Old Exam Questions – Old Course

Differential Equations

(C4 Summer 2005)

8. The size P of a population of bacteria at time t days is to be modelled as a continuous variable such that the rate of increase of P is directly proportional to P.

(a) Write down a differential equation that is satisfied by P. [1]

(b) Given that the initial size of the population is P_0, show that $P = P_0\,e^{kt}$, where k is a positive constant. [5]

(c) Two days after the start, the population is $1{\cdot}2P_0$. Find when the population will be $2P_0$. [4]

(C4 Summer 2006)

8. Water leaks from a hole at the bottom of a large water tank. The depth of the water at time t minutes is x metres. The rate of decrease of x is directly proportional to $\sqrt{x}$.

(a) Write down a differential equation that is satisfied by x. [1]

(b) Given that the depth of water in the tank when $t = 0$ is 9 metres, show that

$$kt = 6 - 2\sqrt{x} \;,$$

where k is a positive constant. [4]

(c) Given that the depth of water in the tank is 4 metres when $t = 20$, find the time taken for the tank to empty. [3]

(C4 Summer 2007)

8. The price £P of an item at time t years is to be modelled as a continuous variable such that the rate of increase of P is directly proportional to P.

(a) Write down a differential equation that is satisfied by P. [1]

(b) Given that the price of the item at $t = 0$ is £50, show that $P = 50e^{kt}$, where k is a positive constant. [5]

(c) After seven years the price of the item is £65. Find the price of the item after sixteen years. [4]

(C4 Summer 2008)

7. A neglected large lawn contains a certain type of weed. The area of the lawn covered by the weed at time t years is $W\,\mathrm{m}^2$. The rate of increase of W is directly proportional to W.

(a) Write down a differential equation that is satisfied by W. [1]

(b) The area of the lawn covered by the weed initially is $0{\cdot}10\,\mathrm{m}^2$ and one year later the area covered is $2{\cdot}01\,\mathrm{m}^2$. Find an expression for W in terms of t. [6]

(C4 Summer 2009)

7. The value of an electronic component may be modelled as a continuous variable. The value of the component at time t years is £P. The rate of decrease of P is directly proportional to P^3.

(a) Write down a differential equation that is satisfied by P. [1]

(b) The value of the component when $t = 0$ is £20. Show that

$$\frac{1}{P^2} = \frac{1}{400} + At,$$

where A is a positive constant. [5]

(c) Given that the value of the component when $t = 1$ is £10, find the time when the value is £5. [4]

(C4 Summer 2010)

8. The value, £V, of a car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^2.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that $V = 12000$ when $t = 0$, show that

$$V = \frac{12000}{at+1},$$

where a is a constant. [4]

(c) The value of the car at the end of two years is £9000. Find the value of the car at the end of four years. [4]

(C4 Summer 2011)

8. The size N of the population of a small island may be modelled as a continuous variable. At time t, the rate of increase of N is directly proportional to the value of N.

(a) Write down the differential equation that is satisfied by N. [1]

(b) Show that $N = Ae^{kt}$, where A and k are constants. [3]

(c) Given that $N = 100$ when $t = 2$ and that $N = 160$ when $t = 12$,

(i) show that $k = 0{\cdot}047$, correct to three decimal places,

(ii) find the size of the population when $t = 20$. [7]

(C4 Summer 2012)

8. Water is leaking from a hole at the bottom of a large tank. The volume of the water in the tank at time t hours is $V\,\text{m}^3$. The rate of decrease of V is directly proportional to V^3.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that $V = 60$ when $t = 0$, show that

$$V^2 = \frac{3600}{at + 1},$$

where a is a constant. [4]

(c) When $t = 2$, the volume of the water in the tank is $50\,\text{m}^3$. Find the value of t when the volume of the water in the tank is $27\,\text{m}^3$. Give your answer correct to one decimal place. [4]

(C4 Summer 2013)

8. Part of the surface of a small lake is covered by green algae. The area of the lake covered by the algae at time t years is $A\,\text{m}^2$. The rate of increase of A is directly proportional to $\sqrt{A}$.

(a) Write down a differential equation satisfied by A. [1]

(b) The area of the lake covered by the algae at time $t = 3$ is $64\,\text{m}^2$ and the area covered at time $t = 5{\cdot}5$ is $196\,\text{m}^2$. Find an expression for A in terms of t. [6]

(C4 Summer 2014)

8. The value £V of a long term investment may be modelled as a continuous variable. At time t years, the rate of increase of V is directly proportional to the value of V.

(a) Write down a differential equation satisfied by V. [1]

(b) Show that $V = Ae^{kt}$, where A and k are constants. [3]

(c) The value of the investment after 2 years is £292 and its value after 28 years is £637.

(i) Show that $k = 0{\cdot}03$, correct to two decimal places.

(ii) Find the value of A correct to the nearest integer.

(iii) Find the initial value of the investment. Give your answer correct to the nearest pound. [6]

(C4 Summer 2015)

9. A bookseller values a rare book at £A on August 1st 2010. The value, £P, of the book t years after this date may be modelled as a continuous variable. The rate of increase of P may be assumed to be directly proportional to P^2.

(a) Write down a differential equation satisfied by P. [1]

(b) Show that

$$\frac{1}{k}\left(\frac{P-A}{PA}\right) = t,$$

where k is a constant. [4]

(c) The value of the book is £800 on August 1st 2013 and £900 on August 1st 2014. Find the value of A. [3]

(C4 Summer 2016)

7. The value, £V, of a particular car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^3.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that the initial value of the car is £A, show that

$$V^2 = \frac{A^2}{bt+1},$$

where b is a constant. [4]

(c) When $t = 2$, the value of the car has fallen to a half of its initial value. Find the value of t when the value of the car will have fallen to a quarter of its initial value. [4]

(C4 Summer 2017)

8. The size N of the population of a small island may be modelled as a continuous variable. At time t years, the rate of increase of N is assumed to be directly proportional to the value of $\sqrt{N}$.

(a) Write down a differential equation satisfied by N. [1]

(b) When t = 5, the size of the population was 256. When t = 7, the size of the population was 400. Find an expression for N in terms of t. [6]

(C4 Summer 2018)

8. The value of a painting on January 1st 2000 was £900. The value, £V, of the painting t years after this date may be modelled as a continuous variable. The rate of increase of V may be assumed to be directly proportional to $V^{\frac{3}{2}}$.

(a) Write down a differential equation satisfied by V. [1]

(b) The value of the painting on January 1st 2003 was £1600.
Find its value on January 1st 2008. [8]