GCE AS/A Level – LEGACY

0981/01

S18-0981-01

MATHEMATICS – M2
Mechanics

TUESDAY, 19 JUNE 2018 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A vehicle moves in a straight line so that its velocity v ms^{-1} at time t seconds is given by

$$v = \frac{1}{20}(80 + 16t - t^2) \qquad 0 \leqslant t \leqslant 20.$$

At time $t = 0$, the vehicle is at the point *A*.

(a) Find an expression for the acceleration of the vehicle at time t seconds. [2]

(b) Determine the maximum velocity of the vehicle, showing that the value you have found is a maximum. [4]

(c) Calculate the displacement of the vehicle from *A* when $t = 20$. [4]

2. A particle *P*, of mass 8 kg, moves in a plane such that its position vector at time t seconds is given by

$$\mathbf{r} = (3t^2 + 1)\mathbf{i} + (t\cos 4t)\mathbf{j} \text{ metres.}$$

(a) Find the momentum vector of *P* when $t = 0$. [3]

(b) Calculate the kinetic energy of *P* when $t = \pi$. [3]

(c) Determine the force acting on *P* when $t = \pi$ and find a vector that is perpendicular to this force. [4]

(d) Find the rate of work of the force acting on *P* when $t = \pi$. [3]

3. A car of mass 1500 kg is towing a trailer of mass M kg up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{21}$. The resistance to motion acting on the car is constant at 120 N and that acting on the trailer is constant at 60 N. The car's engine is working at 50 kW. Given that the acceleration is 0·4 ms^{-2} when the speed of the car and the trailer is 25 ms^{-1}, calculate the value of M and determine the tension in the rigid tow-bar connecting the car and the trailer at this instant. [8]

4. A rough circular plate rotates horizontally, with constant angular velocity ω rad s^{-1}, about a fixed smooth vertical axis through its centre. An object of mass m kg lies on the plate at a distance 0·25 m from the axis and is connected to the axis by a light horizontal spring of natural length 0·2 m and modulus $3mg$ N. The coefficient of friction between the object and the plate is 0·4. Find the greatest and the least value of ω if the object is to remain at rest on the plate. [10]

5. A light elastic string, of natural length 1·2 m and modulus of elasticity 60 N, has one end *A* attached to a fixed point and the other end *B* attached to a particle *P* of mass 3 kg. Initially *P* is held at a point which is 0·6 m vertically below *A*. It is then released and allowed to fall. Calculate the speed of *P* when the length of the string is 1·5 m. [8]

6. A ball is projected from a point *A* on horizontal ground with initial velocity V ms^{-1} at an angle θ above the horizontal. The ball just clears a vertical wall 3 m high which is at a horizontal distance of 6 m from *A*.

(a) Show that V and θ satisfy the equation

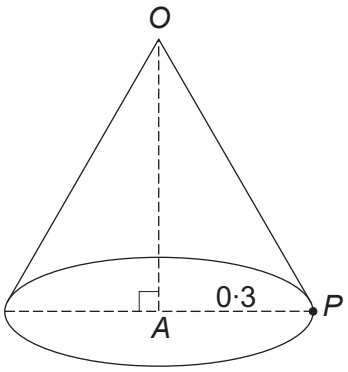
$$3 = 6\tan\theta - \frac{18g}{V^2\cos^2\theta},$$

where g is the acceleration due to gravity. [3]

The ball also just clears the top of a building 10 m high which is at a horizontal distance of 24 m from *A*.

(b) Find the value of θ and the value of V. [6]

7. A particle *P*, of mass 3 kg, is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point *O*. The particle *P* moves with a uniform angular velocity of 4 rad s^{-1} in a horizontal circular path with centre *A* and radius 0·3 m, as shown in the diagram.



(a) Find,

(i) the angle *AOP*,

(ii) the tension in the string. [7]

(b) Determine the length of the string. [1]

8. A particle rests on the inside of a fixed smooth vertical circular hoop at its lowest point. The hoop has centre *O* and radius a m. The particle is given a horizontal velocity of u ms^{-1} such that it leaves the hoop at the point *P*, where *OP* makes an angle $\theta = \cos^{-1}\left(\frac{2}{3}\right)$ with the **upward** vertical. Show that $u = 2\sqrt{ag}$. [9]

END OF PAPER