Ehangiad Binomial

Binomial Expansion

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Adolygu Uned 1 / *Revision of Unit 1:*

Os yw n yn gyfanrif positif, mae / *If n is a positive integer, then*

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + x^n$$

$$(a+x)^n = a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \binom{n}{3}a^{n-3}x^3 + \cdots + x^n$$

$$(a+bx)^n = a^n + \binom{n}{1}a^{n-1}(bx) + \binom{n}{2}a^{n-2}(bx)^2 + \binom{n}{3}a^{n-3}(bx)^3 + \cdots + (bx)^n$$

ble mae $\binom{n}{r}$ yn dod o driongl Pascal neu o'r botwm **nCr** ar gyfrifiannell. /
*where $\binom{n}{r}$ comes from Pascal's triangle or from the **nCr** button on a calculator.*

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Enghraifft / *Example:*

Ehangwch / *Expand* $(1+3x)^4$.

Triongl Pascal / *Pascal's Triangle:*

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

$$
\begin{aligned}
(1+3x)^4 &= 1(1)^4(3x)^0+4(1)^3(3x)^1+6(1)^2(3x)^2+4(1)^1(3x)^3+1(1)^0(3x)^4 \\
&= 1(1)^4+4(1)^3(3x)+6(1)^2(3x)^2+4(1)^1(3x)^3+1(3x)^4 \\
&= 1+4(3x)+6(9x^2)+4(27x^3)+1(81x^4) \\
&= 1+12x+54x^2+108x^3+81x^4
\end{aligned}
$$

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Os **nad** yw n yn gyfanrif positif, yna

- Nid yw'n bosib ffeindio $(a + bx)^n$ trwy luosi allan $(a + bx)$ n o weithiau;
- Nid yw'n bosib defnyddio'r botwm **nCr** ar gyfrifiannell i gyfrifo'r cyfernodau.

Rhaid yn hytrach defnyddio cyfres anfeidredd a diffiniad cyffredinol $\binom{n}{r}$.

*If n is **not** a positive integer, then*

- *It is not possible to find $(a + bx)^n$ by multiplying out $(a + bx)$ n times;*
- *It is not possible to use the **nCr** button on a calculator to calculate the coefficients.*

Instead, we must use an infinite sequence and the general definition of $\binom{n}{r}$.

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Yn gyffredinol / *In general,*

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Felly, er enghraifft, / *Therefore, for example,*

$$\binom{n}{4} = \frac{n!}{4!\,(n-4)!}$$

$$\binom{n}{4} = \frac{n \times (n-1) \times (n-2) \times (n-3) \times \cancel{(n-4)} \times \cancel{(n-5)} \times \cdots \times \cancel{3} \times \cancel{2} \times \cancel{1}}{[4 \times 3 \times 2 \times 1] \times [\cancel{(n-4)} \times \cancel{(n-5)} \times \cdots \times \cancel{3} \times \cancel{2} \times \cancel{1}]}$$

$$\binom{n}{4} = \frac{n(n-1)(n-2)(n-3)}{4 \times 3 \times 2 \times 1}$$

Felly, ar gyfer cyfanrif positif n, / *Therefore, for a positive integer* n,

$$(a+bx)^n = a^n + na^{n-1}(bx) + \frac{n(n-1)}{2 \times 1}a^{n-2}(bx)^2 + \frac{n(n-1)(n-2)}{3 \times 2 \times 1}a^{n-3}(bx)^3 + \cdots + (bx)^n$$

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Os **nad** yw n yn gyfanrif positif, yna / *If* n *is* ***not*** *a positive integer, then*

$$(a+bx)^n = a^n + na^{n-1}(bx) + \frac{n(n-1)}{2\times 1}a^{n-2}(bx)^2 + \frac{n(n-1)(n-2)}{3\times 2\times 1}a^{n-3}(bx)^3 + \cdots$$

Mae'r dotiau'n dangos mai cyfres anfeidredd yw hwn. / *The dots show that this is an infinite sequence.*

Mae'r gyfres yn synhwyrol dim ond os yw'r termau'n mynd yn **llai** wrth fynd ymlaen. Felly mae'r ehangiad yn ddilys dim ond os yw / *The expansion is only sensible if the terms get progressively* ***smaller****. Therefore the expansion is only valid if*

$$\left|\frac{bx}{a}\right| < 1$$

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Enghraifft / Example:

Ehangwch $(3+2x)^{-\frac{1}{2}}$ hyd at y term mewn x^2. Nodwch pryd fydd eich ehangiad yn ddilys. /

Expand $(3+2x)^{-\frac{1}{2}}$ *up to and including the term in* x^2*. Note when your expansion is valid.*

$$(3+2x)^{-\frac{1}{2}} = 3^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) \times 3^{-\frac{1}{2}-1} \times (2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2\times1} \times 3^{-\frac{1}{2}-2} \times (2x)^2$$

$$= 3^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) \times 3^{-\frac{3}{2}} \times (2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2\times1} \times 3^{-\frac{5}{2}} \times (2x)^2$$

$$= 3^{-\frac{1}{2}} - 3^{-\frac{3}{2}}x + \frac{3}{8} \times 3^{-\frac{5}{2}} \times 4x^2$$

$$= 3^{-\frac{1}{2}} - 3^{-\frac{3}{2}}x + 3^{-\frac{5}{2}} \times \frac{3}{2}x^2$$

$$= \frac{1}{3^{\frac{1}{2}}} - \frac{1}{3^{\frac{3}{2}}}x + \frac{1}{3^{\frac{5}{2}}} \times \frac{3}{2}x^2$$

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Enghraifft (parhad) / Example (continued):

$$(3+2x)^{-\frac{1}{2}} = \frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}x + \frac{1}{9\sqrt{3}} \times \frac{3}{2}x^2$$
$$= \frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}x + \frac{1}{6\sqrt{3}}x^2$$
$$= \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{9}x + \frac{\sqrt{3}}{18}x^2$$
$$= \frac{\sqrt{3}}{3}\left(1 - \frac{x}{3} + \frac{x^2}{6}\right)$$

Mae'r ehangiad yn ddilys os yw $\left|\frac{2x}{3}\right| < 1$, hynny yw $|x| < \frac{3}{2}$.

The expansion is valid if $\left|\frac{2x}{3}\right| < 1$, *that is* $|x| < \frac{3}{2}$.