Ffwythiannau Cyfansawdd a Gwrthdro

Composite and Inverse Functions

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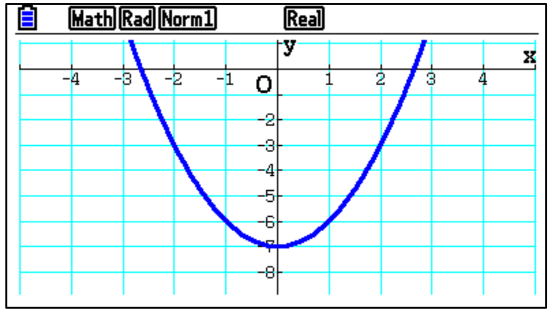
Mae **ffwythiant** f yn mapio elfennau o'r set X (y **parth**) i elfennau o'r set Y (yr **amrediad**).
Mae'r mewnbwn $x \in X$ yn mapio i un (a dim ond un) allbwn $y \in Y$.

*The **function** f maps elements of the set X (the **domain**) to elements of the set Y (the **range**).*
The input $x \in X$ maps to one (and only one) output $y \in Y$.

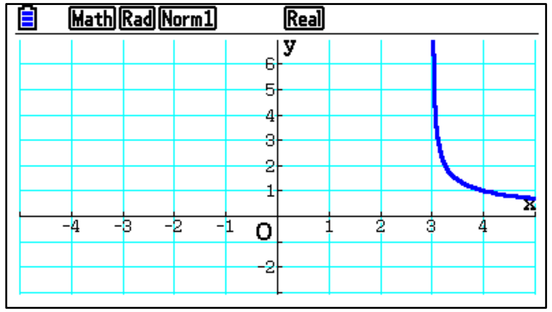
Ar gyfer $f(x) = x^2 - 7$, y parth yw $(-\infty, \infty)$ a'r amrediad yw $[-7, \infty)$.

For $f(x) = x^2 - 7$, the domain is $(-\infty, \infty)$ and the range is $[-7, \infty)$.



Ar gyfer $f(x) = \frac{1}{\sqrt{x-3}}$, y parth yw $[3, \infty)$ a'r amrediad yw $(0, \infty)$.

For $f(x) = \frac{1}{\sqrt{x-3}}$, the domain is $[3, \infty)$ and the range is $(0, \infty)$.



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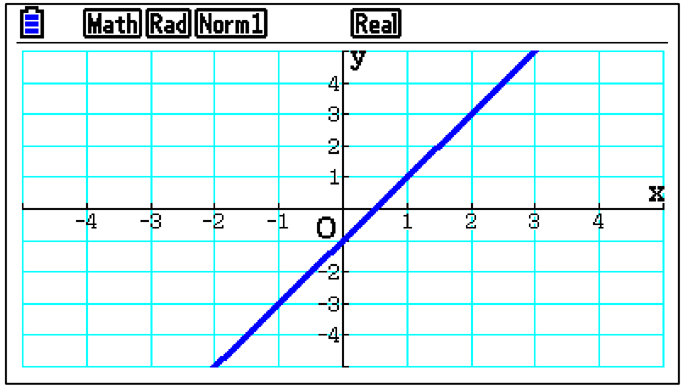
Mewn **ffwythiant un-i-un**, mae pob elfen o'r amrediad yn cyfateb i **union un elfen** o'r parth.

In a ***one-to-one function****, each element of the range corresponds to* ***exactly one element*** *of the domain.*

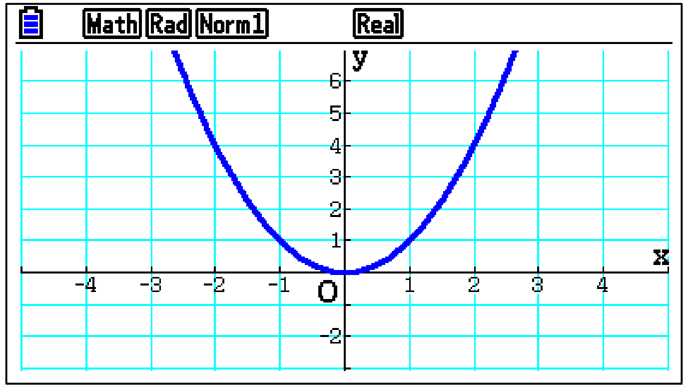
Enghraifft / *Example:* $f(x) = 2x + 1$.



Mewn **ffwythiant llawer-i-un**, mae pob elfen o'r amrediad yn gallu cyfateb i **mwy nag un elfen** o'r parth.

In a ***many-to-one function****, each element of the range can correspond to* ***more than one element*** *of the domain.*

Enghraifft / *Example:* $f(x) = x^2$.



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Mae **ffwythiant cyfansawdd** yn cyfuno dau neu fwy o ffwythiannau.

*A **composite function** combines two or more functions.*

O gael ffwythiannau $f(x)$ a $g(x)$, mae'r ffwythiant cyfansawdd $fg(x)$ yn golygu 'gweithredu f ar ganlyniadau $g(x)$'. Mae'r ffwythiant cyfansawdd $gf(x)$ yn golygu 'gweithredu g ar ganlyniadau $f(x)$'.

Given two functions $f(x)$ and $g(x)$, the composite function $fg(x)$ means 'apply f to the results of $g(x)$'. The composite function $gf(x)$ means 'apply g to the results of $f(x)$'.

Mae'r drefn yn bwysig. Nid yw $fg(x)$ o angenrheidrwydd yr un peth â $gf(x)$.

The order is important. $fg(x)$ is not necessarily the same as $gf(x)$.

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Beth yw parth $fg(x)$?

(1) Darganfyddwch barth g a pharth f.
(2) Darganfyddwch yr holl werthoedd x o barth g fel bod $g(x)$ ym mharth f.
(3) Parth $fg(x)$ yw'r gwerthoedd o gam (2) fel bod y ffwythiant $fg(x)$ wedi'i ddiffinio.

What is the domain of $fg(x)$?

(1) Find the domain of g and the domain of f.
(2) Find all values of x from the domain of g so that $g(x)$ is in the domain of f.
(3) The domain of $fg(x)$ is all the values from step (2) so that the function $fg(x)$ is defined.

Ffwythiannau Cyfansawdd a Gwrthdro
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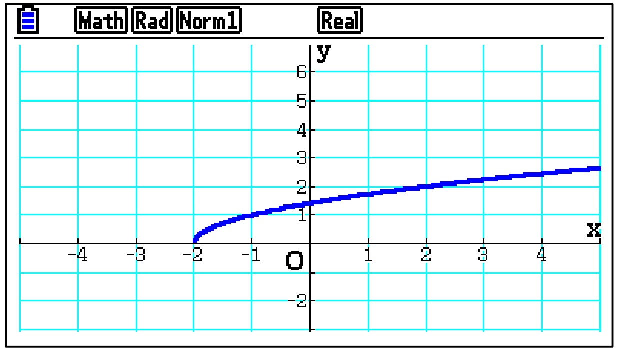
Enghraifft / *Example:*

$f(x) = \sqrt{x+2},\ g(x) = x^2 + 3$

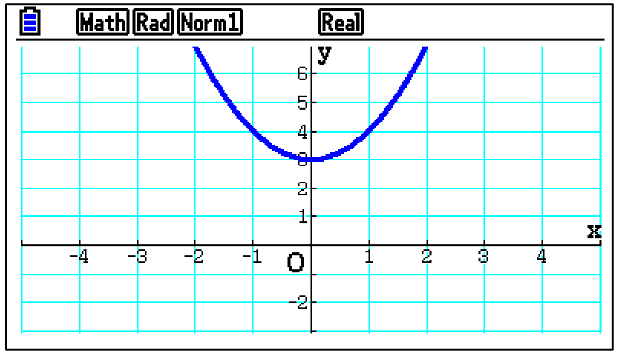
Ffwythiant / *Function*	Parth / *Domain*	Amrediad / *Range*
$f(x)$	$[-2, \infty)$	$[0, \infty)$
$g(x)$	$(-\infty, \infty)$	$[3, \infty)$
$fg(x)$	$(-\infty, \infty)$	$[\sqrt{5}, \infty)$

$$fg(x) = \sqrt{g(x) + 2}$$
$$fg(x) = \sqrt{(x^2 + 3) + 2}$$
$$fg(x) = \sqrt{x^2 + 5}$$

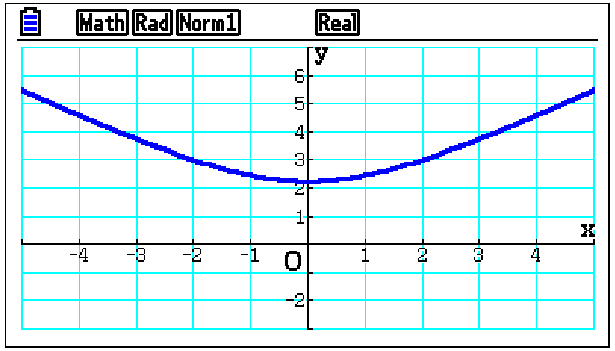
$f(x)$



$g(x)$



$fg(x)$



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Enghraifft / *Example:*

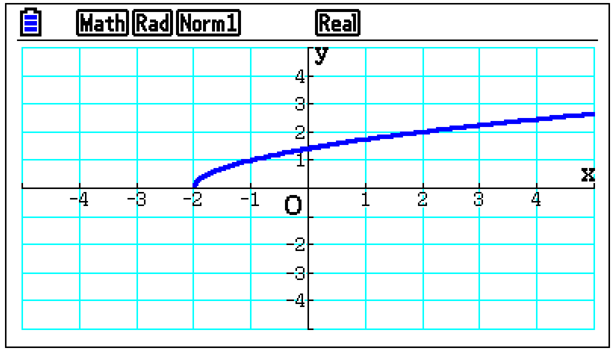
$f(x) = \sqrt{x + 2},\ g(x) = x^2 - 3$

Ffwythiant / *Function*	Parth / *Domain*	Amrediad / *Range*
$f(x)$	$[-2, \infty)$	$[0, \infty)$
$g(x)$	$(-\infty, \infty)$	$[-3, \infty)$
$fg(x)$	$(-\infty, -1] \cup [1, \infty)$	$[0, \infty)$

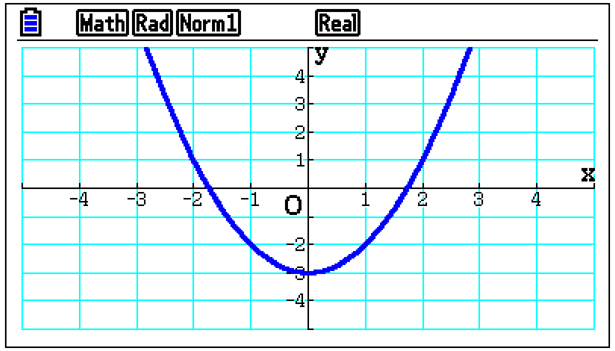
$$fg(x) = \sqrt{g(x) + 2}$$
$$fg(x) = \sqrt{(x^2 - 3) + 2}$$
$$fg(x) = \sqrt{x^2 - 1}$$

$fg(x)$ heb ei ddiffinio yn y parth / *not defined in the domain* $(-1, 1)$.

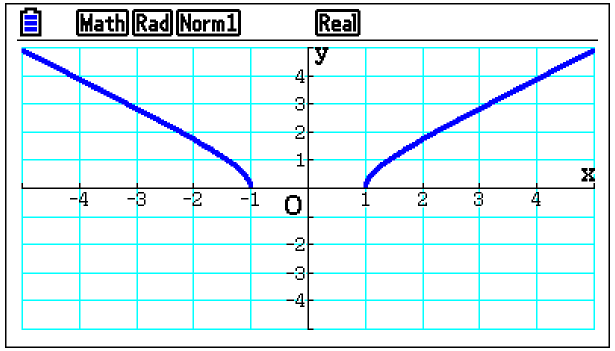
$f(x)$



$g(x)$



$fg(x)$



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O gael ffwythiant $f(x)$, mae'r **ffwythiant gwrthdro** $f^{-1}(x)$ yn gwrthdroi effaith y ffwythiant $f(x)$.

*Given the function $f(x)$, the **inverse function** $f^{-1}(x)$ reverses the effect of the original function $f(x)$.*

Mae'n bosib darganfod y ffwythiant gwrthdro trwy newid testun y ffwythiant $f(x)$ i fod yn x.

We can find the inverse function by changing the subject of $f(x)$ to be x.

Enghraifft / *Example:*

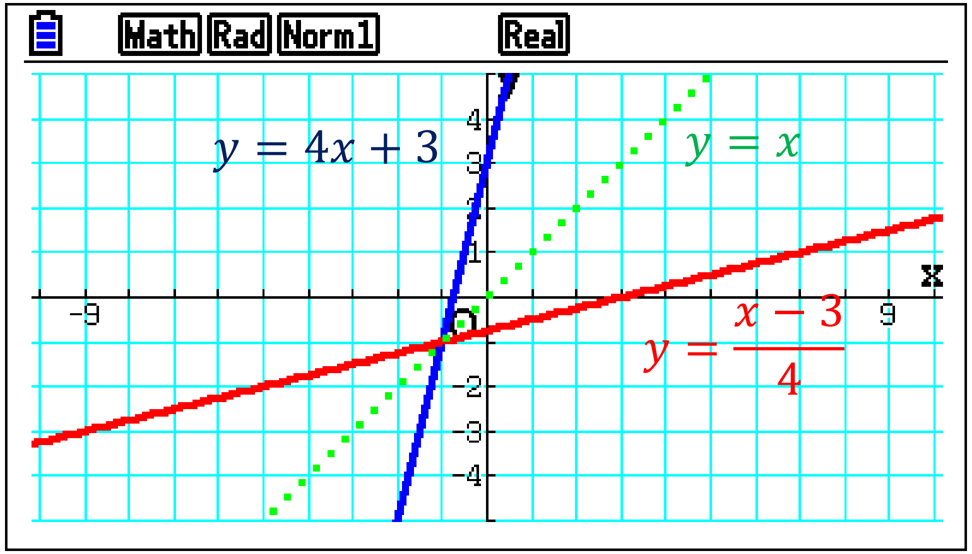
$f(x) = 4x + 3$

$4x + 3 = f(x)$ [Cyfnewid ochrau / *Swap sides*]

$4x = f(x) - 3$ [Tynnu 3 / *Subtract 3*]

$x = \frac{f(x)-3}{4}$ [Rhannu efo 4 / *Divide by 4*]

Felly / *Therefore* $f^{-1}(x) = \frac{x-3}{4}$.



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Mae'r graff ar gyfer $f^{-1}(x)$ yn adlewyrchiad o'r graff ar gyfer $f(x)$ yn y llinell $y = x$.

The graph of $f^{-1}(x)$ is a reflection of the graph of $f(x)$ in the line $y = x$.

Parth $f^{-1}(x)$ yw amrediad $f(x)$, a pharth $f(x)$ yw amrediad $f^{-1}(x)$.

The domain of $f^{-1}(x)$ is the range of $f(x)$, and the domain of $f(x)$ is the range of $f^{-1}(x)$.

O gael eu gwrthdroi, mae ffwythiannau llawer-i-un yn troi'n berthnasau un-i-lawer. Felly, heb gwtogi'r parth, nid oes gan ffwythiannau llawer-i-un ffwythiant gwrthdro.

When inverted, many-to-one functions become one-to-many relationships. Therefore, without curtailing the domain, many-to-one functions do not have an inverse function.

Os yw ffwythiant efo ffwythiant gwrthdro, yna $ff^{-1}(x) = f^{-1}f(x) = x$.

If a function has an inverse function, then $ff^{-1}(x) = f^{-1}f(x) = x$.