WELSH JOINT EDUCATION COMMITTEE CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education **Tystysgrif Addysg Gyffredinol**

Advanced Level/Advanced Subsidiary **Safon Uwch/Uwch Gyfrannol**

MATHEMATICS M3

Mechanics

Specimen Paper 2005/2006

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9·8 ms^{-2}.

INFORMATION FOR CANDIDATES

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle, of mass 4 kg, is projected vertically upwards with initial speed 6 ms^{-1} from a point *A*. The air resistance is of magnitude $\frac{gv^2}{9}$ N, where v ms^{-1} is the speed of the particle at time t s.

(a) Show that when the particle is moving upward with speed v ms^{-1}, its retardation is of magnitude $\frac{g}{36}(36+v^2)$N. [3]

(b) Find the time taken for the particle to reach its maximum height. [7]

(c) Calculate the maximum height of the particle. [7]

2. A particle moves in a straight line with Simple Harmonic Motion with centre *O*. The amplitude of the motion is 5 m. When the particle is at *O*, its speed is 8 ms^{-1}.

(a) Show that the period of the motion is $\frac{5\pi}{4}$, and find the time taken to make 9 complete oscillations. [5]

(b) Determine the speed of the particle when it is at a distance of 4 m from *O*. [3]

(c) Find the magnitude of the acceleration of the particle when it is at a distance of 3 m from *O*. [3]

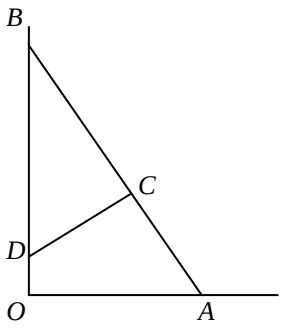
(d) Two points *A* and *B* are on different sides of *O* on the path of the particle. The point *A* is 2·4 m from *O* and the point *B* is 3·6 m from *O*. Calculate the shortest time for the particle to travel from *A* to *B*. [5]

3. A particle moves in a straight line such that at time t s, its displacement x m, from a fixed point *O*, satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+8\frac{\mathrm{d}x}{\mathrm{d}t}+12x=12t+20.$$

Given that when $t = 0$, $x = 0$ and the particle is moving with velocity 3 ms^{-1}, find its displacement at time $t = 2$ s. [14]

4. A uniform ladder *AB*, of length 10 m and mass 15 kg, stands on smooth horizontal ground and leans on a smooth vertical wall. It is kept in equilibrium by a light inextensible rope, attached at one end to the ladder at the point *C* and attached at the end to a point *D* on the wall. The rope *CD* is perpendicular to and in the same vertical plane as *AB*, as shown in the diagram.



The point *O* is the corner of the wall and the ground. Distances *OA* = 6 m, *OB* = 8 m and *AC* = 4 m.

(a) Calculate the magnitudes of the tension in the rope and the reactions of the wall and the ground on the ladder. [10]

(b) A man, of mass 80 kg, climbs the ladder. The rope will break when the tension exceeds 2000 N.

Determine whether the man will be able to reach the top of the ladder.

(c) State **one** modelling assumption which you have made in your solution. [1]

5. A light inextensible string, of length 1 m, connects particles *A* and *B*, of masses 2 kg and 5 kg respectively. Initially, the particles are lying on a smooth horizontal surface 0·6 m apart. Particle *B* is projected with a velocity of 7 ms^{-1} in a direction perpendicular to the line joining the initial positions of *A* and *B*.

(a) Determine the speeds of *A* and *B* immediately after the string becomes taut, and find the impulsive tension in the string. [9]

(b) Calculate the energy lost by the system when the string becomes taut. [4]