Fformiwlâu Adiad Trigonometreg

Trigonometric Addition Formulae

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Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

Nid yw'r ffwythiannau trigonometreg yn ddosbarthol.

Mae hyn yn golygu fod (er enghraifft) $\sin(A+B) \neq \sin A + \sin B$.

Gellid profi hyn trwy ystyried $A = B = 45°$:

Ochr chwith $= \sin(45° + 45°) = \sin 90° = 1$.

Ochr dde $= \sin 45° + \sin 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \neq 1$.

Y fformiwla gywir ar gyfer $\sin(A+B)$ yw

$$\sin(A+B) = \sin A \cos B + \cos A \sin B.$$

Os yw A a B yn onglau llym, gellid profi'r fformiwla hon trwy ystyried y trionglau ar y sleid nesaf.

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The trigonometric functions are not distributive.

This means that (for example) $\sin(A + B) \neq \sin A + \sin B$.

We can prove this by considering $A = B = 45°$:

Left hand side $= \sin(45° + 45°) = \sin 90° = 1$.

Right hand side $= \sin 45° + \sin 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \neq 1$.

The correct formula for $\sin(A + B)$ is

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

If A and B are acute angles, then we can prove this formula by considering the triangles on the next slide.

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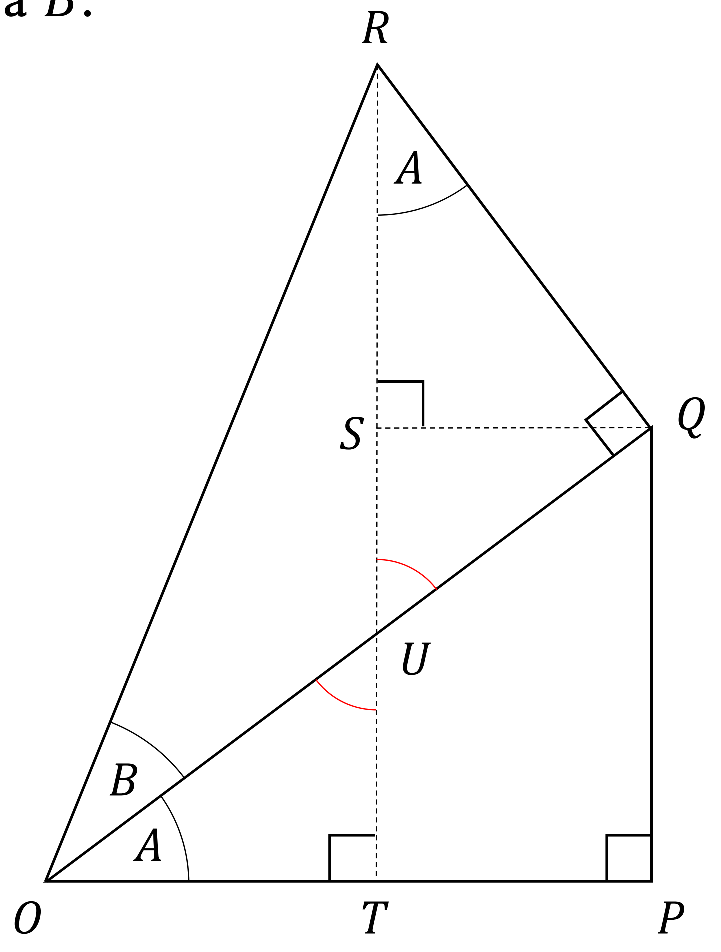
Mae OPQ ag OQR yn drionglau ongl sgwâr sy'n cynnwys yr onglau A a B.

Mae RT ag SQ yn llinellau sy'n ffurfio'r onglau sgwâr $O\hat{T}R$ ag $R\hat{S}Q$.

Mae $O\hat{U}T = R\hat{U}Q$ felly mae OTU ag URQ yn drionglau cyflun.

Felly mae $U\hat{R}Q = A$.

$$\begin{aligned}\sin(A+B) &= \frac{TR}{OR} \\ &= \frac{TS+SR}{OR} \\ &= \frac{PQ+SR}{OR} \\ &= \frac{PQ}{OR} + \frac{SR}{OR} \\ &= \frac{PQ}{OQ} \times \frac{OQ}{OR} + \frac{SR}{QR} \times \frac{QR}{OR} \\ &= \sin A \cos B + \cos A \sin B\end{aligned}$$



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Trigonometric Addition Formulae

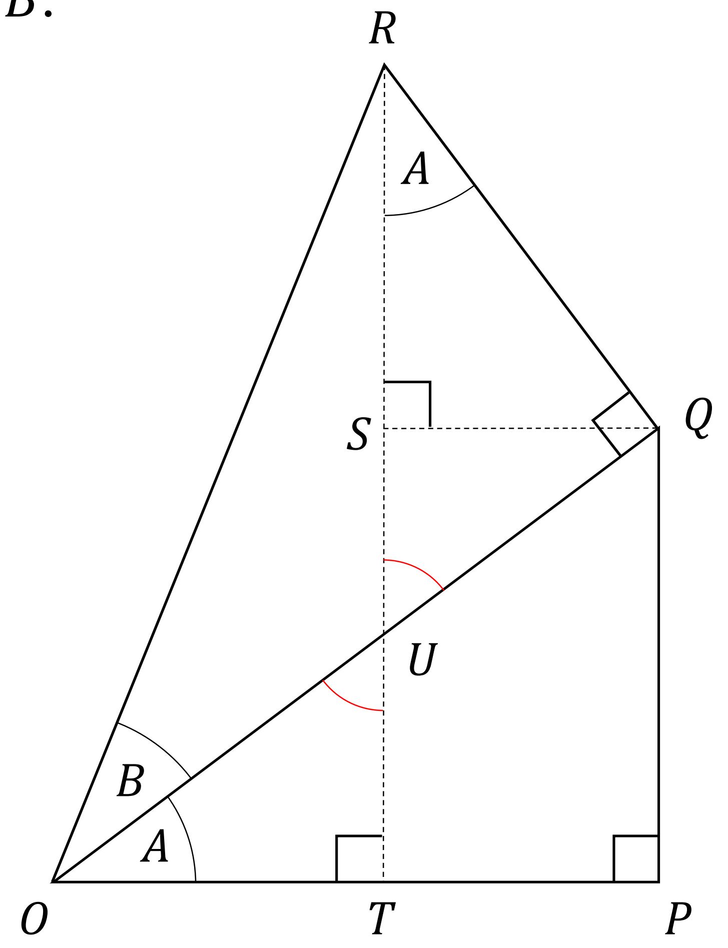
OPQ and OQR are right-angled triangles containing the angles A and B.

RT and SQ are lines forming the right angles $O\hat{T}R$ and $R\hat{S}Q$.

$O\hat{U}T = R\hat{U}Q$ therefore OTU and URQ are similar triangles.

Therefore $U\hat{R}Q = A$.

$$\begin{aligned}\sin(A+B) &= \frac{TR}{OR}\\ &= \frac{TS+SR}{OR}\\ &= \frac{PQ+SR}{OR}\\ &= \frac{PQ}{OR}+\frac{SR}{OR}\\ &= \frac{PQ}{OQ}\times\frac{OQ}{OR}+\frac{SR}{QR}\times\frac{QR}{OR}\\ &= \sin A\cos B+\cos A\sin B\end{aligned}$$



Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

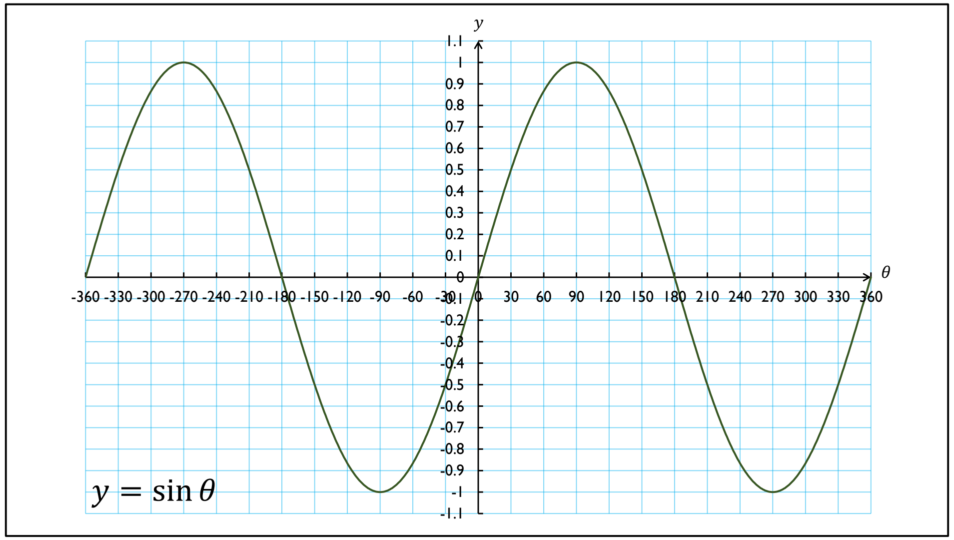
Os ydym yn amnewid $-B$ yn lle B yn y fformiwla

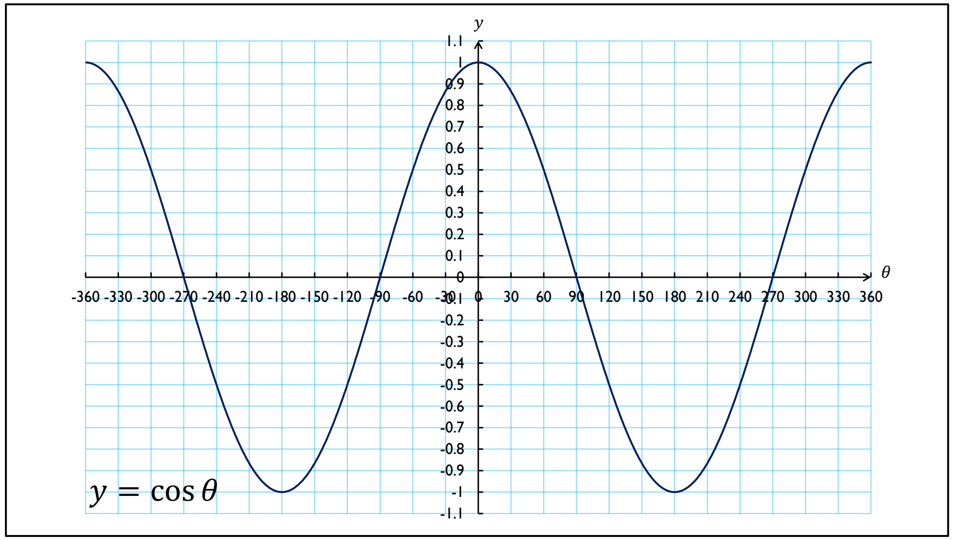
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

cawn

$$\sin(A-B) = \sin A\cos(-B) + \cos A\sin(-B)$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

(trwy gymesuredd graffiau $\sin$ a $\cos$.)





Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

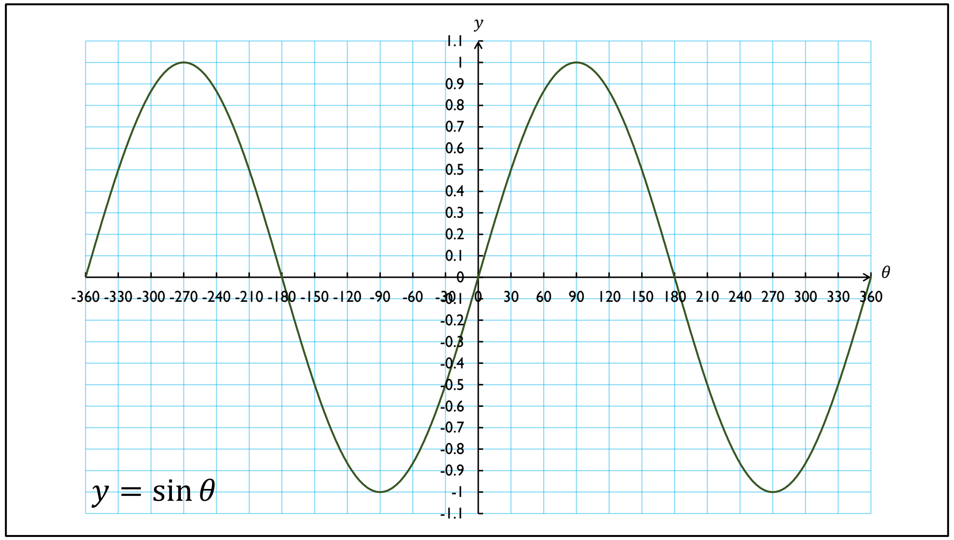
If we substitute $-B$ instead of B in the formula

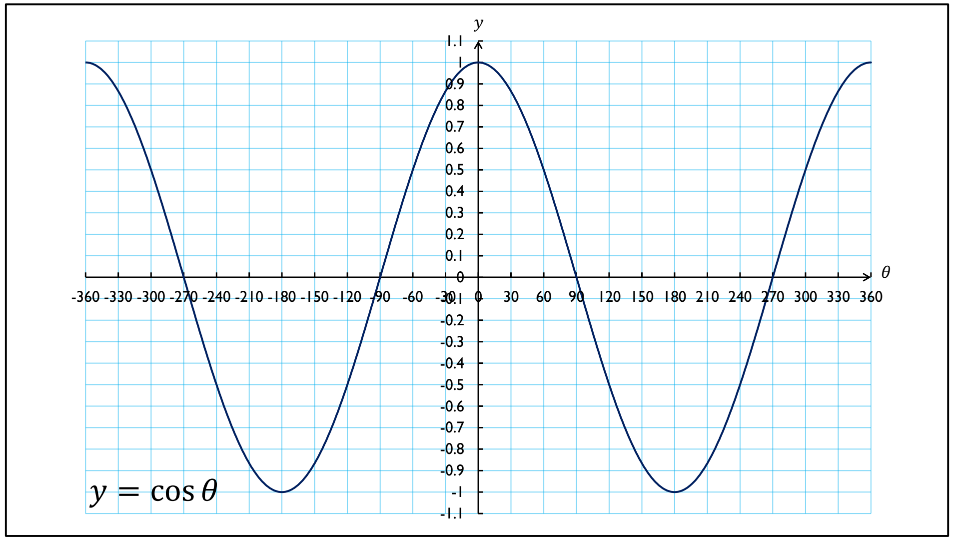
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

we obtain

$$\sin(A-B) = \sin A\cos(-B) + \cos A\sin(-B)$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

(through the symmetry of the graphs of $\sin$ and $\cos$.)





Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

Os ydym yn amnewid $\left(\frac{\pi}{2} - A\right)$ yn lle A yn y fformiwla

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

cawn

$$\sin\left(\frac{\pi}{2} - A - B\right) = \sin\left(\frac{\pi}{2} - A\right)\cos B - \cos\left(\frac{\pi}{2} - A\right)\sin B$$

$$\sin\left(\frac{\pi}{2} - A - B\right) = \cos A \cos B - \sin A \sin B$$

(trwy'r cysylltiad rhwng graffiau sin a cos)

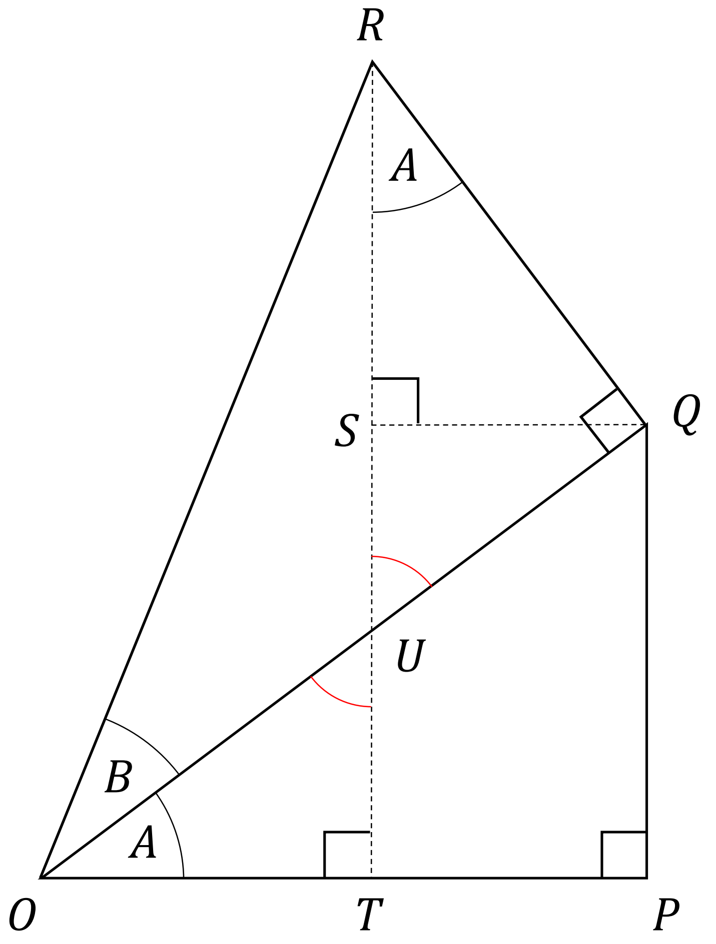
$$\sin\left(O\hat{R}U\right) = \cos A \cos B - \sin A \sin B$$

(yn ystyried y triongl ORQ)

$$\cos\left(\frac{\pi}{2} - O\hat{R}U\right) = \cos A \cos B - \sin A \sin B$$

(trwy'r cysylltiad rhwng graffiau sin a cos)

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$



Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

If we substitute $\left(\frac{\pi}{2} - A\right)$ instead of A in the formula

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

we obtain

$$\sin\left(\frac{\pi}{2} - A - B\right) = \sin\left(\frac{\pi}{2} - A\right)\cos B - \cos\left(\frac{\pi}{2} - A\right)\sin B$$

$$\sin\left(\frac{\pi}{2} - A - B\right) = \cos A \cos B - \sin A \sin B$$

(through the connection between the graphs of $\sin$ and $\cos$)

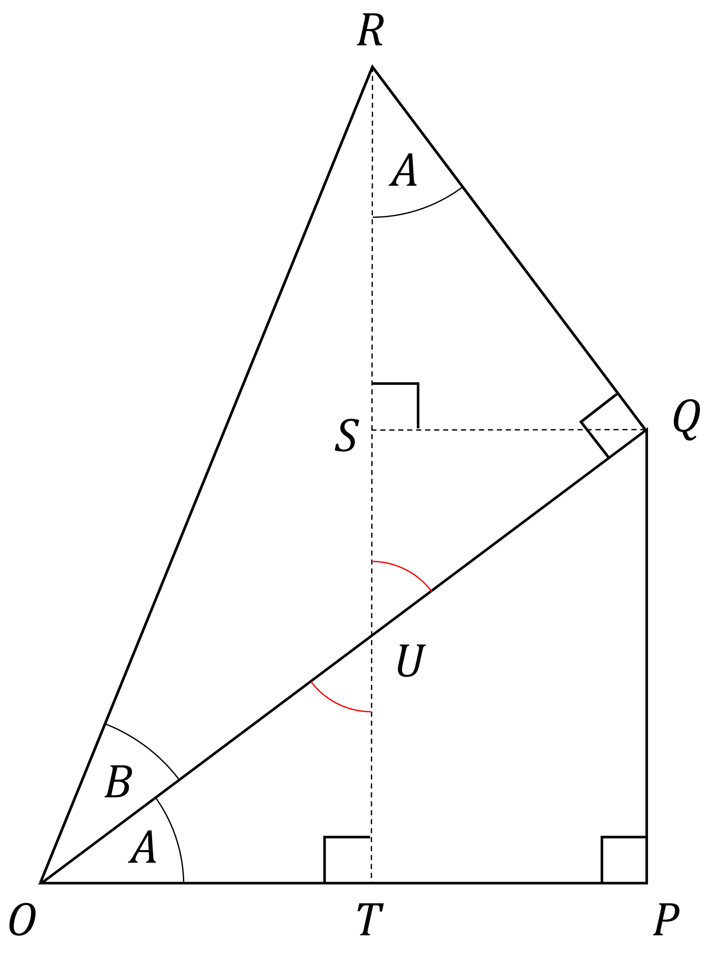
$$\sin\left(O\hat{R}U\right) = \cos A \cos B - \sin A \sin B$$

(considering the triangle ORQ)

$$\cos\left(\frac{\pi}{2} - O\hat{R}U\right) = \cos A \cos B - \sin A \sin B$$

(through the connection between the graphs of $\sin$ and $\cos$)

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$



Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

Os ydym yn amnewid $-B$ yn lle B yn y fformiwla

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

cawn

$$\cos(A - B) = \cos A \cos(-B) - \sin A \sin(-B)$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

(trwy gymesuredd graffiau $\sin$ a $\cos$.)

Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

If we substitute $-B$ instead of B in the formula

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

we obtain

$$\cos(A-B) = \cos A \cos(-B) - \sin A \sin(-B)$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

(through the symmetry of the graphs of $\sin$ and $\cos$.)

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Trigonometric Addition Formulae

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$\tan(A+B) = \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

$$\tan(A+B) = \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}$$

(yn rhannu bob term efo / dividing each term by $\cos A\cos B$)

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

Os ydym yn amnewid $-B$ yn lle B yn y fformiwla

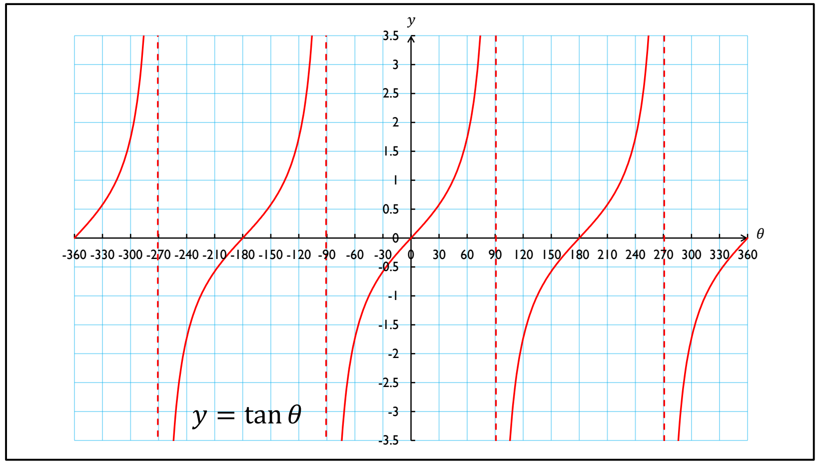
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

cawn

$$\tan(A-B) = \frac{\tan A + \tan(-B)}{1 - \tan A \tan(-B)}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(trwy gymesuredd graff tan.)



Fformiwlâu Adiad Trigonometreg
Trigonometric Addition Formulae

If we substitute $-B$ instead of B in the formula

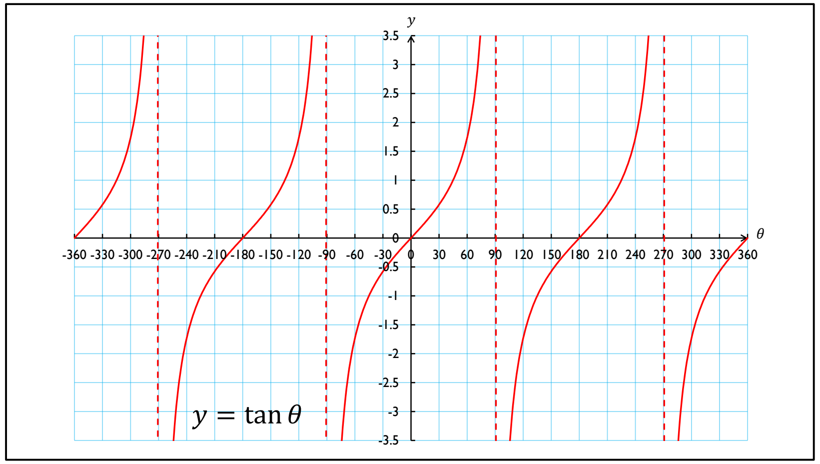
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

we obtain

$$\tan(A-B) = \frac{\tan A + \tan(-B)}{1 - \tan A \tan(-B)}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(through the symmetry of the graph of tan.)



Fformiwlâu Adiad Trigonometreg
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Crynodeb / Summary:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$