Old Exam Questions – Old Course

Composite Functions

(C3 Summer 2005)

10. The functions f and g have domains $(0, \infty)$ and $(5, \infty)$ respectively, and are defined by

$$f(x) = x^2 + 1,$$
$$g(x) = 2x - 3.$$

(a) Write down the ranges of f and g. [2]

(b) Give the reason why $gf(1)$ cannot be formed. [1]

(c) Solve the equation [4]

$$fg(x) = 3x^2 - 6x + 17.$$

(C3 Winter 2006)

9. The function f has domain $(-\infty, \infty)$ and is defined by $f(x) = e^x$.

The function g has domain $(2, \infty)$ and is defined by $g(x) = \ln(x^2 - 4)$.

(a) State the domain of fg. [1]

(b) Solve the equation $fg(x) = 5$. [4]

(C3 Summer 2006)

8. The function f has domain $x \geqslant 1$ and is defined by

$$f(x) = x - \frac{1}{x}.$$

(a) Show that $f'(x)$ is always positive. Deduce the least value of $f(x)$. [3]

(b) Find the range of f. [1]

(c) The function g has domain $[0, \infty)$ and is defined by

$$g(x) = 3x^2 + 2.$$

Solve the equation

$$gf(x) = \frac{3}{x^2} + 8.$$ [4]

(C3 Winter 2007)

10. The functions f and g are defined for all values of x by

$$f(x) = x + 5,$$

$$g(x) = |2x + 1| + 2.$$

Solve the inequality $fg(x) > 10$. [5]

(C3 Summer 2007)

8. The functions f and g have domains $[0, \infty)$ and $(-\infty, \infty)$ respectively, and are defined by

$$f(x) = e^x,$$

$$g(x) = x^2 + 1.$$

(a) Find the range of f and the range of g. [2]

(b) Find an expression for $gf(x)$, simplifying your expression as much as possible. [2]

(c) Write down the domain and range of gf. [2]

(d) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = gf(x)$ indicating where the graphs meet the y-axis. [5]

(C3 Winter 2008)

9. The functions f and g have domains $(0, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = \ln x,$$
$$g(x) = e^{4x}.$$

Find and simplify an expression for

(a) $fg(x)$, [2]

(b) $gf(x)$. [3]

(C3 Summer 2008)

10. The function f has domain $(-\infty, \infty)$ and is defined by

$$f(x) = 2e^x.$$

The function g has domain $[1, \infty)$ and is defined by

$$g(x) = 3 \ln x.$$

(a) Explain why $gf(-1)$ does not exist. [2]

(b) Find in its simplest form an expression for $fg(x)$. State the domain and range of fg. [5]

(C3 Winter 2009)

10. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = 2x - k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[0, \infty)$ and is defined by

$$g(x) = 3x^2 + 4.$$

(b) Find the largest value of k that allows the function gf to be formed. [2]

(c) Given that $gf(2) = 31$, find the value of k. [4]

(C3 Summer 2009)

9. The function f has domain $(-\infty, \infty)$ and is defined by

$$f(x) = 3e^{2x}.$$

The function g has domain $(0, \infty)$ and is defined by

$$g(x) = \ln 4x.$$

(a) Write down the domain and range of fg. [2]

(b) Solve the equation

$$fg(x) = 12.$$ [5]

(C3 Winter 2010)

10. The functions f and g have domains $(0, \infty)$ and $(2, \infty)$ respectively and are defined by

$$f(x) = x^2 - 1,$$
$$g(x) = 2x - 1.$$

(a) Write down the ranges of f and g. [2]

(b) Give the reason why $gf(1)$ cannot be formed. [1]

(c) (i) Find an expression for $fg(x)$. Simplify your answer.

(ii) Write down the domain and range of fg. [4]

(C3 Summer 2010)

10. The functions f and g have domains $[-3, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = \sqrt{x+4},$$
$$g(x) = 2x^2 - 3.$$

(a) Write down the range of f and the range of g. [2]

(b) Find an expression for $gf(x)$. Simplify your answer. [2]

(c) Solve the equation $fg(x) = 17$. [4]

(C3 Winter 2011)

10. The functions f and g have domains $[0, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = e^x,$$
$$g(x) = 4x^3 + 7.$$

(a) Find and simplify an expression for $gf(x)$. [2]

(b) Find the domain and range of gf. [2]

(c) (i) Solve the equation $gf(x) = 18$. Give your answer correct to three decimal places.

(ii) Giving a reason, write down a value for k so that $gf(x) = k$ has no solution. [3]

(C3 Summer 2011)

10. The functions f and g have domains $(-\infty, 0)$ and $(6, \infty)$ respectively and are defined by

$$f(x) = x^2 - 19,$$
$$g(x) = 1 - \frac{1}{2}x.$$

(a) Write down the range of f and the range of g. [2]

(b) Write down the domain and range of fg. [2]

(c) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 2x - 26.$$ [4]

(C3 Winter 2012)

10. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = 3x + k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[-2, \infty)$ and is defined by

$$g(x) = x^2 - 6.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k, for $gf(x)$.

(ii) Given that $gf(2) = 3$, find the value of k. [5]

(C3 Summer 2012)

10. The function g has domain $(-\infty, \infty)$ and is defined by

$$g(x) = \sqrt{3x^2 + 7}.$$

Solve the equation

$$gg(x) = 8.$$ [5]

(C3 Winter 2013)

9. *(a)* The functions f and g have domains $(-\infty, \infty)$ and $(0, \infty)$ respectively and are defined by

$$f(x) = x^2 - 25,$$
$$g(x) = 2x - 3.$$

(i) Write down the domain of fg.

(ii) Write down the range of fg.

(iii) Write down an expression for $fg(x)$.

(iv) Solve the equation $fg(x) = 0$. [7]

(b) The function h is defined by

$$h(x) = \frac{2x + 7}{5x - 2}.$$

(i) Show that $hh(x) = x$.

(ii) **Hence** write down an expression for $h^{-1}(x)$. [3]

(C3 Summer 2013)

11. The functions f and g have domains $(0, \infty)$ and $\left(0, \frac{\pi}{4}\right]$ respectively and are defined by

$$f(x) = \ln x,$$
$$g(x) = \tan x.$$

(a) (i) Write down the domain of fg.

(ii) Write down the range of fg. [3]

(b) (i) Solve the equation $fg(x) = -0{\cdot}4$. Give your answer correct to two decimal places.

(ii) Giving a reason, write down a value for k so that $fg(x) = k$ has no solution. [3]

(C3 Winter 2014)

10. The functions f and g have domains $(0, \infty)$ and $(-\infty, -2)$ respectively and are defined by

$$f(x) = \sqrt{x^2 + 5},$$

$$g(x) = \frac{-4}{x + 1}.$$

(a) By considering $g'(x)$, show that g is an increasing function. [2]

(b) Write down the range of g. [2]

(c) Write down the domain and range of fg. [2]

(d) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 3.$$ [5]

(C3 Summer 2014)

10. The functions f and g have domains $[-2, \infty)$ and $[2, \infty)$ respectively and are defined by

$$f(x) = x^2 + kx - 8,$$
$$g(x) = kx - 4,$$

where k is a positive constant.

(a) Write down, in terms of k, the range of g. [1]

(b) (i) Find the least value of k so that the function fg can be formed.

(ii) Write down an expression in terms of k for $fg(x)$.

(iii) Given that $fg(3) = 0$, find the value of k. [7]

(C3 Summer 2015)

10. *(a)* Show, by counter-example, that the following statement is false.

'If two functions h and k are such that their derivatives h' and k' are equal, then the functions h and k must themselves be equal.' [2]

(b) The functions f and g have domains [7, 60] and [9, ∞) respectively and are defined by

$$f(x) = 2\ln(4x + 5) + 3,$$
$$g(x) = e^x.$$

(i) Find an expression for $f^{-1}(x)$.

(ii) Write down the domain of f^{-1}, giving the end-points of your domain correct to the nearest integer.

(iii) Write down an expression for $gf(x)$ and simplify your answer. [9]

(C3 Summer 2016)

9. The function f has domain (– ∞, 12] and is defined by

$$f(x) = e^{4-\frac{x}{3}} + 8.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1}. [2]

(C3 Summer 2017)

9. The function f has domain $[2, \infty)$ and is defined by

$$f(x) = 4x + k,$$

where k is a constant.

(a) Write down, in terms of k, the range of f. [1]

The function g has domain $[-3, \infty)$ and is defined by

$$g(x) = x^2 - 9.$$

(b) Find the least value of k so that the function gf can be formed. [2]

(c) (i) Write down an expression, in terms of k, for $gf(x)$.

(ii) Given that $gf(2) = 7$, find the value of k.

[5]

(C3 Summer 2018)

10. The functions f and g have domains $(-\infty, \infty)$ and $(0, \infty)$ respectively and are defined by

$$f(x) = x^2 + 2x - 24,$$
$$g(x) = 5 - 3x.$$

(a) Write down the domain of fg. [1]

(b) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 200.$$ [5]