



**GCE**

**FURTHER MATHEMATICS**

**UNIT 5: FURTHER STATISTICS B**

**SAMPLE ASSESSMENT MATERIALS**

**(1 hour 45 minutes)**

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Alun does the crossword in the Daily Bugle every day. The time that he takes to complete the crossword,  $X$  minutes, is modelled by the normal distribution  $N(32, 4^2)$ . You may assume that the times taken to complete the crossword on successive days are independent.

- (a) (i) Find the upper quartile of  $X$  and explain its meaning in context.
- (ii) Find the probability that the total time taken by Alun to complete the crosswords on five randomly chosen days is greater than 170 minutes. [7]
- (b) Belle also does the crossword every day and the time that she takes to complete the crossword,  $Y$  minutes, is modelled by the normal distribution  $N(18, 2^2)$ . Find the probability that, on a randomly chosen day, the time taken by Alun to complete the crossword is more than twice the time taken by Belle to complete the crossword. [6]

2. A factory manufactures a certain type of string. In order to ensure the quality of the product, a random sample of 10 pieces of string is taken every morning and the breaking strength of each piece, in Newtons, is measured. One morning, the results are as follows.

68.1 70.4 68.6 67.7 71.3 67.6 68.9 70.2 68.4 69.8

You may assume that this is a random sample from a normal distribution with unknown mean  $\mu$  and unknown variance  $\sigma^2$ .

- (a) Determine a 95% confidence interval for  $\mu$ . [9]
- (b) The factory manager is given these results and he asks 'Can I assume that the confidence interval that you have given me contains  $\mu$  with probability 0.95?' Explain why the answer to this question is no and give a correct interpretation. [2]

3. A motoring organisation wishes to determine whether or not the petrol consumption of two different car models A and B are the same. A trial is therefore carried out in which 6 cars of each model are given 10 litres of petrol and driven at a predetermined speed around a track until the petrol is used up. The distances travelled, in miles, are shown below

Model A: 86.3 84.2 85.8 83.1 84.7 85.3

Model B: 84.9 85.9 84.8 86.5 85.2 85.5

It is proposed to use a test with significance level 5% based on the Mann-Whitney statistic  $U$ .

- (a) State suitable hypotheses. [2]
- (b) Find the critical region for the test. [3]
- (c) Determine the value of  $U$  for the above data and state your conclusion in context. You must justify your answer. [4]
4. (a) In an opinion poll of 1800 people, 1242 said that they preferred red wine to white wine. Calculate a 95% confidence interval for the proportion of people in the population who prefer red wine to white wine. [6]

- (b) In another opinion poll of 1000 people on the same subject, the following confidence interval was calculated.

[0.672, 0.732].

Determine

- (i) the number of people in the sample who stated that they prefer red wine to white wine,
- (ii) the confidence level of the confidence interval, giving your answer as a percentage correct to three significant figures. [6]

5. A new species of animal has been found on an uninhabited island. A zoologist wishes to investigate whether or not there is a difference in the mean weights of males and females of the species. She traps some of the animals and weighs them with the following results.

Males (kg) 5.3, 4.6, 5.2, 4.5, 4.3, 5.5, 5.0, 4.8  
 Females (kg) 4.9, 5.0, 4.1, 4.6, 4.3, 5.3, 4.2, 4.5, 4.8, 4.9

You may assume that these are random samples from normal populations with a common standard deviation of 0.5 kg.

- (a) State suitable hypotheses for this investigation. [1]
- (b) Determine the  $p$ -value of these results and state your conclusion in context. [9]
6. A medical student is investigating two different methods, A and B, of measuring a patient's blood pressure. He believes that Method B gives, on average, a higher reading than Method A so he defines the following hypotheses.

$H_0$ : There is on average no difference in the readings obtained using Methods A and B;

$H_1$ : The reading obtained using Method B is on average higher than the reading obtained using Method A.

He selects 10 patients at random and he measures their blood pressures using both methods. He obtains the following results.

Patient	A	B	C	D	E	F	G	H	I	J
Method A	121	133	119	142	151	139	161	148	151	125
Method B	126	131	127	152	145	151	157	155	160	126

- (a) Carry out an appropriate Wilcoxon signed rank test on this data set, using a 5% significance level. [6]
- (b) State what conclusion the medical student should reach, justifying your answer. [2]

7. The discrete random variable  $X$  has the following probability distribution, where  $\theta$  is an unknown parameter belonging to the interval  $\left(0, \frac{1}{3}\right)$ .

Value of $X$	1	3	5
Probability	$\theta$	$1 - 3\theta$	$2\theta$

- (a) Obtain an expression for  $E(X)$  in terms of  $\theta$  and show that

$$\text{Var}(X) = 4\theta(3 - \theta). \quad [4]$$

In order to estimate the value of  $\theta$ , a random sample of  $n$  observations on  $X$  was obtained and  $\bar{X}$  denotes the sample mean.

- (b) (i) Show that

$$V = \frac{\bar{X} - 3}{2}$$

is an unbiased estimator for  $\theta$ .

- (ii) Find an expression for the variance of  $V$ . [4]

- (c) Let  $Y$  denote the number of observations in the random sample that are equal to 1.

Show that

$$W = \frac{Y}{n}$$

is an unbiased estimator for  $\theta$  and find an expression for  $\text{Var}(W)$ . [5]

- (d) Determine which of  $V$  and  $W$  is the better estimator, explaining your method clearly. [4]