

Old Exam Questions – Old Course
Inverse Functions

(C3 Summer 2005)

9. The function f has domain $(2, \infty)$ and is defined by

$$f(x) = \ln(x - 2) + 3.$$

Find an expression for $f^{-1}(x)$. [4]

(C3 Winter 2006)

8. The function f is defined for $x \geq 0$ by $f(x) = 3x^2 + 4$.

(a) Find an expression for f^{-1} , stating the range and domain of f^{-1} . [6]

(b) Sketch the graphs of f and f^{-1} using the same axes. [3]

(C3 Summer 2006)

10. The function f has domain $[0, \infty)$ and is defined by

$$f(x) = \sqrt{x+1}.$$

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Write down the domain and range of f^{-1} . [2]

(c) Sketch the graph of $y = f^{-1}(x)$. Using the same diagram, sketch the graph of $y = f(x)$. [3]

(C3 Winter 2007)

9. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = \ln(5x - 4) + 2.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) State the domain and range of f^{-1} . [2]

(C3 Summer 2007)

9. The function f has domain $x \geq 0$ and is defined by

$$f(x) = \frac{8}{x+2}.$$

Find an expression for $f^{-1}(x)$ and write down the domain of f^{-1} . [4]

(C3 Winter 2008)

10. The function f has domain $(2, \infty)$ and is defined by

$$f(x) = \frac{1}{\sqrt{x-2}} .$$

(a) Write down the range of f . [1]

(b) Find an expression for $f^{-1}(x)$, stating the domain and range of f^{-1} . [5]

(c) Show that the equation

$$f^{-1}(x) = -\frac{3}{x}$$

has no solutions. [4]

(C3 Summer 2008)

9. The function f has domain $x \leq -1$ and is defined by

$$f(x) = (x + 1)^2 - 2.$$

(a) Find the range of f . [1]

(b) Find an expression for $f^{-1}(x)$. State the domain and range of f^{-1} . [6]

(C3 Winter 2009)

9. The function f has domain $x \leq 0$ and is defined by $f(x) = 5x^2 + 4$.

(a) Find an expression for $f^{-1}(x)$. [5]

(b) Write down the domain and range of f^{-1} . [1]

(C3 Summer 2009)

10. The function f has domain $(0, \infty)$ and is defined by

$$f(x) = 1 - \frac{2}{3x^2 + 2} .$$

(a) Show that $f'(x)$ is always positive. [4]

(b) Write down the range of f . [1]

(c) Find an expression for $f^{-1}(x)$. State the domain and range of f^{-1} . [6]

(C3 Winter 2010)

9. The function f has domain $[4, \infty)$ and is defined by

$$f(x) = \frac{1}{2}\sqrt{x-3} .$$

(a) Find an expression for $f^{-1}(x)$. Write down the range and domain of f^{-1} . [5]

(b) Sketch the graph of $y = f^{-1}(x)$. On the same diagram, sketch the graph of $y = f(x)$. [3]

(C3 Summer 2010)

9. The function f has domain $[1, \infty)$ and is defined by

$$f(x) = \ln(3x - 2) + 5.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) State the domain of f^{-1} . [1]

(C3 Winter 2011)

9. The function f has domain $(-\infty, -1]$ and is defined by

$$f(x) = 4x^2 - 3.$$

(a) Write down the range of f . [1]

(b) Find an expression for $f^{-1}(x)$ and write down the range and domain of f^{-1} . [5]

(c) (i) Evaluate $f^{-1}(6)$.

(ii) By carrying out an appropriate calculation involving f , verify that your answer to part (i) is correct. [3]

(C3 Summer 2011)

9. The function f has domain $(-\infty, -\frac{1}{2})$ and is defined by

$$f(x) = e^{2x+1} - 3.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1} . [2]

(C3 Winter 2012)

9. The function f has domain $[6, \infty)$ and is defined by

$$f(x) = 3 - \frac{1}{\sqrt{x-2}}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1} . [2]

(C3 Summer 2012)

9. The function f has domain $(-\infty, 0)$ and is defined by

$$f(x) = \frac{x^2 + 3}{x^2 + 5}.$$

- (a) (i) Show that $f'(x)$ is always negative.
(ii) Write down the range of f . [6]
- (b) (i) Find an expression for $f^{-1}(x)$.
(ii) Write down the range and domain of f^{-1} . [5]

(C3 Winter 2013)

8. The function f has domain $[-1, \infty)$ and is defined by

$$f(x) = \ln(4x + 5) - 2.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
(b) State the domain of f^{-1} . [1]

(C3 Summer 2013)

10. The function f has domain $(-\infty, 10]$ and is defined by

$$f(x) = e^{5-\frac{x}{2}} + 6.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
(b) Write down the domain of f^{-1} . [2]

(C3 Winter 2014)

9. The function f has domain $[7, \infty)$ and is defined by

$$f(x) = 1 + \frac{2}{\sqrt{3x-5}}.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
(b) Write down the domain of f^{-1} . [2]

(C3 Summer 2014)

9. The function f has domain $(-\infty, 4)$ and is defined by

$$f(x) = x^2 - 8x + 7.$$

- (a) Express $f(x)$ in the form

$$f(x) = (x + a)^2 + b,$$

where a, b are constants whose values are to be found. [1]

- (b) Hence or otherwise, find an expression for $f^{-1}(x)$. [4]

(C3 Summer 2015)

10. (a) Show, by counter-example, that the following statement is false.

'If two functions h and k are such that their derivatives h' and k' are equal, then the functions h and k must themselves be equal.'

[2]

- (b) The functions f and g have domains $[7, 60]$ and $[9, \infty)$ respectively and are defined by

$$\begin{aligned} f(x) &= 2\ln(4x + 5) + 3, \\ g(x) &= e^x. \end{aligned}$$

- (i) Find an expression for $f^{-1}(x)$.
 (ii) Write down the domain of f^{-1} , giving the end-points of your domain correct to the nearest integer.
 (iii) Write down an expression for $gf(x)$ and simplify your answer. [9]

(C3 Summer 2016)

9. The function f has domain $(-\infty, 12]$ and is defined by

$$f(x) = e^{4 - \frac{x}{3}} + 8.$$

- (a) Find an expression for $f^{-1}(x)$. [4]

- (b) Write down the domain of f^{-1} . [2]

10. The function h is defined by

$$h(x) = \frac{4x + 3}{5x - 4}.$$

- (a) Show that $hh(x) = x$. [3]

- (b) Use the result of part (a) to write down an expression for $h^{-1}(x)$.
 Hence evaluate $h^{-1}(-1)$. [2]

(C3 Summer 2017)

8. The function f has domain $[8, \infty)$ and is defined by

$$f(x) = 2 + \frac{3}{\sqrt{5x-4}}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1} . [2]

(C3 Summer 2018)

9. The function f has domain $(-\infty, 0]$ and is defined by

$$f(x) = 4 - \frac{7}{2-3x}.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Write down the domain of f^{-1} . [2]