Cyfres Rifyddol

Arithmetic Series

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Mae cyfres o rifau yn **gyfres rifyddol** os ydym yn adio'r un cysonyn i gael y rhif nesaf yn y gyfres.
*A sequence of numbers is an **arithmetic sequence** if we add the same constant to obtain the next number in the sequence.*

Er enghraifft, mae'r gyfres 5, 8, 11, 14, 17, ... yn gyfres rifyddol ble mae'r term cyntaf yn 5 ac mae'r gwahaniaeth cyffredin yn 3.
For example, the sequence 5, 8, 11, 14, 17, ... is an arithmetic sequence where the first term is 5 and the common difference is 3.

Terminoleg / *Terminology*

a	Term cyntaf y gyfres	*First term of the sequence*
d	Y gwahaniaeth cyffredin	*The common difference*
l	Term olaf y gyfres, os oes un yn bodoli	*Last term of the sequence, if one exists*
t_n	$n$fed term y gyfres	*The nth term of the sequence*

Cyfres Rifyddol
Arithmetic Series

Ar gyfer cyfres rifyddol efo term cyntaf a a gwahanaieth cyffredin d:
For an arithmetic sequence with first term a and common difference d:

Term cyntaf / *First term*	$t_1 = a$
Ail derm / *Second term*	$t_2 = a + d$
Trydydd term / *Third term*	$t_3 = a + 2d$
Pedwerydd term / *Fourth term*	$t_4 = a + 3d$
Nfed term / *Nth term*	$t_n = a + (n - 1)d$

Ar gyfer pob cyfanrif n / *For all integers n*,

$$d = t_{n+1} - t_n$$

Cyfres Rifyddol
Arithmetic Series

Mae'n bosib ystyried cyfanswm S_n yr n term cyntaf mewn cyfres rifyddol.
The arithmetic series S_n is the sum of the first n terms of an arithmetic sequence.

Ar gyfer y gyfres rifyddol 5, 8, 11, 14, 17, ... / *For the arithmetic sequence 5, 8, 11, 14, 17, ...*

$$S_3 = 5 + 8 + 11 = 24$$
$$S_6 = 5 + 8 + 11 + 14 + 17 + 20 = 75$$

Swm n term cyntaf cyfres rifyddol

Profwch mai swm n term cyntaf cyfres rifyddol efo term cyntaf a a gwahaniaeth cyffredin d yw $S_n = \frac{1}{2}n(a + l)$.

Prawf

Gadewch i ni ysgrifennu termau'r gyfres mewn trefn ac yna mewn trefn gwrthdro.

$$S_n = t_1 + t_2 + t_3 + \cdots + t_{n-1} + t_n$$
$$S_n = t_n + t_{n-1} + t_{n-2} + \cdots + t_2 + t_1$$

Trwy adio'r uchod cawn

$$S_n + S_n = (t_1 + t_n) + (t_2 + t_{n-1}) + (t_3 + t_{n-2}) + \cdots + (t_{n-1} + t_2) + (t_n + t_1).$$

Mae pob un o'r mynegiadau mewn cromfachau yr un peth â $t_1 + t_n$. Er enghraifft, mae $t_2 + t_{n-1} = (t_1 + d) + (t_n - d) = t_1 + t_n$ ac mae $t_3 + t_{n-2} = (t_1 + 2d) + (t_n - 2d) = t_1 + t_n$.

Felly mae $$2S_n = \underbrace{(t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n) + \cdots + (t_1 + t_n)}_{n \text{ gwaith}}$$

$$2S_n = n(t_1 + t_n)$$
$$S_n = \frac{1}{2}n(t_1 + t_n).$$

Ond t_1 yw term cyntaf a y gyfres rifyddol a t_n yw term olaf l y gyfres rifyddol, felly $S_n = \frac{1}{2}n(a + l)$. **QED**

Swm n term cyntaf cyfres rifyddol

Profwch mai swm n term cyntaf cyfres rifyddol efo term cyntaf a a gwahaniaeth cyffredin d yw $S_n = \frac{1}{2}n(2a + (n-1)d)$.

Prawf

Gadewch i ni ysgrifennu termau'r gyfres mewn trefn ac yna mewn trefn gwrthdro.

$$S_n = t_1 + t_2 + t_3 + \cdots + t_{n-1} + t_n$$

$$S_n = t_n + t_{n-1} + t_{n-2} + \cdots + t_2 + t_1$$

Trwy adio'r uchod cawn

$$S_n + S_n = (t_1 + t_n) + (t_2 + t_{n-1}) + (t_3 + t_{n-2}) + \cdots + (t_{n-1} + t_2) + (t_n + t_1).$$

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$$2S_n = n(t_1 + t_n)$$

$S_n = \frac{1}{2}n(t_1 + t_n)$. Ond $n$fed term cyfres rifyddol yw $t_n = a + (n-1)d$, felly

$$S_n = \frac{1}{2}n\big(a + (a + (n-1)d)\big)$$

$$S_n = \frac{1}{2}n(2a + (n-1)d).$$

QED

The sum of the first n terms of an arithmetic sequence

Prove that the sum of the first n terms of an arithmetic sequence with first term a and common difference d is given by $S_n = \frac{1}{2}n(a + l)$.

Proof

Let us write the terms of the arithmetic series in order and then in reverse order.

$$S_n = t_1 + t_2 + t_3 + \cdots + t_{n-1} + t_n$$
$$S_n = t_n + t_{n-1} + t_{n-2} + \cdots + t_2 + t_1$$

By adding the above we obtain

$$S_n + S_n = (t_1 + t_n) + (t_2 + t_{n-1}) + (t_3 + t_{n-2}) + \cdots + (t_{n-1} + t_2) + (t_n + t_1).$$

Each of the expressions in brackets is equivalent to $t_1 + t_n$. For example,
$t_2 + t_{n-1} = (t_1 + d) + (t_n - d) = t_1 + t_n$ and
$t_3 + t_{n-2} = (t_1 + 2d) + (t_n - 2d) = t_1 + t_n$.

Therefore $$2S_n = \underbrace{(t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n) + \cdots + (t_1 + t_n)}_{n \text{ times}}$$

$$2S_n = n(t_1 + t_n)$$
$$S_n = \frac{1}{2}n(t_1 + t_n).$$

But t_1 is the first term a of the arithmetic sequence and t_n is the last term l of the arithmetic sequence, so
$S_n = \frac{1}{2}n(a + l)$.

QED

The sum of the first n terms of an arithmetic sequence

Prove that the sum of the first n terms of an arithmetic sequence with first term a and common difference d is given by $S_n = \frac{1}{2}n(2a + (n-1)d)$.

Proof

Let us write the terms of the arithmetic series in order and then in reverse order.

$$S_n = t_1 + t_2 + t_3 + \cdots + t_{n-1} + t_n$$
$$S_n = t_n + t_{n-1} + t_{n-2} + \cdots + t_2 + t_1$$

By adding the above we obtain

$$S_n + S_n = (t_1 + t_n) + (t_2 + t_{n-1}) + (t_3 + t_{n-2}) + \cdots + (t_{n-1} + t_2) + (t_n + t_1).$$

Each of the expressions in brackets is equivalent to $t_1 + t_n$. For example,
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Therefore $$2S_n = \underbrace{(t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n) + \cdots + (t_1 + t_n)}_{n \text{ times}}$$

$$2S_n = n(t_1 + t_n)$$

$S_n = \frac{1}{2}n(t_1 + t_n)$. But the nth term of an arithmetic sequence is $t_n = a + (n-1)d$, so

$$S_n = \frac{1}{2}n\big(a + (a + (n-1)d)\big)$$

$$S_n = \frac{1}{2}n(2a + (n-1)d).$$

QED