GCE AS/A Level – LEGACY

0978/01

S18-0978-01

MATHEMATICS – FP2
Further Pure Mathematics

MONDAY, 25 JUNE 2018 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find the three cube roots of the complex number $3+4\mathrm{i}$. Give your answers in Cartesian form with the real and imaginary parts correct to two decimal places. [8]

2. The function f is defined by

$$f(x)=\sqrt{-x} \qquad \text{for } x<0,$$

$$f(x)=-\sqrt{x} \qquad \text{for } x \geqslant 0.$$

Determine whether f is an even function, an odd function or neither even nor odd. [3]

3. *(a)* Express $3+2x-x^2$ in the form $a-(x-b)^2$, where a, b are positive integers. [2]

(b) Hence evaluate the integral

$$\int_0^2 \frac{1}{\sqrt{3+2x-x^2}}\,\mathrm{d}x,$$

giving your answer in the form $\dfrac{\pi}{n}$, where n is a positive integer to be determined. [3]

4. *(a)* By putting $t=\tan\dfrac{x}{2}$, show that the equation

$$\sec x + \tan x = 2$$

can be written in the form

$$3t^2 + 2t - 1 = 0.$$ [3]

(b) Hence find the general solution of the equation

$$\sec x + \tan x = 2.$$ [7]

5. *(a)* Express $\dfrac{5}{(x+1)(x^2+4)}$ in partial fractions. [4]

(b) Hence evaluate the integral

$$\int_1^2 \frac{5}{(x+1)(x^2+4)}\,\mathrm{d}x,$$

giving your answer correct to three significant figures. [6]

6. *(a)* Given that $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$$

and find a similar expression for $z^n + \frac{1}{z^n}$. [4]

(b) Hence show that

$$\sin^3\theta\cos\theta = a\sin 4\theta + b\sin 2\theta,$$

where a, b are constants whose values are to be determined. [5]

7. The equation of the ellipse *E* is

$$\frac{x^2}{9} + \frac{y^2}{4} = 1\,.$$

(a) Determine

(i) the eccentricity of *E*,

(ii) the coordinates of the foci of *E*,

(iii) the equations of the directrices of *E*. [4]

(b) Determine the equation of the normal to *E* at the point $(3\cos\theta,\ 2\sin\theta)$, simplifying your answer. [5]

(c) This normal meets the x and y axes at the points *A* and *B* respectively. Show that the locus of the midpoint of *AB* as θ varies is an ellipse. [5]

8. The function f is defined by

$$f(x) = \frac{1+x+x^2}{1-x+x^2}\,.$$

(a) Find the equation of the asymptote on the graph of f. [1]

(b) (i) Find the coordinates of the two stationary points on the graph of f.

(ii) By considering the signs of $f'(x)$ in the vicinity of these stationary points, classify each as a maximum or a minimum. [8]

(c) Sketch the graph of f. [2]

(d) The set *S* = (2, 3). Determine $f^{-1}(S)$. [5]

END OF PAPER

