Old Exam Questions – Old Course
Roots of Equations

(C3 Summer 2005)

2. *(a)* Sketch the graphs of $y = x^4$ and $y = 1 - 3x$. Deduce the number of real roots of the equation

$$x^4 + 3x - 1 = 0.$$ [3]

(b) Show that the equation

$$x^4 + 3x - 1 = 0$$

has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \frac{1 - x_n^4}{3}$$

with $x_0 = 0{\cdot}3$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. [7]

(C3 Winter 2006)

4. *(b)* Show that the equation

$$4x^3 + 10x - 1 = 0$$

has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \frac{1 - 4x_n^3}{10}$$

with $x_0 = 0{\cdot}1$ may be used to find α. Calculate and record the values of x_1, x_2, x_3. Write down the value of x_3 correct to six decimal places and show that it is the value of α correct to six decimal places. [7]

(C3 Summer 2006)

4. *(b)* Show that the equation

$$e^{2a} - a - 10 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$a_{n+1} = \frac{1}{2}\ln(a_n + 10)$$

with $a_0 = 1{\cdot}2$ can be used to find α. Find and record the values of a_1, a_2, a_3, a_4.

Write down the value of a_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. [7]

(C3 Winter 2007)

3. Show that the equation

$$\cos x + 2x - 2 = 0$$

has a root α between 0 and $\frac{\pi}{2}$.

The recurrence relation

$$x_{n+1} = 1 - \frac{1}{2}\cos x_n$$

with $x_0 = 0{\cdot}5$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to three decimal places and prove that this is the value of α correct to three decimal places. [7]

(C3 Summer 2007)

3. *(b)* Show that the equation

$$t^3 + 4t - 2 = 0$$

has a root α between 0 and 1.

The recurrence relation

$$t_{n+1} = \frac{2 - t_n^3}{4}$$

with $t_0 = 0{\cdot}5$ can be used to find α. Find and record the values of t_1, t_2, t_3, t_4. Write down the value of t_4 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

(C3 Winter 2008)

4. Show that the equation

$$2\ln(70 + x) - x = 0$$

has a root α between 8 and 9.

The recurrence relation

$$x_{n+1} = 2\ln(70 + x_n)$$

with $x_0 = 8{\cdot}8$ can be used to find α.

Find and record the values of x_1, x_2, x_3. Write down the value of x_3 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

(C3 Summer 2008)

5. *(b)* Show that the equation

$$9x^3 - 9x + 1 = 0$$

has a root α between 0 and 0·2.

The recurrence relation

$$x_{n+1} = x_n^3 + \frac{1}{9}$$

with $x_0 = 0{\cdot}1$ can be used to find α. Find and record the values of x_1, x_2, x_3.

Write down the value of x_3 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

(C3 Winter 2009)

4. *(a)* By sketching the graphs of $y = x^3$ and $y = 4 - x$, determine the number of real roots of the equation $x^3 + x - 4 = 0$. [3]

(b) **You may assume** that the equation $x^3 + x - 4 = 0$ has a root α between 1 and 2. The recurrence relation

$$x_{n+1} = (4 - x_n)^{\frac{1}{3}}$$

with $x_0 = 1{\cdot}4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [5]

(C3 Summer 2009)

4. *(b)* Show that the equation

$$(x-1)e^{2x} - 1 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$x_{n+1} = 1 + e^{-2x_n}$$

with $x_0 = 1{\cdot}1$ may be used to find α. Find and record the values of x_1, x_2, x_3. Write down the value of x_3 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

(C3 Winter 2010)

4. Show that the equation

$$2 - 10x + \sin x = 0$$

has a root α between 0 and $\frac{\pi}{8}$.

The recurrence relation

$$x_{n+1} = \frac{1}{10}\left(2 + \sin x_n\right),$$

with $x_0 = 0{\cdot}2$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

(C3 Summer 2010)

4. Show that the equation

$$4x^3 - 2x - 5 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$x_{n+1} = \left(\frac{2x_n + 5}{4}\right)^{\frac{1}{3}},$$

with $x_0 = 1{\cdot}2$, may be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. [7]

(C3 Winter 2011)

4. **You may assume** that the equation $6x^4 + 7x - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \frac{3 - 6x_n^4}{7}$$

with $x_0 = 0{\cdot}4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.
Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. [5]

(C3 Summer 2011)

4. *(a)* Show that $f(x) = 11\tan^{-1} 2x - 3x^2$ has a stationary value when x satisfies

$$12x^3 + 3x - 11 = 0\ .$$ [3]

(b) **You may assume** that the equation $12x^3 + 3x - 11 = 0$ has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \left(\frac{11 - 3x_n}{12}\right)^{\frac{1}{3}}$$

with $x_0 = 0{\cdot}9$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

(C3 Winter 2012)

3. *(b)* Show that the equation

$$2t^4 - 4t - 7 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$t_{n+1} = \left(\frac{4t_n + 7}{2}\right)^{\frac{1}{4}}$$

with $t_0 = 1{\cdot}6$ can be used to find α. Find and record the values of t_1, t_2, t_3, t_4. Write down the value of t_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

(C3 Summer 2012)

4. Show that the equation

$$\cos x - 5x + 2 = 0$$

has a root α between 0 and $\frac{\pi}{4}$.

The recurrence relation

$$x_{n+1} = \frac{1}{5}(2 + \cos x_n)$$

with $x_0 = 0{\cdot}6$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

(C3 Winter 2013)

4. *(a)* On the same diagram, sketch the graphs of $y = \ln x$ and $y = 11 - 2x$.
Deduce the number of roots of the equation

$$\ln x + 2x - 11 = 0.$$ [3]

(b) **You may assume** that the equation

$$\ln x + 2x - 11 = 0$$

has a root α between 4 and 5.
The recurrence relation

$$x_{n+1} = \frac{11 - \ln x_n}{2},$$

with $x_0 = 4{\cdot}7$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

(C3 Summer 2013)

8. **You may assume** that the equation

$$x^2 + e^x - 3 = 0$$

has a root α between -2 and -1.

The recurrence relation

$$x_{n+1} = -\left(3 - e^{x_n}\right)^{\frac{1}{2}}$$

with $x_0 = -1{\cdot}5$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

(C3 Winter 2014)

5. **You may assume** that the equation $x^3 + 7x^2 - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \sqrt{\frac{3}{x_n + 7}}$$

with $x_0 = 1$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.

Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

(C3 Summer 2014)

5. *(a)* Show that $f(x) = \ln(3x^2 - 2x - 1) - 4x^2$ has a stationary value when x satisfies

$$12x^3 - 8x^2 - 7x + 1 = 0.$$ [4]

(b) **You may assume** that the equation $12x^3 - 8x^2 - 7x + 1 = 0$ has a root α between -1 and 0.
The recurrence relation

$$x_{n+1} = \left(\frac{8x_n^2 + 7x_n - 1}{12}\right)^{\frac{1}{3}}$$

with $x_0 = -0{\cdot}6$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. [5]

(C3 Summer 2015)

5. *(a)* On the same diagram, sketch the graphs of $y = \cos^{-1} x$ and $y = 5x - 1$. [2]

(b) **You may assume** that the equation

$$\cos^{-1} x - 5x + 1 = 0$$

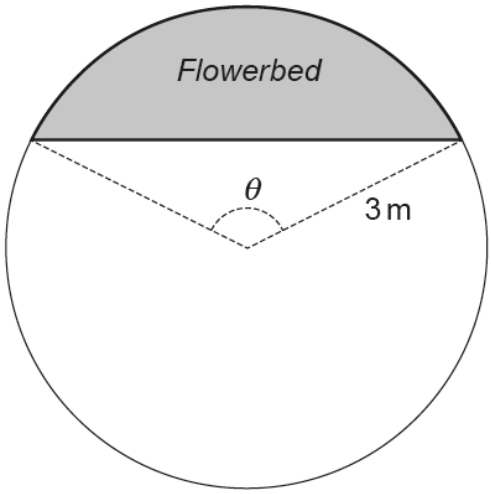
has a root α between $0{\cdot}4$ and $0{\cdot}5$.
The recurrence relation

$$x_{n+1} = \frac{1}{5}\left(1 + \cos^{-1} x_n\right)$$

with $x_0 = 0{\cdot}4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and prove that this is the value of α correct to four decimal places. [5]

(C3 Summer 2016)

5. The diagram shows a circular garden plot of radius 3 m. Alun wants to use a minor segment of the plot as a flowerbed and has a 13·5 m length of edging, all of which he intends to use to form the perimeter of the shaded area below. The angle subtended at the centre of the circular plot is denoted by θ radians.



(a) Show that θ satisfies the equation

$$\theta + 2\sin\left(\frac{\theta}{2}\right) = 4{\cdot}5\,.$$ [3]

(b) Alun believes that the value of θ will turn out to be approximately 2·5.
Starting with θ_0 = 2·5, use the recurrence relation

$$\theta_{n+1} = 4{\cdot}5 - 2\sin\left(\frac{\theta_n}{2}\right)$$

to find the values of θ_1, θ_2, θ_3. Write down the value of θ_3 correct to two decimal places and prove that this is the value of θ correct to two decimal places. [5]

(C3 Summer 2017)

4. A large tank in the form of a cuboid is used to store water. The width of the tank is denoted by x m. The length of the tank is 4 m **greater** than its width, whilst the height of the tank is 2 m **less** than its width. The volume of the tank is 150 m^3.

(a) (i) Show that $x^3 + 2x^2 - 8x - 150 = 0$.

(ii) Show that $5 < x < 6$. [4]

(b) The recurrence relation

$$x_{n+1} = \left(150 + 8x_n - 2x_n^2\right)^{\frac{1}{3}},$$

with x_0 = 6, can be used to find the value of x. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to two decimal places and prove that this is the value of x correct to two decimal places. [5]

(C3 Summer 2018)

5. *(a)* Show that $f(x) = (2x - 5)e^{2x} + 12x + 7$ has a stationary value when x satisfies

$$(x - 2)e^{2x} + 3 = 0.$$ [4]

(b) **You may assume** that the equation

$$(x - 2)e^{2x} + 3 = 0$$

has a root α between 1 and 2.
The recurrence relation

$$x_{n+1} = 2 - 3e^{-2x_n}$$

with $x_0 = 2$ may be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and prove that this is the value of α correct to four decimal places. [5]