WELSH JOINT EDUCATION COMMITTEE CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education Tystysgrif Addysg Gyffredinol

Advanced Level/Advanced Subsidiary Safon Uwch/Uwch Gyfrannol

MATHEMATICS S3

Statistics

Specimen Paper 2005/2006

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Four cards numbered 1,2,3,3 respectively are placed in a bag. Two of these cards are chosen at random. Find the sampling distribution of the sum of the two numbers on the chosen cards when the sampling is done

(a) without replacement, [5]

(b) with replacement. [6]

2. A market research organisation is asked to estimate the proportion p of the electorate supporting a recent government initiative. It therefore questions a random sample of 1500 electors and finds that 930 of them support the initiative.

(a) Calculate an unbiased estimate of p. [1]

(b) Calculate, approximately, the standard error of your estimate. Explain briefly why you cannot calculate this standard error exactly. [3]

(c) Calculate an approximate 90 % confidence interval for p. [3]

(d) State, giving a reason, whether or not you are able to conclude, **definitely**, that the value of p exceeds 0·5. [2]

3. A farmer grows 200 tomato plants in his greenhouse, 120 of Variety A and 80 of Variety B. He records the weight x kg of tomatoes obtained from each plant of Variety A and the weight y kg of tomatoes obtained from each plant of Variety B. He does the following calculations at the end of the season:

<u>Variety A</u>

$$\text{Sample mean } (\bar{x}) = 2{\cdot}45$$
$$\text{Unbiased variance estimate } (\hat{\sigma}_x^2) = 0{\cdot}082$$

<u>Variety B</u>

$$\text{Sample mean } (\bar{y}) = 2{\cdot}51$$
$$\text{Unbiased variance estimate } (\hat{\sigma}_y^2) = 0{\cdot}094$$

The farmer wishes to investigate whether or not there is any difference in mean yield of tomatoes between the two varieties.

(a) State suitable hypotheses. [1]

(b) Calculate the p-value of these results and state your conclusion in context. [8]

4. *(a)* The weights (in kg) of the eight male babies born last month in a certain hospital were as follows:

3·3 4·6 3·2 3·9 3·4 4·2 3·8 4·0

Assuming that these weights can be regarded as a random sample from a $N(\mu, \sigma^2)$ distribution,

(i) calculate unbiased estimates of μ and σ^2, [5]

(ii) determine a 99% confidence interval for μ. [4]

(b) In an attempt to obtain a more accurate estimate of μ, the weights (x kg) of all babies born during the last year are now taken into account. It is found that there were 110 such births and that $\Sigma x = 431 \cdot 2$ and $\Sigma x^2 = 1712 \cdot 1$. Determine a 95% confidence interval based on these data. [6]

5. Jim is investigating the relationship between the length of a wire, y cm, and the temperature, x°C, of the wire. He obtains the following experimental results.

x	10	15	20	25	30	35	40	45
y	74·3	76·1	77·2	78·6	80·4	82·1	83·9	85·5

[You are given that Σy = 638·1; Σxy = 17882·5]

(a) Assuming a linear relationship $y = \alpha + \beta x$, calculate a and b, the least squares estimates of α and β. [5]

The values of x are exact whereas the measured values of y are subject to independent normally distributed errors with zero mean and standard deviation 0·2.

(b) Test the null hypothesis that β = 0·3 against a two-sided alternative, using a significance level of 5%. [7]

(c) (i) Use your values of a and b to estimate the true length of the wire when the temperature is 35°C. Determine the standard error of your estimate.

(ii) Hence find a 95% confidence interval for the true length of the wire when the temperature is 35°C. [6]

6. The continuous random variable X is uniformly distributed on the interval $[0, \theta]$, where θ is an unknown positive constant. In order to estimate θ, a random sample of 48 observations on X is taken and $\overline{X}$ denotes the mean of these observations.

(a) Given that

$$\hat{\theta} = 2\overline{X} \ ,$$

show that $\hat{\theta}$ is an unbiased estimator for θ and find its standard error. [7]

(b) (i) Using the Central Limit Theorem, write down an approximation to the distribution of $\hat{\theta} - \theta$.

(ii) Find, approximately, the probability that $\hat{\theta}$ differs from θ by more than $0{\cdot}05\theta$. [6]