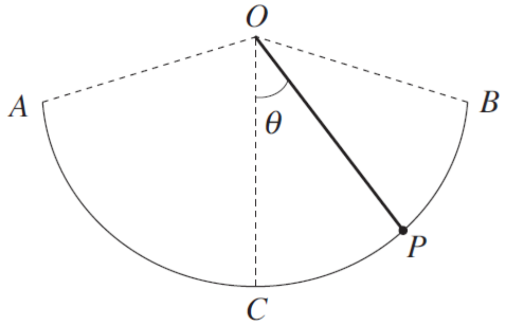
Old Exam Questions – Old Course

Motion in a Vertical Circle

(Haf 2006)

8. One end of a light rod of length l m is attached to a fixed point O and the other end is attached to a particle P of mass m kg. The particle P is set in motion so that it moves back and forth along the minor arc AB of a vertical circle with centre O and radius l m, as shown in the diagram.



When P is at its lowest point C, its speed is u ms^{-1} and the tension in the rod is $2mg$ N.

(a) Show that $u = \sqrt{gl}$. [4]

(b) The speed of P when OP makes an angle θ with the vertical is denoted by v ms^{-1}. Show that $v^2 = gl\,(2\cos\theta - 1)$. [3]

(c) Find the greatest value of θ. [2]

(d) Find the value of θ when the tension in the rod is mg N. [4]

(Haf 2007)

7. A particle, of mass 3 kg, is attached to one end of a light rod of length 0·9 m. The other end of the rod is freely pivoted at a fixed point O. The particle moves in a vertical circle with centre O, such that its speed at the lowest point of its path is three times its speed at the highest point of its path.

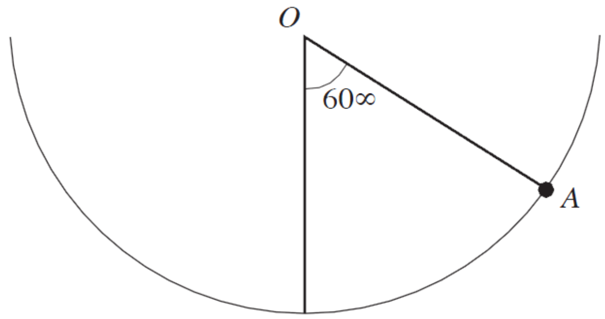
(a) Show that the speed of the particle at the lowest point of its path is 6·3 ms^{-1}. [5]

(b) Calculate the thrust in the rod when the particle is at the highest point of its path. [4]

(c) If a string replaced the rod, state, with a reason, whether the particle would still move in complete circles. [2]

(Haf 2008)

9. A ball, of mass 2 kg, is attached to one end of a light inextensible string of length 0·5 m. The other end of the string is attached to a fixed point O. Initially, the ball is held at rest at a point A such that OA is inclined at an angle of 60° to the downward vertical through O, as shown in the diagram.



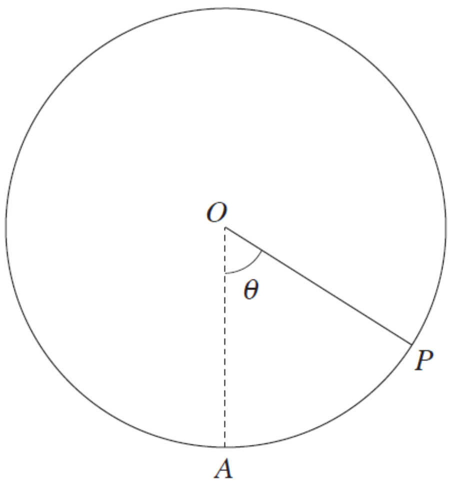
The ball is projected downwards from A with velocity $4\,\text{ms}^{-1}$ perpendicular to OA so that it starts describing a vertical circle centre O. When the string is inclined at an angle θ to the downward vertical, the speed of the ball is $v\,\text{ms}^{-1}$.

(a) Show that $v^2 = 9{\cdot}8\cos\theta + 11{\cdot}1$. [4]

(b) Find, in terms of θ, the tension in the string. [4]

(Haf 2009)

8. In the diagram below, A is the lowest point on the smooth inside surface of a sphere, with centre O and radius 2m. The point P is on the inside surface of the sphere such that $A\hat{O}P = \theta$. A particle, of mass 5 kg, is projected horizontally from A with speed $9\ \text{ms}^{-1}$ so that it moves in the vertical circle with centre O which passes through P.



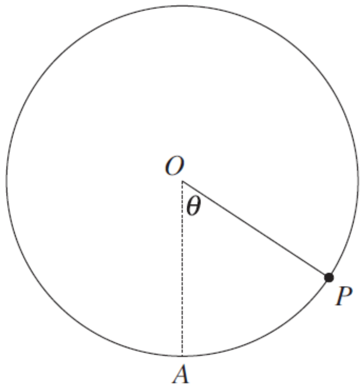
(a) Calculate, in terms of θ, the speed of the particle at P. [4]

(b) Find, in terms of θ, the reaction between the particle and the sphere at P. [4]

(c) Will the particle move in complete circles? Give a reason for your answer. [2]

(Haf 2010)

7. The diagram shows a particle P, of mass 3 kg, attached by a light inextensible string of length 2·5 m to a fixed point O. Initially, P is projected from its lowest point A with a horizontal speed of 13 ms^{-1} so that it starts to move in a vertical circle with centre O.



(a) Find an expression, in terms of θ, for the speed of P when OP makes an angle θ with OA.

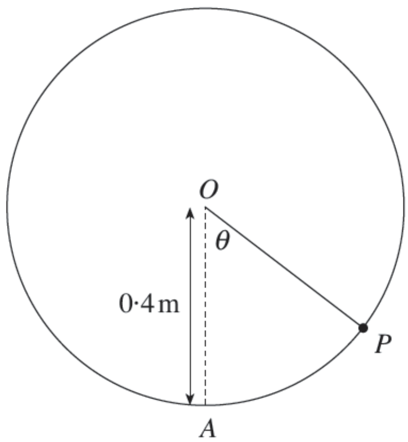
Find the speed of P when $\cos\theta = \frac{1}{2}$. [5]

(b) Find an expression, in terms of θ, for the tension in the string when OP makes an angle θ with OA. [4]

(c) Determine whether or not P describes complete circles. [2]

(Haf 2011)

8. The diagram shows a particle P, of mass 3 kg, attached by a light inextensible string of length 0·4 m to a fixed point O. Initially, P is projected from the point A, which is vertically below O, with a horizontal speed of 4 ms^{-1}.



(a) The speed of P when OP makes an angle θ with OA is $v \text{ ms}^{-1}$.
Show that $v^2 = 8{\cdot}16 + 7{\cdot}84\cos\theta$. [4]

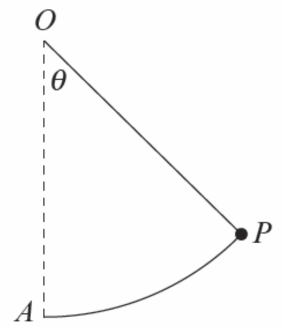
(b) Find an expression, in terms of θ, for the tension in the string when OP makes an angle θ with OA. [4]

(c) Determine whether or not P describes complete circles. [3]

(d) Would your conclusion to *(c)* be different if the string was replaced by a light rigid rod? Justify your answer. [2]

(Haf 2012)

9. A particle of mass 3 kg is attached to one end of a light inextensible string of length 1·2 m. The other end of the string is attached to a fixed point O. Initially, the particle hangs vertically below O at the point A. The particle is then projected horizontally with speed u ms^{-1} from A. When the particle is at the point P, the string makes an angle θ with the vertical OA as shown in the diagram.



The particle comes to instantaneous rest when $\cos\theta = \frac{2}{3}$.

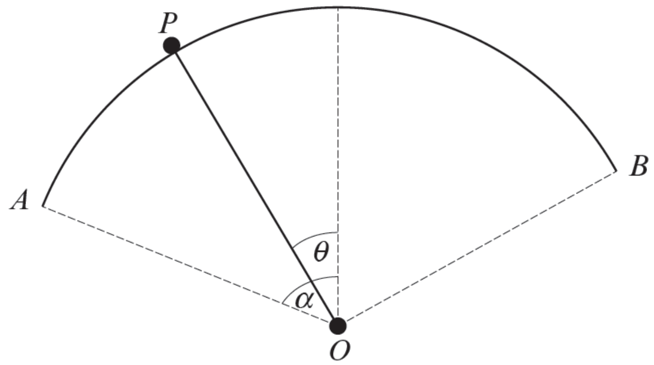
(a) Calculate the value of u and find an expression for v^2 in terms of $\cos\theta$, where v is the velocity of the particle at P. [6]

(b) Find an expression, in terms of θ, for the tension in the string when the particle is at P. [4]

(c) Determine the greatest value and the least value of the tension in the string. [2]

(Haf 2013)

8. The diagram shows a particle of mass 3 kg at a point P on the smooth outer surface AB of a sphere centre O and radius 4 m. The points O, A, P and B are in the same vertical plane.



Initially, the particle is held at rest at the point A, where OA makes an angle α with the upwards vertical and $\cos\alpha = 0{\cdot}8$. The particle is then projected with velocity 5 ms^{-1} in a direction which is perpendicular to OA, so that the particle moves along the arc AB. When the particle is at P, OP makes an angle θ with the upwards vertical.

(a) Find, in terms of θ, the speed of the particle at P. [4]

(b) Determine, in terms of θ, the reaction between the particle and the sphere at P. [4]

(Haf 2014)

7. One end of a light rod of length l metres is freely jointed to a fixed point *O* and the other end is attached to a particle of mass m kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, u ms^{-1}, is a quarter of its speed at the lowest point of the circle.

(a) Show that $u^2 = \frac{4}{15}gl$. [3]

(b) When the rod is inclined at an angle θ to the **downward** vertical,
(i) find an expression for the tension in the rod in terms of m, g and θ.
(ii) determine the value of θ when the tension in the rod becomes zero. [9]

(Haf 2015)

8. One end of a light inextensible string of length 0·8 m is attached to a fixed point. The other end of the string is attached to a particle *P* of mass 3 kg. Initially *P* hangs at rest with the string vertical. The particle *P* is then projected horizontally with speed 5 ms^{-1}, so that it starts to describe a vertical circle. When the string is inclined at an angle θ to the downwards vertical, *P* has speed v ms^{-1} and the tension in the string is T N.

(a) Find, in terms of θ,

(i) an expression for v^2,

(ii) an expression for T. [8]

(b) Find the greatest possible value of θ and briefly describe the subsequent motion of *P*. [3]

(Haf 2016)

9. A smooth sphere, with centre *O* and radius 4 m, is fixed. A particle *P*, of mass m, resting on the sphere at its highest point, is given a horizontal speed of magnitude $\sqrt{g}$ ms^{-1}, where g is the magnitude of the acceleration due to gravity. At the instant the line *OP* makes an angle θ with the upwards vertical, the speed of *P* is v ms^{-1}.

(a) Determine an expression for v^2 in terms of g and θ while *P* remains in contact with the sphere. [4]

(b) Find, in terms of m, g and θ, the magnitude of the force exerted by the sphere on *P*. Hence calculate the value of $\cos\theta$ and the value of v^2 when *P* leaves the surface of the sphere. [7]

(Haf 2017)

6. A particle P, of mass 5 kg, is attached to one end of a light inextensible string of length 0·8 m. The other end of the string is attached to a fixed point O. Initially, the particle P is held at rest with the string OP taut and inclined at an angle of 60° to the downward vertical through O. The particle P is then projected with speed u ms^{-1} in a downward direction perpendicular to the string, so that P starts to describe a vertical circle with centre O. When the string OP is inclined at an angle θ to the downward vertical, the speed of P is v ms^{-1}.

(a) Find, in terms of u and θ, an expression for v^2. [4]

(b) Find, in terms of u and θ, an expression for the tension in the string when OP makes an angle θ with the downward vertical. [4]

(c) Determine the least value of u so that the particle describes complete circles. [2]

(d) Suppose that the string is replaced by a light rod. Determine the least value of u so that the particle describes complete circles. [2]

(Haf 2018)

8. A particle rests on the inside of a fixed smooth vertical circular hoop at its lowest point. The hoop has centre O and radius a m. The particle is given a horizontal velocity of u ms^{-1} such that it leaves the hoop at the point P, where OP makes an angle $\theta = \cos^{-1}\left(\frac{2}{3}\right)$ with the **upward** vertical. Show that $u = 2\sqrt{ag}$. [9]