2.3 A2 UNIT 3

Unit 3: Pure Mathematics B
Written examination : 2 hours 30 minutes
35% of A level qualification
120 marks

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Unit 1.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Topics	Guidance
2.3.1 Proof	
Proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs).	
2.3.2 Algebra and Functions	
Simplify rational expressions, including by factorising and cancelling and by algebraic division (by linear expressions only).	
Sketch curves defined by the modulus of a linear function.	Be able sketch graphs of the form $y = \|ax + b\|$. To include solving equations involving the modulus function.
Understand and use composite functions; inverse functions and their graphs.	Understand and use the definition of a function. Understand and use the domain and range of functions. In the case of a function defined by a formula (with unspecified domain) the domain is taken to be the largest set such that the formula gives a unique image for each element of the set. The notation fg will be used for composition.

Topics	Guidance
Understand the effect of combinations of transformations on the graph of $y = f(x)$, as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$ and $y = f(ax)$.	
Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).	With denominators of the form $(ax + b)(cx + d)$, $(ax+b)(cx+d)(ex+f)$ and $(ax + b)(cx + d)^2$. Learners will **not** be expected to sketch the graphs of rational functions.
Use of functions in modelling, including consideration of limitations and refinements of the models.	
2.3.3 Coordinate geometry in the (x, y) plane	
Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.	To include finding the equations of tangents and normals to curves defined parametrically or implicitly. Knowledge of the properties of curves other than the circle will **not** be expected.
Use parametric equations in modelling in a variety of contexts.	

Topics	Guidance
2.3.4 Sequences and Series	
Understand and use the binomial expansion of $(a+bx)^n$, for any rational n, including its use for approximation. Be aware that the expansion is valid for $\left\|\frac{bx}{a}\right\|<1$ (proof not required).	To include the expansion, in ascending powers of x, of expressions such as $(2-x)^{\frac{1}{2}}$ and $\frac{(4-x)^{\frac{3}{2}}}{(1+2x)}$.
Work with sequences, including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$. Increasing sequences, decreasing sequences, periodic sequences.	
Understand and use sigma notation for sums of series.	
Understand and work with arithmetic sequences and series, including the formulae for the nth term and the sum to n terms.	Use of $u_n = a+(n-1)d$. Use and proof of $S_n = \frac{n}{2}[2a+(n-1)d]$ and $S_n = \frac{n}{2}[a+l]$.
Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series. The sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation.	Use of $u_n = ar^{n-1}$. Use and proof of $S_n = \frac{a(1-r^n)}{1-r}$. Use of $S_\infty = \frac{a}{1-r}$ for $\|r\| < 1$.
Use sequences and series in modelling.	

Topics	Guidance
2.3.5 Trigonometry	
Work with radian measure, including use for arc length, area of sector and area of segment.	
Understand and use the standard small angle approximations of sine, cosine and tangent. $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$ and $\tan\theta \approx \theta$, where θ is in radians.	
Know and use exact values of $\sin$ and $\cos$ for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof.	
Understand and use the definitions of sec, cosec, cot, $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. Understand the relationships of all of these to sin, cos and tan and understand their graphs, ranges and domains.	
Understand and use $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$.	The solution of trigonometric equations such as $\sec^2\theta + 5 = 5\tan\theta$.
Understand and use double angle formulae. Use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$. Understand geometric proofs of these formulae.	Use of these formulae to solve equations in a given range, e.g. $\sin 2\theta = \sin\theta$, Applications to integration, e.g. $\int \cos^2 x \,dx$.

Topics	Guidance
Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$.	Use of these to solve equations in a given range, e.g. $3\cos\theta + \sin\theta = 2$. Application to finding greatest and least values, e.g. the least value of $\dfrac{1}{3\cos\theta + 4\sin\theta + 10}$.
Construct proofs involving trigonometric functions and identities.	
2.3.6 Differentiation	
Differentiation from first principles for $\sin x$ and $\cos x$.	
Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves, and points of inflection.	Points of inflection to include stationary and non-stationary points.
Differentiate e^{kx}, a^{kx} , $\sin kx$, $\cos kx$, $\tan kx$, and related sums, differences and constant multiples. Understand and use the derivative of $\ln x$.	
Apply differentiation to find points of inflection.	
Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.	To include the use of $\dfrac{dy}{dx} = \dfrac{1}{\left(\dfrac{dx}{dy}\right)}$.
Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only.	
Construct simple differential equations in pure mathematics.	

2.3.7 Integration	
Integrate e^{kx}, $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples.	Use of the results: 1) if $\int f(x)dx = F(x) + k$ then $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c.$ 2) $\int f'(g(x))g'(x)dx = f(g(x)) + c$
Use a definite integral to find the area between two curves.	
Understand and use integration as the limit of a sum.	
Carry out simple cases of integration by substitution and integration by parts. Understand these methods as the reverse processes of the chain rule and the product rule respectively. Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated. Integration by parts includes more than one application of the method but excludes reduction formulae.	
Integrate using partial fractions that are linear in the denominator.	
Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions. (Separation of variables may require factorisation involving a common factor.)	Questions will be set in pure mathematics only.

2.3.8 Numerical Methods	
Locate roots of $f(x) = 0$ by considering changes in sign of $f(x)$ in an interval of x in which $f(x)$ is sufficiently well-behaved. Understand how change of sign methods can fail.	
Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams. Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$. Understand how such methods can fail.	The iterative formula will be given. Consideration of the conditions for convergence will **not** be required.
Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between.	Learners will be expected to use the trapezium rule to estimate the area under a curve and to determine whether it gives an overestimate or an underestimate of the area under a curve. Simpson's rule is excluded.
Use numerical methods to solve problems in context.	To solve problems in context which lead to equations that cannot be solved analytically.