

**WELSH JOINT EDUCATION COMMITTEE CYD-BWYLLGOR ADDYSG CYMRU**

**General Certificate of Education**

**Tystysgrif Addysg Gyffredinol**

**Advanced Level/Advanced Subsidiary**

**Safon Uwch/Uwch Gyfrannol**

**MATHEMATICS FP1**

**Further Pure Mathematics**

**Specimen Paper 2005/2006**

**(1  $\frac{1}{2}$  hours)**

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

A calculator may be used for this paper.

A formula booklet is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that  $2 - i$  is a root of the equation

$$x^4 - 6x^3 + 18x^2 - 30x + 25 = 0,$$

find the other three roots of the equation. [8]

2. Differentiate  $\frac{1}{x^3}$  from first principles. [6]

3. The complex number  $z$  satisfies the equation

$$3z - \bar{z} = \frac{8 - 6i}{1 - 2i}.$$

Evaluate  $z$  in the form  $a + bi$ , where  $a, b$  are integers. [6]

4. The sum of the first  $n$  terms of a series is given by  $n^2 + 2n$ .

(a) Show that the  $n$ th term of the series is given by

$$T_n = 2n + 1. [3]$$

(b) Obtain an expression in terms of  $n$  for

$$\sum_{r=1}^n T_r^2,$$

simplifying your answer as far as possible. [6]

5. Use mathematical induction to prove that  $3^{2n} + 7$  is divisible by 8 for all positive integers  $n$ . [7]

6. The function  $f$  is defined by

$$f(x) = \frac{(x^3 + 1)^{1/2}}{(x^2 + 1)^{3/2}}$$

Using logarithmic differentiation, or otherwise, evaluate  $f'(2)$ . [7]

7. The roots of the quadratic equation

$$x^2 + 3x + 3 = 0$$

are denoted by  $\alpha, \beta$ . Find the cubic equation whose roots are  $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$  and  $\alpha\beta$ .

[12]

8. (a) Find the value of  $\lambda$  for which the following matrix is singular.

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ \lambda & 1 & 5 \end{bmatrix}$$

[3]

- (b) (i) Find the value of  $\mu$  for which the following system of equations is consistent.

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ \mu \end{bmatrix}$$

- (ii) For this value of  $\mu$ , find the general solution to this system of equations. [8]

9. The transformation  $T$  of the plane consists of a translation in which the point  $(x, y)$  is transformed to the point  $(x + 2, y - 1)$ , followed by an anti-clockwise rotation through  $\pi/2$  about the origin.

- (a) Find the  $3 \times 3$  matrix representing  $T$ . [5]

- (b) Find the equation of the image under  $T$  of the curve with equation  $y = x^2$ . [4]