C4: Hafaliadau Differol

Haf 2005

⑧ (a) $\frac{dP}{dt} \propto P$

$\frac{dP}{dt} = kP$

(b) $dP = kP\,dt$

$\frac{dP}{P} = k\,dt$

$\int \frac{1}{P}\,dP = \int k\,dt$

$\ln|P| = kt + c$

Os yw $t=0$ yna $P=P_0$ felly

$\ln|P_0| = k(0) + c$

$c = \ln|P_0|$

Felly $\ln|P| = kt + \ln|P_0|$

$\ln|P| - \ln|P_0| = kt$

$\ln\left|\frac{P}{P_0}\right| = kt$

$\frac{P}{P_0} = e^{kt}$

$P = P_0 e^{kt}$ ✓

(c) Os yw $t=2$, $P = 1.2P_0$.

Felly

$1.2P_0 = P_0 e^{k(2)}$

$1.2 = e^{2k}$

$\ln(1.2) = 2k$

$k = \frac{\ln(1.2)}{2}$

Felly $P = P_0 e^{\left(\frac{\ln(1.2)}{2}\right)t}$

Os yw $P = 2P_0$ yna

$2P_0 = P_0 e^{\left(\frac{\ln 1.2}{2}\right)t}$

$2 = e^{\left(\frac{\ln(1.2)}{2}\right)t}$

$\ln(2) = \frac{\ln(1.2)}{2}t$

$2\times\ln(2) = \ln(1.2)t$

$t = \frac{2\ln(2)}{\ln(1.2)}$

$t = 7.604$

diwrnod i 3 lle degol.

Haf 2006

(8) (a) $\frac{dx}{dt} \propto -\sqrt{x}$

$\frac{dx}{dt} = -k\sqrt{x}$

(b) $dx = -k\sqrt{x}\,dt$

$\frac{dx}{\sqrt{x}} = -k\,dt$

$\int \frac{1}{\sqrt{x}}\,dx = \int -k\,dt$

$\int x^{-\frac{1}{2}}\,dx = \int -k\,dt$

$\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = -kt + C$

$2\sqrt{x} = -kt + C$

Os yw $t=0$, mae $x=9$

Felly

$2\sqrt{9} = -k(0) + C$

$2 \times 3 = C$

$C = 6$

Felly $2\sqrt{x} = -kt + 6$

$kt = 6 - 2\sqrt{x}$ ✓

(c) Os yw $t=20$, mae $x=4$

Felly

$20k = 6 - 2\sqrt{4}$

$20k = 6 - 2 \times 2$

$20k = 2$

$k = \frac{2}{20}$

$k = \frac{1}{10}$

Felly $\frac{1}{10}t = 6 - 2\sqrt{x}$

Os gw'r tanc yn wag mae $x=0$:

$\frac{1}{10}t = 6 - 2\sqrt{0}$

$\frac{1}{10}t = 6$

$t = 60$ munud

Haf 2007

(8) (a) $\frac{dP}{dt} \propto P$

$\frac{dP}{dt} = kP$

(b) $dP = kP\,dt$

$\frac{dP}{P} = k\,dt$

$\int \frac{1}{P}\,dP = \int k\,dt$

$\ln|P| = kt + c$

Os yw $t = 0$, mae $P = 50$

Felly

$\ln(50) = k(0) + c$

$c = \ln(50)$

Felly $\ln|P| = kt + \ln(50)$

$\ln|P| - \ln(50) = kt$

$\ln\left|\frac{P}{50}\right| = kt$

$\frac{P}{50} = e^{kt}$

$P = 50e^{kt}$ ✓

(c) Os yw $t = 7$, mae $P = 65$

Felly

$65 = 50e^{7k}$

$\frac{65}{50} = e^{7k}$

$1.3 = e^{7k}$

$\ln(1.3) = 7k$

$k = \frac{\ln(1.3)}{7}$

Felly

$P = 50e^{\left(\frac{\ln(1.3)}{7}\right)t}$

Os yw $t = 16$ yna

$P = 50e^{\left(\frac{\ln(1.3)}{7}\right) \times 16}$

$P = £91.08$

ir geiniog agosaf.

Haf 2008

(7) (a) $\frac{dW}{dt} \propto W$

$\frac{dW}{dt} = kW$

(b) $dW = kW\,dt$

$\frac{dW}{W} = k\,dt$

$\int \frac{1}{W}\,dW = \int k\,dt$

$\ln|W| = kt + c$

Os yw $t=0$, mae $W = 0.10$

Felly

$\ln|0.1| = k(0) + c$

$c = \ln|0.1|$

Felly

$\ln|W| = kt + \ln|0.1|$

$\ln|W| - \ln|0.1| = kt$

$\ln\left|\frac{W}{0.1}\right| = kt$

$\frac{W}{0.1} = e^{kt}$

$W = 0.1e^{kt}$

Os yw $t = 1$, mae $W = 2.01$

Felly

$2.01 = 0.1e^{k(1)}$

$\frac{2.01}{0.1} = e^{k}$

$20.1 = e^{k}$

$\ln(20.1) = k$

Felly

$W = 0.1e^{\ln(20.1)t}$

$W = 0.1 \times (e^{\ln(20.1)})^{t}$

$W = 0.1 \times (20.1)^{t}$

$W = 0.1(20.1)^{t}$

Haf 2009

(7) (a) $\frac{dP}{dt} \propto -P^3$

$\frac{dP}{dt} = -KP^3$

(b) $dP = -KP^3 dt$

$\frac{dP}{P^3} = -K\,dt$

$\int \frac{1}{P^3} dP = \int -K\,dt$

$\int P^{-3} dP = \int -K\,dt$

$\frac{P^{-2}}{-2} = -Kt + C$

$-\frac{1}{2P^2} = -Kt + C$

Os yw $t=0$, mae $P=20$

Felly

$-\frac{1}{2(20)^2} = -K(0) + C$

$-\frac{1}{800} = C$

$C = -\frac{1}{800}$

Felly

$-\frac{1}{2P^2} = -Kt - \frac{1}{800}$

$\frac{1}{P^2} = 2Kt + \frac{1}{400}$ (Felly $A = 2K$)

(c) Os yw $t=1$, mae $P=10$

Felly

$\frac{1}{10^2} = \frac{1}{400} + A(1)$

$\frac{1}{100} = \frac{1}{400} + A$

$A = \frac{1}{100} - \frac{1}{400}$

$A = \frac{3}{400}$

Felly $\frac{1}{P^2} = \frac{3}{400}t + \frac{1}{400}$.

Os yw $P=5$ yna

$\frac{1}{5^2} = \frac{3}{400}t + \frac{1}{400}$

$\frac{1}{25} - \frac{1}{400} = \frac{3}{400}t$

$\frac{3}{80} = \frac{3}{400}t$

$\frac{3 \times 400}{3 \times 80} = t$

$t = 5$ mlynedd

Haf 2010

(8) (a) $\frac{dV}{dt} \propto -V^2$

$\frac{dV}{dt} = -KV^2$

(b) $dV = -KV^2\,dt$

$\frac{dV}{V^2} = -K\,dt$

$\int \frac{1}{V^2}\,dV = \int -K\,dt$

$\int V^{-2}\,dV = \int -K\,dt$

$-V^{-1} = -Kt + C$

os yw $t=0$, $V=12000$

felly

$\frac{-1}{12000} = -K(0) + C$

$C = \frac{-1}{12000}$

Felly

$-\frac{1}{V} = -Kt - \frac{1}{12000}$

$\frac{1}{V} = Kt + \frac{1}{12000}$

$\frac{12000}{V} = 12000Kt + 1$

$12000 = V(12000Kt + 1)$

$\frac{12000}{12000Kt+1} = V$

$V = \frac{12000}{12000Kt+1}$

felly $a = 12000K$.

(c) os yw $t=2$, $V=9000$

felly

$9000 = \frac{12000}{2a+1}$

$9000(2a+1) = 12000$

$2a + 1 = \frac{12000}{9000}$

$2a + 1 = \frac{4}{3}$

$2a = \frac{1}{3}$

$a = \frac{1}{6}$

Felly $V = \frac{12000}{\frac{1}{6}t + 1}$

os yw $t=4$:

$V = \frac{12000}{\frac{1}{6}\times 4 + 1}$

$V = \frac{12000}{\frac{5}{3}}$

$V = £7200$

Haf 2011

(8) (a) $\frac{dN}{dt} \propto N$

$\frac{dN}{dt} = kN$

(b) $dN = kN\,dt$

$\frac{dN}{N} = k\,dt$

$\int \frac{1}{N}\,dN = \int k\,dt$

$\ln|N| = kt + c$

$N = e^{kt+c}$

$N = e^{kt} \times e^{c}$

$N = e^{c} \times e^{kt}$

$N = Ae^{kt}$

ble mae $A = e^{c}$.

(c) (i) Os yw $t=2$, $N=100$

Felly

$100 = Ae^{2k}$

$\frac{100}{e^{2k}} = A$ ——— (1)

Os yw $t=12$, $N=160$

Felly

$160 = Ae^{12k}$

$\frac{160}{e^{12k}} = A$ ——— (2)

Mae (1) a (2) yn awgrymu

$\frac{100}{e^{2k}} = \frac{160}{e^{12k}}$

$100e^{12k} = 160e^{2k}$

$e^{12k} = 1.6e^{2k}$

$12k = \ln(1.6e^{2k})$

$12k = \ln(1.6) + \ln(e^{2k})$

$12k = \ln(1.6) + 2k$

$10k = \ln(1.6)$

$k = \frac{\ln(1.6)}{10}$

$k = 0.047$ i 3 ll. d.

Felly, yn ôl yn (1),

$A = \frac{100}{e^{2\times 0.047}}$

$A = \frac{100}{e^{0.094}}$

Felly $N = \frac{100}{e^{0.094}} e^{0.047t}$

(ii) Os yw $t = 20$

$N = \frac{100}{e^{0.094}} e^{0.047 \times 20}$

$N = 233$ ir cyfanrif agosaf.

Haf 2012

(8) (a) $\frac{dV}{dt} \propto -V^3$

$\frac{dV}{dt} = -KV^3$

(b) $dV = -KV^3 dt$

$\frac{dV}{V^3} = -K\,dt$

$\int \frac{1}{V^3}\,dV = \int -K\,dt$

$\int V^{-3}\,dV = \int -K\,dt$

$\frac{V^{-2}}{-2} = -Kt + C$

$\frac{-1}{2V^2} = -Kt + C$

Os yw $t=0$, $V=60$

felly

$\frac{-1}{2(60)^2} = -K(0) + C$

$C = \frac{-1}{7200}$

felly

$\frac{-1}{2V^2} = -Kt - \frac{1}{7200}$

$\frac{1}{2V^2} = Kt + \frac{1}{7200}$

$\frac{7200}{2V^2} = 7200Kt + 1$

$3600 = (7200Kt + 1)V^2$

$\frac{3600}{7200Kt+1} = V^2$

$V^2 = \frac{3600}{7200Kt + 1}$

$V^2 = \frac{3600}{at + 1}$

ble mae $a = 7200K$.

(c) Os yw $t=2$, $V=50$

felly

$50^2 = \frac{3600}{2a + 1}$

$2500(2a+1) = 3600$

$2a + 1 = \frac{3600}{2500}$

$2a + 1 = \frac{36}{25}$

$2a = \frac{11}{25}$

$a = 0.22$

Felly $V^2 = \frac{3600}{0.22t + 1}$

Os yw $V = 27$ yna

$27^2 = \frac{3600}{0.22t + 1}$

$0.22t + 1 = \frac{3600}{729}$

$0.22t = \frac{319}{81}$

$t = \frac{1450}{81}$

$t = 17.9$ awr i unlle degol

Haf 2013

(8) (a) $\frac{dA}{dt} \propto \sqrt{A}$

$\frac{dA}{dt} = K\sqrt{A}$

(b) $dA = K\sqrt{A}\,dt$

$\frac{dA}{\sqrt{A}} = K\,dt$

$\int \frac{dA}{\sqrt{A}} = \int K\,dt$

$\int A^{-\frac{1}{2}}\,dA = \int K\,dt$

$\frac{A^{\frac{1}{2}}}{\frac{1}{2}} = Kt + C$

$2\sqrt{A} = Kt + C$

Os yw $t=3$, $A=64$

Felly

$2\sqrt{64} = K(3) + C$

$16 = 3K + C$

$16 - 3K = C$ ——— (1)

Os yw $t=5.5$, $A=196$

Felly

$2\sqrt{196} = K(5.5) + C$

$28 = 5.5K + C$

Yn amnewid o (1):

$28 = 5.5K + 16 - 3K$

$12 = 2.5K$

$K = 4.8$

Felly, yn ôl yn (1):

$C = 16 - 3K$

$C = 16 - 3 \times 4.8$

$C = 1.6$

Felly $2\sqrt{A} = 4.8t + 1.6$

$\sqrt{A} = 2.4t + 0.8$

$A = (2.4t + 0.8)^2$

Haf 2014

(8) (a) $\frac{dV}{dt} \propto V$

$\frac{dV}{dt} = KV$

(b) $dV = KV\,dt$

$\frac{dV}{V} = K\,dt$

$\int \frac{1}{V}\,dV = \int K\,dt$

$\ln|V| = Kt + C$

$V = e^{Kt+C}$

$V = e^{Kt} \times e^{C}$

$V = e^{C} \times e^{Kt}$

$V = Ae^{Kt}$

ble mae $A = e^{C}$.

(c) (i) Os yw $t = 2$, $V = 292$

Felly

$292 = Ae^{2K}$

$\frac{292}{e^{2K}} = A$ —— (1)

Os yw $t = 28$, $V = 637$.

Felly

$637 = Ae^{28K}$

$\frac{637}{e^{28K}} = A$ —— (2)

↑

↑ Mae (1) a (2) yn awgrymu

$\frac{292}{e^{2K}} = \frac{637}{e^{28K}}$

$292e^{28K} = 637e^{2K}$

$e^{28K} = \frac{637}{292}e^{2K}$

$28K = \ln\left(\frac{637}{292}e^{2K}\right)$

$28K = \ln\left(\frac{637}{292}\right) + \ln(e^{2K})$

$28K = \ln\left(\frac{637}{292}\right) + 2K$

$26K = \ln\left(\frac{637}{292}\right)$

$K = \frac{1}{26}\ln\left(\frac{637}{292}\right)$

$K = 0.0300000609\ldots4$

$K = 0.03$ i 2 le degol ✓

(ii) Yn ôl yn (1):

$A = \frac{292}{e^{2\times 0.03}}$

$A = 274.99\ldots$

$A = 275$ i'r cyfannif agosaf

(iii) $V = 275e^{0.03t}$

Os yw $t = 0$

$V = 275e^{0.03\times 0}$

$V = £275$

C4 Haf 2015

9) a) $\frac{dP}{dt} \propto P^2$

$\frac{dP}{dt} = kP^2$

b) $dP = kP^2\,dt$

$\frac{dP}{P^2} = k\,dt$

$\int \frac{1}{P^2}\,dP = \int k\,dt$

$\int P^{-2}\,dP = \int k\,dt$

$-P^{-1} = kt + C$

OS yw $t=0$, mae $P = A$, felly

$-A^{-1} = k(0) + C$

$C = -\frac{1}{A}$

Felly $-\frac{1}{P} = kt - \frac{1}{A}$

$\frac{1}{A} - \frac{1}{P} = kt$

$\frac{P}{AP} - \frac{A}{AP} = kt$

$\frac{P-A}{PA} = kt$

$\frac{1}{k}\left(\frac{P-A}{PA}\right) = t$ ✓

c) Os yw $t = 3$, $P = 800$, felly

$$\frac{1}{K}\left(\frac{800-A}{800A}\right) = 3$$

$$\frac{800-A}{800A} = 3K$$

$$\frac{1}{3}\left(\frac{800-A}{800A}\right) = K \quad \text{——} (1)$$

Os yw $t = 4$, $P = 900$, felly

$$\frac{1}{K}\left(\frac{900-A}{900A}\right) = 4$$

$$\frac{1}{4}\left(\frac{900-A}{900A}\right) = K \quad \text{——} (2)$$

(1), (2) $\Rightarrow$ $\frac{1}{3}\left(\frac{800-A}{800A}\right) = \frac{1}{4}\left(\frac{900-A}{900A}\right)$

$$4(900A)(800-A) = 3(800A)(900-A)$$

$$2880000A - 3600A^2 = 2160000A - 2400A^2$$

$$720000A - 1200A^2 = 0$$

$$7200A - 12A^2 = 0$$

$$600A - A^2 = 0$$

$$A(600 - A) = 0$$

Unai $A = 0$ neu $600 - A = 0$

$A = £600$

C4 Haf 2016

(7) a) $\frac{dV}{dt} \propto -V^3$

$\frac{dV}{dt} = -KV^3$

b) $dV = -KV^3\,dt$

$\frac{dV}{V^3} = -K\,dt$

$\int \frac{1}{V^3}\,dV = \int -K\,dt$

$\int V^{-3}\,dV = \int -K\,dt$

$\frac{V^{-2}}{-2} = -Kt + C$

$-\frac{1}{2V^2} = -Kt + C$

Os yw $t=0$, mae $V=A$,

Felly $-\frac{1}{2A^2} = -K(0) + C$

$C = -\frac{1}{2A^2}$

Felly $-\frac{1}{2V^2} = -Kt - \frac{1}{2A^2}$

$-\frac{1}{2V^2} = \frac{-2A^2Kt - 1}{2A^2}$

$\frac{1}{2V^2} = \frac{2A^2Kt + 1}{2A^2}$

$2V^2 = \frac{2A^2}{2A^2Kt + 1}$

$\Rightarrow V^2 = \frac{A^2}{2A^2Kt + 1}$

$V^2 = \frac{A^2}{bt + 1}$

(gan ddiffinio cysonyn $b = 2A^2K$)

c) Amnewid $t = 2$:

$V^2 = \frac{A^2}{2b + 1}$

ond mae gwerth y car yn hanner ei werth cychwynnol, felly

$\left(\frac{A}{2}\right)^2 = \frac{A^2}{2b+1}$

$\frac{A^2}{4} = \frac{A^2}{2b+1}$

$2b + 1 = \frac{4A^2}{A^2}$

$2b + 1 = 4$

$2b = 3$

$b = 1.5$

Felly $V^2 = \frac{A^2}{1.5t + 1}$

Os yw $V = \left(\frac{A}{4}\right)$,

$\left(\frac{A}{4}\right)^2 = \frac{A^2}{1.5t + 1}$

$\frac{A^2}{16} = \frac{A^2}{1.5t + 1}$

Rhannu efo A^2; cilydd:

$16 = 1.5t + 1$

$15 = 1.5t$

$t = 10$ mlynedd

C4 Haf 2017

8) $\frac{dN}{dt} \propto \sqrt{N}$

$\frac{dN}{dt} = K\sqrt{N}$

b) $dN = K\sqrt{N}\,dt$

$\frac{dN}{\sqrt{N}} = K\,dt$

$\int \frac{1}{\sqrt{N}}\,dN = \int K\,dt$

$\int N^{-\frac{1}{2}}\,dN = \int K\,dt$

$\frac{N^{\frac{1}{2}}}{\frac{1}{2}} = Kt + C$

$2\sqrt{N} = Kt + C$

Os yw $t=5$, mae $N = 256$

$2\sqrt{256} = 5K + C$

$2(16) = 5K + C$

$32 - 5K = C$

Os yw $t=7$, mae $N = 400$

$2\sqrt{400} = 7K + C$

$2(20) = 7K + C$

$40 - 7K = C$

$32 - 5K = 40 - 7K$

$7K - 5K = 40 - 32$

$2K = 8$

$K = 4$

Felly $c = 32 - 5(4)$

$c = 12$

Felly $2\sqrt{N} = 4t + 12$

$\sqrt{N} = 2t + 6$

$N = (2t + 6)^2$

C4 Haf 2018

8) a) $\frac{dV}{dt} \propto V^{\frac{3}{2}}$

$\frac{dV}{dt} = kV^{\frac{3}{2}}$

b) $dV = kV^{\frac{3}{2}}dt$

$\frac{dV}{V^{\frac{3}{2}}} = k\,dt$

$\int \frac{1}{V^{\frac{3}{2}}}dV = \int k\,dt$

$\int V^{-\frac{3}{2}}dV = \int k\,dt$

$\frac{V^{-\frac{1}{2}}}{-\frac{1}{2}} = kt + c$

$-2V^{-\frac{1}{2}} = kt + c$

Os yw $t = 0$ yna $V = 900$

Felly $-2(900)^{-\frac{1}{2}} = k(0) + c$

$-2\left(\frac{1}{30}\right) = c$

$c = -\frac{1}{15}$

Felly $-2V^{-\frac{1}{2}} = kt - \frac{1}{15}$

Os yw $t = 3$ yna $V = 1600$

Felly $-2(1600)^{-\frac{1}{2}} = k(3) - \frac{1}{15}$

$-2\left(\frac{1}{40}\right) = 3k - \frac{1}{15}$

$-\frac{2}{40} + \frac{1}{15} = 3k$

$\frac{1}{60} = 3k$

$k = \frac{1}{180}$

Felly $-2V^{-\frac{1}{2}} = \frac{1}{180}t - \frac{1}{15}$

Os yw $t = 8$ yna

$-2V^{-\frac{1}{2}} = \frac{1}{180}(8) - \frac{1}{15}$

$-2V^{-\frac{1}{2}} = -\frac{1}{45}$

$V^{-\frac{1}{2}} = \frac{1}{90}$

$\frac{1}{V^{\frac{1}{2}}} = \frac{1}{90}$

$V^{\frac{1}{2}} = 90$

$\sqrt{V} = 90$

$V = 90^2$

$V = £8,100$