GCE A LEVEL – NEW

1300U30-1

S18-1300U30-1

MATHEMATICS – A2 unit 3
PURE MATHEMATICS B

WEDNESDAY, 6 JUNE 2018 – MORNING

2 hours 30 minutes

1300U301
01

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Write only within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, e.g. | 0 | 1 | .

Leave at least two line spaces between each answer.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

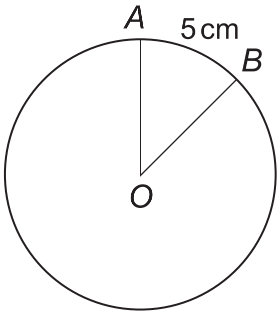
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: *Sufficient working must be shown to demonstrate the* ***mathematical*** *method employed.*

0 1 Solve the equation

$$|2x + 1| = 3|x - 2|. \qquad [4]$$

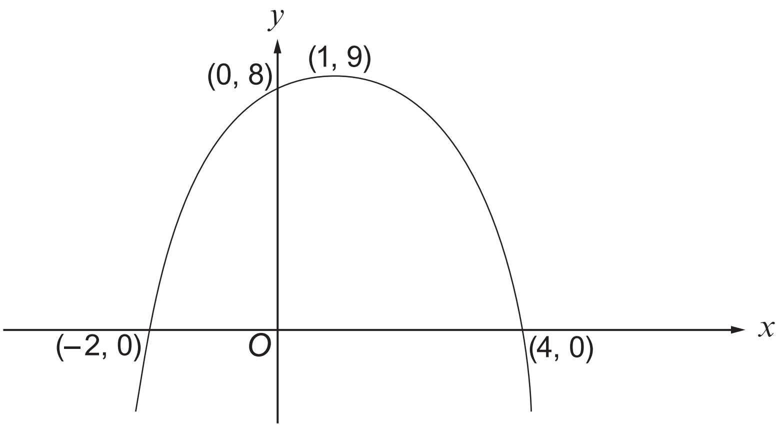
0 2 The diagram below shows a circle centre O, radius 4 cm. Points A and B lie on the circumference such that arc AB is 5 cm.



a) Calculate the angle subtended at O by the arc AB. [2]

b) Determine the area of the sector OAB. [2]

0 3 The diagram below shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-2, 0)$, $(0, 8)$, $(4, 0)$ and has a maximum point at $(1, 9)$.



a) Sketch the graph of $y = 2f(x + 3)$. Indicate the coordinates of the stationary point and the points where the graph crosses the x-axis. [3]

b) Sketch the graph of $y = 5 - f(x)$. Indicate the coordinates of the stationary point and the point where the graph crosses the y-axis. [3]

0 4 Solve the equation

$$2\tan^2\theta + 2\tan\theta - \sec^2\theta = 2,$$

for values of θ between 0° and 360°. [5]

0 5 **a)** Show that

$$\frac{3x}{(x-1)(x-4)^2} \equiv \frac{A}{(x-1)} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2},$$

where A, B and C are constants to be found. [3]

b) Evaluate $\int_5^7 \frac{3x}{(x-1)(x-4)^2}\,dx$, giving your answer correct to 3 decimal places. [5]

0 6 Write down the first three terms in the binomial expansion of $(1-4x)^{-\frac{1}{2}}$ in ascending powers of x. State the range of values of x for which the expansion is valid. By writing $x = \frac{1}{13}$ in your expansion, find an approximate value for $\sqrt{13}$ in the form $\frac{a}{b}$, where a, b are integers. [5]

0 7 Use small angle approximations to find the small negative root of the equation

$$\sin x + \cos x = 0{\cdot}5.$$ [3]

0 8 Find seven numbers which are in arithmetic progression such that the last term is 71 and the sum of all of the numbers is 329. [5]

0 9 **a)** Explain why the sum to infinity of a geometric series with common ratio r only converges when $|r| < 1$. [1]

b) A geometric progression V has first term 2 and common ratio r. Another progression W is formed by squaring each term in V. Show that W is also a geometric progression. Given that the sum to infinity of W is 3 times the sum to infinity of V, find the value of r. [6]

c) At the beginning of each year, a man invests £5000 in a savings account earning compound interest at the rate of 3% per annum. The interest is added at the end of each year. Find the total amount of his savings at the end of the 20th year correct to the nearest pound. [3]

1300U301
03

TURN OVER

1 0 The equation of a curve *C* is given by the parametric equations

$$x = \cos 2\theta, \ \ y = \cos\theta.$$

a) Find the Cartesian equation of *C*. [2]

b) Show that the line $x - y + 1 = 0$ meets *C* at the point *P*, where $\theta = \frac{\pi}{3}$, and at the point Q, where $\theta = \frac{\pi}{2}$. Write down the coordinates of *P* and Q. [5]

c) Determine the equations of the tangents to *C* at *P* and *Q*. Write down the coordinates of the point of intersection of the two tangents. [7]

1 1 Prove by contradiction that, for every real number x such that $0 \leqslant x \leqslant \frac{\pi}{2}$,

$$\sin x + \cos x \geqslant 1.$$ [4]

1 2 **a)** Given that f is a function,

i) state the condition for f^{-1} to exist,

ii) find $ff^{-1}(x)$. [2]

b) The functions g and h, are given by

$$g(x) = x^2 - 1,$$

$$h(x) = e^x + 1.$$

i) Suggest a domain for g such that g^{-1} exists.

ii) Given the domain of h is $(-\infty, \infty)$, find an expression for $h^{-1}(x)$ and sketch, using the same axes, the graphs of $h(x)$ and $h^{-1}(x)$. Indicate clearly the asymptotes and the points where the graphs cross the coordinate axes.

iii) Determine an expression for $gh(x)$ in its simplest form. [8]

1 3 **a)** Express $8\sin\theta - 15\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$. [3]

b) Find all values of θ in the range $0° < \theta < 360°$ satisfying

$$8\sin\theta - 15\cos\theta - 7 = 0.$$ [3]

c) Determine the greatest value and the least value of the expression

$$\frac{1}{8\sin\theta - 15\cos\theta + 23}.$$ [2]

14 Evaluate

a) $\int_1^2 x^3 \ln x \, dx$. [6]

b) $\int_0^1 \frac{2+x}{\sqrt{4-x^2}} \, dx$. [6]

15 The variable y satisfies the differential equation

$$2\frac{dy}{dx} = 5 - 2y, \qquad \text{where } x \geqslant 0.$$

Given that $y = 1$ when $x = 0$, find an expression for y in terms of x. [5]

16 **a)** Differentiate the following functions with respect to x, simplifying your answer wherever possible.

i) $e^{3\tan x}$,

ii) $\frac{\sin 2x}{x^2}$. [5]

b) A function is defined implicitly by

$$3x^2y + y^2 - 5x = 5.$$

Find the equation of the normal at the point (1, 2). [6]

17 By drawing suitable graphs, show that $x - 1 = \cos x$ has only one root. Starting with $x_0 = 1$, use the Newton-Raphson method to find the value of this root correct to two decimal places. [6]

END OF PAPER

BLANK PAGE

BLANK PAGE