# 2.4 A2 UNIT 4

**Unit 4: Further Pure Mathematics B**
Written examination: 2 hours 30 minutes
35% of A level qualification
120 marks

This unit is **compulsory**.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in A level Mathematics.

| Topics | Guidance |
|---|---|
| **2.4.1 Complex Numbers** | |
| Understand de Moivre's theorem and use it to find multiple angle formulae and sums of series. | To include proof by induction of de Moivre's Theorem for positive integer values of $n$.<br><br>For example, showing that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ and $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$. |
| Know and use the definition $\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$ and the form $z = r\mathrm{e}^{\mathrm{i}\theta}$. | |
| Find the $n$ distinct $n$th roots for $r\mathrm{e}^{\mathrm{i}\theta}$ for $r \neq 0$ and know that they form the vertices of a regular $n$-gon in the Argand diagram. | |
| Use complex roots of unity to solve geometric problems. | |

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| **2.4.2 Further Trigonometry** | |
| Solve trigonometric equations.<br><br>Use the formulae for $\sin A \pm \sin B$, $\cos A \pm \cos B$ and for $\sin x$, $\cos x$ and $\tan x$ in terms of $t$, where $t = \tan\frac{1}{2}x$.<br><br>Find the general solution of trigonometric equations. | Questions aimed solely at proving identities will not be set.<br><br>For example, $\cos\theta + \cos 2\theta + \cos 3\theta = 0$ and $2\sin x - \tan\frac{1}{2}x = 0$. |
| **2.4.3 Matrices** | |
| Calculate determinants up to 3 x 3 matrices and interpret as scale factors, including the effect on orientation. | |
| Calculate and use the inverse of non-singular 3 x 3 matrices. | Including knowledge of the term adjugate matrix. |
| Solve three linear simultaneous equations in three variables by use of the inverse matrix and by reduction to echelon form.<br><br>Understand and use the determinantal condition for the solution of simultaneous equations which have a unique solution. | To include equations which<br>(a) have a unique solution,<br>(b) have non-unique solutions,<br>(c) are not consistent. |
| Interpret geometrically the solution and failure of three simultaneous linear equations. | |

| Topics | Guidance |
|---|---|
| **2.4.4 Further Algebra and Functions** | |
| Find the Maclaurin series of a function (including the general term) | |
| Recognise and use the Maclaurin series for $e^x$, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$, and be aware of the range of values of $x$ for which they are valid. | Proof not required. |
| Understand and use partial fractions with denominators of the form $(ax+b)(cx^2+d)$. | |
| **2.4.5 Further Calculus** | |
| Evaluate improper integrals, where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity. | |
| Derive formulae for and calculate volumes of revolution. | Rotation may be about the $x$-axis or the $y$-axis. |
| Understand and evaluate the mean value of a function. | Mean value of a function $f(x) = \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x$ |
| Integrate using partial fractions (extend to quadratic factors $(ax^2+c)$ in the denominator). | |
| Differentiate inverse trigonometric functions. | |
| Integrate functions of the form $\frac{1}{\sqrt{a^2-x^2}}$ and $\frac{1}{a^2+x^2}$ and be able to choose trigonometric substitutions to integrate associated functions. | |

| Topics | Guidance |
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| **2.4.6 Polar Coordinates** | |
| Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates. | Where $r \geq 0$ and the value of $\theta$ may be taken to be in either $[0, 2\pi)$ or $(-\pi, \pi]$. |
| Sketch curves with $r$ given as a function of $\theta$, including the use of trigonometric functions. | Candidates will be expected to sketch simple curves such as $r = a(b + c\cos\theta)$ and $r = a\cos n\theta$.<br><br>Includes the location of points at which tangents are parallel to, or perpendicular to, the initial line. |
| Find the area enclosed by a polar curve. | Excludes the intersection of curves. |
| **2.4.7 Hyperbolic functions** | |
| Understand the definitions of hyperbolic functions, $\sinh x$, $\cosh x$ and $\tanh x$, including their domains and ranges, and be able to sketch their graphs. | $\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$, $\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$<br>$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)}$<br>Know and use the formulae for $\sinh(A \pm B)$, $\cosh(A \pm B)$, $\tanh(A \pm B)$, $\sinh 2A$, $\cosh 2A$ and $\tanh 2A$.<br>Knowledge and use of the identity $\cosh^2 A - \sinh^2 A \equiv 1$ and its equivalents. |
| Differentiate and integrate hyperbolic functions. | eg. Differentiate $\sinh 2x$, $x\cosh^2 x$ |
| Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges. | $\sinh^{-1} x = \ln\left[x + \sqrt{x^2 + 1}\right]$<br>$\cosh^{-1} x = \ln\left[x + \sqrt{x^2 - 1}\right], \quad x \geq 1$<br>$\tanh^{-1} x = \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right], \quad -1 < x < 1$ |

| Topics | Guidance |
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| Derive and use the logarithmic forms of the inverse hyperbolic functions. | |
| Integrate functions of the form $\frac{1}{\sqrt{x^2+a^2}}$ and $\frac{1}{\sqrt{x^2-a^2}}$, and be able to choose substitutions to integrate associated functions. | |
| **2.4.8 Differential equations** | |
| Find and use an integrating factor to solve differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}+P(x)y=Q(x)$ and recognise when it is appropriate to do so. | |
| Find both general and particular solutions to differential equations. | |
| Use differential equations in modelling in a variety of contexts. | Contexts will **not** include mechanics contexts. |
| Solve differential equations of the form $y''+ay'+by=0$, where $a$ and $b$ are constants, by using the auxiliary equation. | |
| Solve differential equations of the form $y''+ay'+by=f(x)$, where $a$ and $b$ are constants, by solving the homogenous case and adding a particular integral to the complementary function (in cases where $f(x)$ is a polynomial, exponential or trigonometric function). | $f(x)$ will have one of the forms $A+Bx$, $cx^2+dx+e$, $k\mathrm{e}^{qx}$ or $m\cos\omega x+n\sin\omega x$. |
| Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation. | |

| Topics | Guidance |
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| Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled 1st order simultaneous equations and be able to solve them. | For example, predator-prey models.<br><br>Restricted to first order differential equations of the form<br>$\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by + f(t)$<br>$\frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy + g(t)$ |