GCE AS/A Level – LEGACY

0979/01

S18-0979-01

MATHEMATICS – FP3
Further Pure Mathematics

TUESDAY, 26 JUNE 2018 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \cosh^3 x - 2\sinh x.$$

(a) Obtain an expression for $f'(x)$. [2]

(b) Determine the x-coordinate of the stationary point on the graph of f. Give your answer correct to three significant figures. [5]

2. The function f is defined by

$$f(x) = \ln(\cos x).$$

(a) Find the Maclaurin series for $f(x)$ as far as the term in x^4. [7]

(b) Use your series to deduce the first two non-zero terms in the Maclaurin series for

(i) $\ln(\sec^2 x)$,

(ii) $\tan x$. [5]

3. Determine the value of the integral

$$\int_0^{\frac{\pi}{3}} e^{2x} \sin 3x \, dx .$$

Give your answer in the form $a(1 + e^{b\pi})$, where a, b are fractions. [8]

4. *(a)* Sketch the curve *C* with polar equation $r = 1 - \cos\theta$, $0 \leqslant \theta \leqslant \pi$. [1]

(b) Find the polar coordinates of the point on *C*, other than the origin, at which the tangent to *C* is parallel to the initial line. [7]

(c) Find the area of the region enclosed by *C* and the initial line. [5]

5. *(a)* Show that the length *L* of the arc joining the points (1, 2) and (4, 4) on the curve with equation $y^2 = 4x$ is given by

$$\int_1^4 \sqrt{\left(1 + \frac{1}{x}\right)} \, dx .$$ [4]

(b) Use the substitution $x = \sinh^2 u$ to determine the value of *L* correct to three significant figures. [8]

6. The equation $\sinh\theta = \cos\theta$ has a root close to 0·7. In order to obtain an accurate approximation to this root, it is proposed to use one of the following iterative sequences.

$$\text{A} : \theta_{n+1} = \sinh^{-1}(\cos\theta_n), \quad \theta_0 = 0{\cdot}7$$

$$\text{B} : \theta_{n+1} = \cos^{-1}(\sinh\theta_n), \quad \theta_0 = 0{\cdot}7$$

(a) Show by differentiation that one of these sequences is convergent and the other sequence is divergent. [8]

(b) Use the convergent sequence to find the value of the root correct to three decimal places. [4]

7. The integral I_n is given, for $n \geqslant 0$, by

$$I_n = \int_1^2 x^2 (\ln x)^n \mathrm{d}x.$$

(a) Show that, for $n \geqslant 1$,

$$I_n = \frac{8}{3}(\ln 2)^n - \frac{n}{3} I_{n-1}.$$ [5]

(b) Hence determine the value of I_3, giving your answer correct to three significant figures. [6]

END OF PAPER