WELSH JOINT EDUCATION COMMITTEE CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education Tystysgrif Addysg Gyffredinol

Advanced Level/Advanced Subsidiary Safon Uwch/Uwch Gyfrannol

MATHEMATICS FP2

Further Pure Mathematics

Specimen Paper 2005/2006

($1\frac{1}{2}$ hours)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function g is defined by

$$g(x) = x + bx^2 \quad \text{for } x \le 1,$$
$$g(x) = 2 + ax^3 \quad \text{for } x > 1.$$

Given that $g(x)$ and its derivative are continuous for all values of x, find the values of the constants a and b. [6]

2. Find all the values of θ in the interval [0°,180°] satisfying the equation

$$\sin\theta - \sin 3\theta + \sin 5\theta = 0.$$ [8]

3. Find the three cube roots of the complex number 3 – 2i. Give your answers in the form $x + \mathrm{i}y$, with x and y correct to two decimal places. [11]

4. The function f is defined on the domain $(1,\infty)$ by

$$f(x) = \frac{2x+1}{x-1}.$$

(a) Show that f is strictly decreasing. [3]

(b) State the range of f. [2]

(c) Given that S denotes the interval [3,4], determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [6]

5. Given that

$$z = \cos\theta + \mathrm{i}\sin\theta,$$

use de Moivre's Theorem to show that

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta.$$

Hence, by expanding $\left(z - \frac{1}{z}\right)^5$, show that

$$\sin^5\theta = \frac{1}{16}(a\sin 5\theta - b\sin 3\theta + c\sin\theta)$$

where a, b and c are integers to be found. [9]

6. A parabola has equation

$$y^2 + 4y - 8x + 12 = 0.$$

(a) Determine the coordinates of

(i) the vertex,

(ii) the focus. [4]

(b) (i) Verify that the point $P\left(2p^2 + 1, 4p - 2\right)$ lies on the parabola for all values of p.

(ii) Find the equation of the tangent to the parabola at P.

(iii) Hence show that the gradients of the two tangents from the origin to the parabola are

$$\frac{2}{1 \pm \sqrt{3}}.$$ [10]

7. *(a)* The function f is defined by

$$\mathrm{f}(x) = \frac{1}{(x+1)\left(x^2+4\right)} \quad (x \neq -1).$$

(i) Sketch the graph of f.

(ii) State the equations of all the asymptotes. [4]

(b) Express $\mathrm{f}(x)$ in partial fractions. [4]

(c) Hence evaluate the integral

$$\int_0^1 \mathrm{f}(x)\,\mathrm{d}x,$$

giving your answer correct to three significant figures. [8]