Cyfres Geometrig

Geometric Series

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Cyfres Geometrig
Geometric Series

Mae cyfres o rifau yn **gyfres geometrig** os ydym yn lluosi efo'r un cysonyn i gael y rhif nesaf yn y gyfres.
*A sequence of numbers is a **geometric sequence** if we multiply by the same constant to obtain the next number in the sequence.*

Er enghraifft, mae'r gyfres 3, 6, 12, 24, 48, ... yn gyfres geometrig ble mae'r term cyntaf yn 3 ac mae'r gymhareb cyffredin yn 2.
For example, the sequence 3, 6, 12, 24, 48, ... is a geometric sequence where the first term is 3 and the common ratio is 2.

Terminoleg / *Terminology*

a	Term cyntaf y gyfres	*First term of the sequence*
r	Y gymhareb cyffredin	*The common ratio*
l	Term olaf y gyfres, os oes un yn bodoli	*Last term of the sequence, if one exists*
t_n	$n$fed term y gyfres	*The nth term of the sequence*

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Ar gyfer cyfres geometrig efo term cyntaf a a chymhareb cyffredin r:
For a geometric sequence with first term a and common ratio r:

Term cyntaf / *First term*	$t_1 = a$
Ail derm / *Second term*	$t_2 = ar$
Trydydd term / *Third term*	$t_3 = ar^2$
Pedwerydd term / *Fourth term*	$t_4 = ar^3$
Nfed term / *Nth term*	$t_n = ar^{n-1}$

Ar gyfer pob cyfanrif n / *For all integers* n,

$$r = \frac{t_{n+1}}{t_n}$$

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Mae'n bosib ystyried cyfanswm S_n yr n term cyntaf mewn cyfres geometrig.
The geometric series S_n is the sum of the first n terms of a geometric sequence.

Ar gyfer y gyfres geometrig 3, 6, 12, 24, 48, ... / *For the geometric sequence 3, 6, 12, 24, 48, ...*

$$S_3 = 3 + 6 + 12 = 21$$

$$S_6 = 3 + 6 + 12 + 24 + 48 + 96 = 189$$

Swm n term cyntaf cyfres geometrig

Profwch mai swm n term cyntaf cyfres geometrig efo term cyntaf a a chymhareb cyffredin r yw $S_n = \frac{a(1-r^n)}{1-r}$.

Prawf

Term 1af: $t_1 = a$

2il Derm: $t_2 = ar$

Nfed Term: $t_n = ar^{n-1}$

Swm yr n term cyntaf: $S_n = t_1 + t_2 + \cdots + t_{n-1} + t_n$

$S_n = a + ar + \cdots + ar^{n-2} + ar^{n-1}$ ——①

Lluosi efo r: $rS_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n$ ——②

Yn gwneud ① – ②: $S_n - rS_n = a - ar^n$

$S_n(1-r) = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

QED

The sum of the first n terms of a geometric sequence

Prove that the sum of the first n terms of a geometric sequence with first term a and common ratio r is given by $S_n = \frac{a(1-r^n)}{1-r}$.

Proof

1st Term: $t_1 = a$

2nd Term: $t_2 = ar$

Nth Term: $t_n = ar^{n-1}$

Sum of the first n terms: $S_n = t_1 + t_2 + \cdots + t_{n-1} + t_n$

$$S_n = a + ar + \cdots + ar^{n-2} + ar^{n-1} \qquad \text{———①}$$

Multiplying by r:

$$rS_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n \qquad \text{———②}$$

Subtracting ① – ②:

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

QED

Swm i anfeidredd
Sum to infinity

Os yw $|r| < 1$ mae'n bosib ystyried cyfanswm S_∞ holl dermau cyfres geometrig.
If $|r| < 1$ *we can consider the sum* S_∞ *of all terms of a geometric sequence.*

O'r prawf gynt, gwyddom fod $S_n = \frac{a(1-r^n)}{1-r}$.
From the previous proof, we know that $S_n = \frac{a(1-r^n)}{1-r}$.

Os yw $|r| < 1$, hynny yw $-1 < r < 1$, yna bydd r^n yn lleihau wrth i n gynyddu.
If $|r| < 1$*, that is* $-1 < r < 1$*, then as* n *increases* r^n *decreases.*

Dywedwn bod $r^n \to 0$ fel mae $n \to \infty$.
We say that $r^n \to 0$ *as* $n \to \infty$.

Felly os yw $|r| < 1$ mae $S_\infty = \frac{a}{1-r}$.
Therefore if $|r| < 1$ *we have* $S_\infty = \frac{a}{1-r}$.