GCE AS/A Level – LEGACY

0983/01

MATHEMATICS – S1
Statistics

WEDNESDAY, 13 JUNE 2018 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The random variable X has mean 12 and variance 9.

   (a) Find the value of $E(X^2)$. [2]

   (b) If $Y = 4X - 3$, find the mean and the standard deviation of Y. [4]

2. The independent events A and B are such that

$$P(A \cup B) = 0.9, \quad P(A \cap B) = 0.4, \quad P(A) > P(B).$$

   (a) Determine the values of $P(A)$ and $P(B)$. [7]

   (b) Determine the value of $P(A|A \cup B)$. [3]

3. Janet is given a ticket for a pop concert and she has to decide which of her two children, Beti or Gwyn, should have the ticket. She therefore puts five white balls and one red ball into a bag and asks the two children to select a ball at random, alternately, starting with Beti. Once a ball is selected, it is not put back into the bag. The child who selects the red ball wins the ticket. Show that Beti and Gwyn are equally likely to win the ticket. [6]

4. The random variable X has the binomial distribution B(10, p). Find the set of values of p for which the standard deviation of X is greater than the mean of X. [5]

5. Jim uses the following method to decide how to travel to work. He throws a fair six-sided dice and if he obtains a 6, he runs to work; otherwise he cycles to work. If he runs to work, the probability that his journey takes longer than 20 minutes is 0.6. If he cycles to work, the probability that his journey takes longer than 20 minutes is 0.24. On a randomly chosen day,

   (a) calculate the probability that his journey takes longer than 20 minutes, [3]

   (b) given that his journey took longer than 20 minutes, calculate the probability that he cycled to work. [3]

6. Cars arrive at a car wash in such a way that the number arriving during an interval of length t minutes has a Poisson distribution with mean 0.25t.

   (a) Find the probability that

   (i) exactly 4 cars arrive between 9:00 a.m. and 9:15 a.m.,

   (ii) more than 6 cars arrive between 10:00 a.m. and 10:20 a.m. [6]

   (b) The probability that less than 10 cars arrive during an interval of length t minutes is equal to 0.7166. Find the value of t. [3]

7. The discrete random variable X has the following probability distribution.

x	1	2	3	4
$P(X = x)$	0·3	α	β	0·2

    (a)   Given that $E(X) = 2·2$, determine the value of α and the value of β.   [5]

    (b)   Three independent observations X_1, X_2, X_3 are taken from the distribution of X.
        Determine the value of $P(X_1 = X_2 = X_3)$.   [3]

8.   *(a)*   Seeds of a certain variety germinate independently with probability 0·6. A batch of 20 of these seeds is planted and X denotes the number which germinate.

        (i)   State the distribution of X, including any parameters.

        (ii)   Without the use of tables, calculate the probability that exactly 15 of these seeds germinate.

        (iii)   Determine the probability that at least 15 of these seeds germinate.   [8]

    (b)   Seeds of a different variety germinate independently with probability 0·05. A batch of 200 of these seeds is planted and Y denotes the number which germinate. Use an appropriate Poisson approximation to determine, approximately, the probability that

        (i)   exactly 8 of these seeds germinate,

        (ii)   more than 12 of these seeds germinate.   [5]

9. The continuous random variable X has cumulative distribution function F given by

$$F(x) = 0 \qquad \text{for } x < 1,$$
$$F(x) = \frac{1}{10}\left(x^2 + x - 2\right) \qquad \text{for } 1 \leqslant x \leqslant 3,$$
$$F(x) = 1 \qquad \text{for } x > 3.$$

    (a)   (i)   Evaluate $P(2 < X < 2·5)$.

        (ii)   Find the upper quartile of X.   [6]

    (b)   (i)   Find an expression for $f(x)$, valid for $1 \leqslant x \leqslant 3$, where f denotes the probability density function of X.

        (ii)   Calculate $E(X)$.   [6]

END OF PAPER