GCE AS/A Level – LEGACY

0985/01

S18-0985-01

MATHEMATICS – S3
Statistics

FRIDAY, 22 JUNE 2018 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A bag contains six balls, three of which are numbered 1, two of which are numbered 2 and one of which is numbered 3. Three of these balls are selected at random, without replacement. Given that X denotes the largest number on the three selected balls, determine the value of $E(X)$. [7]

2. A dairy sells cheese in packs of nominal weight 500 grams. As a quality control check, 60 packs are chosen at random and the weight, x grams, of each pack is measured. The results are summarised below.

$$\sum x = 30\,060, \quad \sum x^2 = 15\,060\,146$$

(a) Determine an approximate 95% confidence interval for the mean weight of the packs of cheese. [8]

(b) What conclusion should the owner of the dairy reach? [1]

3. A firm produces lengths of cord with a mean breaking strength of 20 Newtons. It is claimed that if the cord is treated with a new chemical, the mean breaking strength increases. In order to test this claim, 10 lengths of cord were selected at random and treated with the new chemical. Their breaking strengths (in Newtons) were determined, as follows.

20·1 20·5 19·4 21·6 20·8 20·0 21·3 20·9 21·6 19·8

You may assume that this is a random sample from a normal distribution with mean μ and variance σ^2.

(a) State suitable hypotheses for testing this claim. [1]

(b) Determine unbiased estimates of μ and σ^2. [5]

(c) Carry out an appropriate test with significance level 1% and state your conclusion in context. Explain how you reached your conclusion. [5]

4. Alan is a cricketer and he claims that the probability p of his hitting the wicket when he throws the ball in from a particular point is 0·6. Successive throws may be assumed to be independent. In order to test his claim, he throws 80 balls in from this point and he hits the wicket 42 times.

(a) Calculate an unbiased estimate of p. [1]

(b) Calculate an approximate value for the standard error of your estimate. [2]

(c) Calculate an approximate 90% confidence interval for p. [3]

(d) Does your result support Alan's claim? Justify your answer. [1]

5. A shop sells eggs that are supplied by two different poultry farms, A and B. The owner of the shop wants to determine whether or not the mean weights of the eggs supplied by the two farms are the same. She therefore collects and weighs random samples of 100 eggs from each farm in order to carry out a suitable test. A summary of the results is shown below where x grams, y grams represent the weights of the eggs from A and B respectively.

$$\sum x = 5970, \qquad \sum x^2 = 356\,630, \qquad \sum y = 6025, \qquad \sum y^2 = 363\,402$$

(a) State suitable hypotheses for this test. [1]

(b) Determine an approximate *p*-value for these results and state your conclusion. [11]

(c) Giving a reason, state whether or not your analysis requires the assumption that the weights of eggs are normally distributed. [1]

6. A market gardener wishes to investigate the effect of a new fertiliser on his tomato yield. He therefore plants the same number of tomato plants in six different greenhouses and he applies different amounts of fertiliser in each greenhouse during the growing season. He records the total yield obtained from each greenhouse. He assumes that the total yield, y kg, and the amount of fertiliser applied, x units, satisfy a relationship of the form $y = \alpha + \beta x$, where α and β are unknown constants. The following table gives the results of his investigation.

x	0	10	20	30	40	50
y	22·6	24·1	26·2	28·5	30·7	32·6

You are given that $\sum x = 150, \quad \sum y = 164{\cdot}7, \quad \sum x^2 = 5500, \quad \sum xy = 4478.$

You may assume that the values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·25.

(a) (i) Calculate least squares estimates for α and β.

(ii) Hence estimate the total yield that would have been obtained with an application of 25 units of fertiliser.

(iii) Determine the standard error of this estimate. [10]

(b) An alternative estimate of the total yield defined in *(a)*(ii) can be found by calculating the arithmetic mean of the recorded values of y corresponding to $x = 0$ and $x = 50$.

(i) Calculate this estimate.

(ii) Determine its standard error. [3]

TURN OVER

7. The random variables $X_1,\ X_2$ are a random sample from $\mathrm{N}(\mu,\ 4\sigma^2)$ and the random variables $Y_1,\ Y_2,\ Y_3$ are a random sample from $\mathrm{N}(2\mu,\ \sigma^2)$. The estimator

$$U = a(X_1 + X_2) + b(Y_1 + Y_2 + Y_3)$$

is to be used to estimate μ, where a, b are constants.

(a) Given that U is unbiased, show that

$$2a + 6b = 1.$$ [3]

(b) (i) Show that

$$\mathrm{Var}(U) = \left(75b^2 - 24b + 2\right)\sigma^2.$$

(ii) Find the value of a and the value of b for which this variance is a minimum. Justify your answer. [7]

(c) The estimator

$$V = \frac{W^2}{N}, \text{ where } W = X_1 - X_2 + 2Y_1 - Y_2 - Y_3 \text{ and } N \text{ is a positive integer,}$$

is to be used to estimate σ^2.

(i) Determine the mean and the variance of the random variable W.

(ii) Given that V is an unbiased estimator for σ^2, find the value of N. [5]

END OF PAPER