GCE AS/A Level – LEGACY

0984/01

S18-0984-01

MATHEMATICS – S2
Statistics

WEDNESDAY, 13 JUNE 2018 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The random variable X has the binomial distribution B(4, 0·25) and the random variable Y has the Poisson distribution with mean 3. The random variables X and Y are independent. The random variable U is defined by $U = XY$.

(a) Evaluate $P(U = 2)$. [7]

(b) Determine the mean and the variance of U. [8]

2. The weights of hens may be assumed to be normally distributed with mean 2·6 kg and standard deviation 0·15 kg. The weights of cockerels may be assumed to be normally distributed with mean 4·2 kg and standard deviation 0·25 kg.

(a) Find the probability that the weight of a randomly chosen cockerel is more than twice the weight of a randomly chosen hen. [7]

(b) If 2 cockerels and 5 hens are chosen at random, find the probability that their total weight is more than 21 kg. [6]

3. A new machine dispenses coffee into cups. The amount dispensed is normally distributed with standard deviation 2 ml but the mean μ ml is unknown. Successive amounts dispensed are independent. The owner wishes to determine a confidence interval for μ, so she measures the amounts of coffee, in ml, dispensed into 9 cups with the following results.

250·1 248·3 251·6 247·7 252·3 250·9 253·4 251·7 249·4

(a) Calculate a 90% confidence interval for μ. [6]

(b) The owner used the same data to calculate a different confidence interval for μ and she obtained [249·4, 251·8]. Determine the confidence level of this interval. Give your answer as a percentage correct to one decimal place. [5]

4. When Alun types a page of a document, the number of errors made may be modelled by a Poisson distribution with mean μ. The numbers of errors on different pages are independent.

(a) He types a 20-page document. Assuming that $\mu = 0{\cdot}5$, determine the probability that the total number of errors in the document is equal to 12. [3]

(b) Alun believes that his typing has improved and the value of μ is now less than 0·5. He therefore defines the following hypotheses.

$$H_0 : \mu = 0{\cdot}5 \quad ; \quad H_1 : \mu < 0{\cdot}5$$

He types a 30-page document and he makes 9 errors.

(i) Calculate the *p*-value of this result.

(ii) Interpret this *p*-value using a 5% significance level. Justify your answer. [4]

(c) He now types a 100-page document and he makes 36 errors.

(i) Calculate the approximate *p*-value of this result.

(ii) What evidence does this *p*-value give regarding whether or not Alun's typing has improved? [5]

5. A machine is used to measure the refractive index of glass fragments. The reading obtained is a normally distributed random variable with mean equal to the true refractive index and standard deviation 0·02. Measurements were made on two samples of glass fragments believed to have come from two windows having the same refractive index. These measurements are shown in the table below.

Window 1	1·51 1·54 1·53 1·49 1·52 1·53
Window 2	1·54 1·56 1·57 1·53 1·52 1·55 1·58

(a) State suitable hypotheses to test the above belief using a two-sided test. [1]

(b) (i) Calculate the *p*-value for this test.

(ii) Interpret the *p*-value in context. [9]

6. The continuous random variable X is uniformly distributed on the interval $[a, b]$.

(a) Given that $E(X) = 34$ and $\text{Var}(X) = 12$, determine the value of a and the value of b. [5]

(b) Determine the 95th percentile of X. [2]

(c) (i) State the Central Limit Theorem.

(ii) A random sample of 120 observations was taken from the distribution of X, and S denotes the sum of these 120 observations. Use the Central Limit Theorem to find an approximate value for $P(S > 4140)$. [7]

END OF PAPER

