variables from ③ *to write equation* ②*:*

$$a = \frac{v - u}{t}$$

Ail-drefnu: / *Rearranging:*

$$at = v - u$$
$$u + at = v$$
$$v = u + at \qquad \text{———④}$$

$S = \left(\frac{u + v}{2}\right) t$

Yn defnyddio newidynnau ③ i ysgrifennu hafaliad ①:
Using the variables from ③ *to write equation* ①*:*

$$\frac{u + v}{2} = \frac{S}{t}$$

Ail-drefnu: / *Rearranging:*

$$\left(\frac{u + v}{2}\right) t = S$$
$$S = \left(\frac{u + v}{2}\right) t \qquad \text{———⑤}$$

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$$S = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2aS$$

Yn amnewid am v o ④ i ⑤:
Substituting for v from ④ into ⑤:

$$S = \left(\frac{u + (u + at)}{2}\right)t$$

Symleiddio: / *Simplifying:*

$$S = \left(\frac{2u + at}{2}\right)t$$

$$S = \frac{2ut + at^2}{2}$$

$$S = ut + \frac{1}{2}at^2$$

Ail-drefnu ④: / *Rearranging ④:*

$$v = u + at$$

$$v - u = at$$

$$\left(\frac{v-u}{a}\right) = t \qquad \text{⑥}$$

Yn amnewid am t o ⑥ i ⑤:
Substituting for t from ⑥ into ⑤:

$$S = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$S = \frac{(u+v)(v-u)}{2a}$$

$$S = \frac{uv - u^2 + v^2 - vu}{2a}$$

$$S = \frac{v^2 - u^2}{2a}$$

$$2aS = v^2 - u^2$$

$$v^2 = u^2 + 2aS$$