GCE AS/A Level – LEGACY

0977/01

S18-0977-01

MATHEMATICS – FP1
Further Pure Mathematics

MONDAY, 14 MAY 2018 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Express $\dfrac{1}{n(n+1)}$ in partial fractions. [2]

(b) Consider the series

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+...+\frac{1}{n(n+1)}.$$

Show that the sum of this series is given by $\dfrac{an}{bn+1}$, where a, b are positive integers to be determined. [4]

2. *(a)* Express $(2 + \mathrm{i})^4$ in the form $a + \mathrm{i}b$, where a, b are real. [2]

(b) Hence show that $2 + \mathrm{i}$ is a root of the equation $x^4 + 2x^2 - 32x + 65 = 0$. [3]

(c) Determine the other three roots of this equation. [6]

3. *(a)* Express $\dfrac{1+17\mathrm{i}}{1+2\mathrm{i}}$ in the form $a + \mathrm{i}b$, where a, b are real. [3]

(b) Hence solve the equation

$$2\mathrm{i}z+3\overline{z}=\frac{1+17\mathrm{i}}{1+2\mathrm{i}},$$

where $\overline{z}$ denotes the complex conjugate of z. Give z in trigonometric form with the values of r and θ correct to three significant figures. [6]

4. The transformation T in the plane consists of a clockwise rotation through 90° about the origin, followed by a translation in which the point (x, y) is transformed to the point $(x - 1, y + 2)$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

[3]

(b) Determine the coordinates of the point which is transformed to the point (1, –1) under T. [3]

5. The roots of the cubic equation $x^3 - 2x^2 + 4x + 3 = 0$ are denoted by α, β, γ.

Determine the cubic equation whose roots are $\dfrac{1}{\beta\gamma}, \dfrac{1}{\gamma\alpha}, \dfrac{1}{\alpha\beta}$. [10]

6. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} \lambda & 1 & 2 \\ 4 & \lambda & 1 \\ 5 & 2 & 3 \end{bmatrix}, \text{ where } \lambda \text{ is a constant.}$$

(a) (i) Find an expression for the determinant of $\mathbf{M}$ in terms of λ.

(ii) Show that $\mathbf{M}$ is singular when $\lambda = 3$ and state the other value of λ for which $\mathbf{M}$ is singular. [4]

(b) Given that $\lambda = 3$, determine the value of μ for which the following system of equations is consistent.

$$\begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & 1 \\ 5 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ \mu \\ 2 \end{bmatrix}$$ [4]

(c) Suppose now that $\lambda = 2$ so that

$$\mathbf{M} = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 1 \\ 5 & 2 & 3 \end{bmatrix}.$$

(i) Determine the adjugate matrix of $\mathbf{M}$.

(ii) Hence determine the inverse matrix $\mathbf{M}^{-1}$. [5]

7. Use mathematical induction to prove that

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

for all positive integers n. [7]

8. The complex number z is represented by the point *P*(x, y) in the Argand diagram and

$$|z + 2\mathrm{i}| = 2|z - 3|.$$

(a) Show that the locus of *P* is a circle. [4]

(b) Find its radius and the coordinates of its centre. [3]

TURN OVER

9. The function f is defined on the domain (0, 2) by

$$f(x) = (\sin x)^x.$$

(a) Show that

$$f'(x) = f(x)\,g(x),$$

where $g(x)$ is to be determined. [3]

(b) (i) Evaluate $g(0{\cdot}1)$, $g(1)$ and $g(1{\cdot}6)$.

(ii) What do your three values tell you about the number of stationary points on the graph of f? [3]

END OF PAPER

