



**GCE AS/A LEVEL – NEW**

2305U10-1



**FURTHER MATHEMATICS – AS unit 1**  
**FURTHER PURE MATHEMATICS A**

MONDAY, 14 MAY 2018 – AFTERNOON

1 hour 30 minutes

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**Reminder:** Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. The matrices **A** and **B** are such that  $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ -1 & -3 \end{bmatrix}$  and  $\mathbf{B} = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ .

(a) Explain why **B** has no inverse. [1]

(b) (i) Find the inverse of **A**. [3]

(ii) Hence, find the matrix **X**, where  $\mathbf{AX} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ . [2]

2. Prove, by mathematical induction, that  $\sum_{r=1}^n r(r+3) = \frac{1}{3}n(n+1)(n+5)$

for all positive integers  $n$ . [6]

3. A cubic equation has roots  $\alpha, \beta, \gamma$  such that

$$\alpha + \beta + \gamma = -9, \quad \alpha\beta + \beta\gamma + \gamma\alpha = 20, \quad \alpha\beta\gamma = 0.$$

(a) Find the values of  $\alpha, \beta$ , and  $\gamma$ . [4]

(b) Find the cubic equation with roots  $3\alpha, 3\beta, 3\gamma$ .  
Give your answer in the form  $ax^3 + bx^2 + cx + d = 0$ , where  $a, b, c, d$  are constants to be determined. [4]

4. A complex number is defined by  $z = -3 + 4i$ .

(a) (i) Express  $z$  in the form  $r(\cos\theta + i\sin\theta)$ , where  $-\pi \leq \theta \leq \pi$ .

(ii) Express  $\bar{z}$ , the complex conjugate of  $z$ , in the form  $r(\cos\theta + i\sin\theta)$ . [4]

Another complex number is defined as  $w = \sqrt{5}(\cos 2.68 + i\sin 2.68)$ .

(b) Express  $zw$  in the form  $r(\cos\theta + i\sin\theta)$ . [3]

5. (a) Show that  $\frac{2}{n-1} - \frac{2}{n+1}$  can be expressed as  $\frac{4}{(n^2-1)}$ . [1]

(b) Hence, find an expression for  $\sum_{r=2}^n \frac{4}{(r^2-1)}$  in the form  $\frac{(an+b)(n+c)}{n(n+1)}$ ,  
where  $a, b, c$  are integers whose values are to be determined. [6]

(c) Explain why  $\sum_{r=1}^{100} \frac{4}{(r^2-1)}$  cannot be calculated. [1]

6. (a) Show that  $1 - 2i$  is a root of the cubic equation  $x^3 + 5x^2 - 9x + 35 = 0$ . [3]

(b) Find the other two roots of the equation. [4]

7. The complex number  $z$  is represented by the point  $P(x, y)$  in the Argand diagram and

$$|z - 4 - i| = |z + 2|.$$

(a) Find the equation of the locus of  $P$ . [4]

(b) Give a geometric interpretation of the locus of  $P$ . [1]

8. The transformation  $T$  in the plane consists of a translation in which the point  $(x, y)$  is transformed to the point  $(x - 1, y + 1)$ , followed by a reflection in the line  $y = x$ .

(a) Determine the  $3 \times 3$  matrix which represents  $T$ . [4]

(b) Find the equation of the line of fixed points of  $T$ . [2]

(c) Find  $T^2$  and hence write down  $T^{-1}$ . [3]

9. The line  $L_1$  passes through the points  $A(1, 2, -3)$  and  $B(-2, 1, 0)$ .

(a) (i) Show that the vector equation of  $L_1$  can be written as

$$\mathbf{r} = (1 - 3\lambda)\mathbf{i} + (2 - \lambda)\mathbf{j} + (-3 + 3\lambda)\mathbf{k}.$$

(ii) Write down the equation of  $L_1$  in Cartesian form. [4]

The vector equation of the line  $L_2$  is given by  $\mathbf{r} = 2\mathbf{i} - 4\mathbf{j} + \mu(4\mathbf{j} + 7\mathbf{k})$ .

(b) Show that  $L_1$  and  $L_2$  do not intersect. [5]

(c) Find a vector in the direction of the common perpendicular to  $L_1$  and  $L_2$ . [5]

**END OF PAPER**