GCE AS/A Level – LEGACY

0976/01

S18-0976-01

MATHEMATICS – C4
Pure Mathematics

FRIDAY, 15 JUNE 2018 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that $f(x)=\dfrac{3x^2-3x-8}{x(x-2)^2}$,

(a) express $f(x)$ in terms of partial fractions, [4]

(b) evaluate

$$\int_6^9 f(x)\,\mathrm{d}x\,,$$

giving your answer correct to two decimal places. [3]

2. The curve *C* has equation

$$x^2 - y^3 - 3xy + 1 = 0.$$

The point *P* has coordinates (–2, –1) and lies on *C*.

(a) Show that the equation of the tangent to *C* at the point *P* is given by

$$x = 3y + 1.$$ [4]

(b) The tangent to *C* at the point *P* intersects *C* again at the point *Q*. Find the coordinates of *Q*. [5]

3. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$2\cos 2\theta = 3\sin^2\theta - 5\cos^2\theta + \cos\theta + 1.$$ [6]

(b) (i) Express $12\sin\phi - 5\cos\phi$ in the form $R\sin(\phi-\alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Hence find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

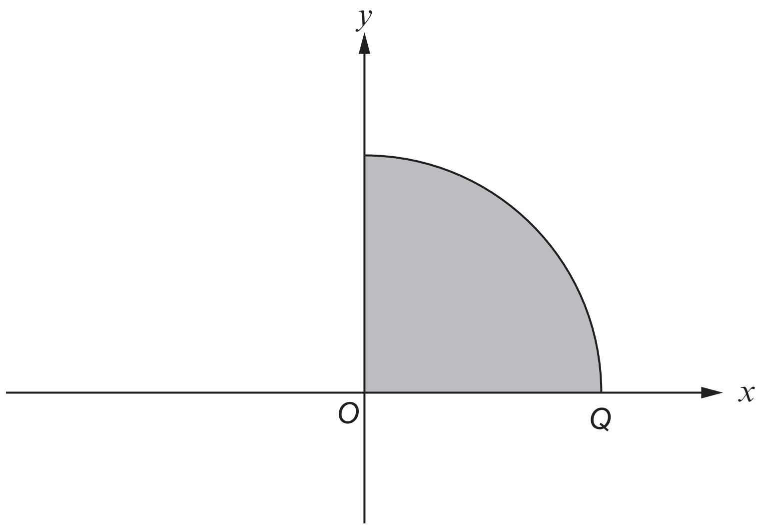
$$12\sin\phi - 5\cos\phi = -2.$$ [6]

4. *(a)* Expand $\dfrac{1}{(1+2x)^2}$ in ascending powers of x up to and including the term in x^2. [2]

(b) (i) **Use your answer to part *(a)*** to expand $\left(\dfrac{1+3x}{1+2x}\right)^2$ in ascending powers of x up to and including the term in x^2.

(ii) State the range of values of x for which your expansion is valid. [4]

5. The region shaded in the diagram below is bounded by the x-axis, the y-axis, and that part of the curve with equation $x^2 + y^2 = a^2$ $(a > 0)$ lying in the first quadrant. The curve intersects the x-axis at the point *Q*.



(a) Write down the x-coordinate of *Q*. [1]

(b) (i) By carrying out an appropriate integration, find the volume generated when the region shaded in the diagram is rotated through four right-angles about the x-axis.

(ii) Give a geometrical interpretation of your answer. [4]

6. The curve *C* has the parametric equations

$$x = \frac{3}{t^2}, \quad y = 4t^3.$$

The point *P* lies on *C* and has parameter p. Find and simplify the equation of the tangent to *C* at the point *P*. [4]

7. *(a)* Find $\int (4x+1)e^{4x-5}dx$. Simplify your answer. [4]

(b) (i) Use the substitution $x = 4\sin\theta$ to show that

$$\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}}dx = \int_0^a b\sin^2\theta\, d\theta,$$

where a and b are constants whose values are to be determined.

(ii) **Hence**, evaluate

$$\int_0^{2\sqrt{2}} \frac{x^2}{\sqrt{(16-x^2)}}dx.$$

Give your answer in the form $c\pi + d$, where c and d are integers whose values are to be determined. [8]

TURN OVER

8. The value of a painting on January 1st 2000 was £900. The value, £V, of the painting t years after this date may be modelled as a continuous variable. The rate of increase of V may be assumed to be directly proportional to $V^{\frac{3}{2}}$.

(a) Write down a differential equation satisfied by V. [1]

(b) The value of the painting on January 1st 2003 was £1600.
Find its value on January 1st 2008. [8]

9. *(a)* The vectors $\mathbf{p}$ and $\mathbf{q}$ are given by

$$\mathbf{p} = 3\mathbf{i} - 2\mathbf{j} + 7\mathbf{k},$$
$$\mathbf{q} = \mathbf{i} + 6\mathbf{j} - 4\mathbf{k}.$$

Find the angle between $\mathbf{p}$ and $\mathbf{q}$. Give your answer in degrees, correct to one decimal place. [4]

(b) The position vectors of the points *A* and *B* are denoted by $\mathbf{a}$ and $\mathbf{b}$ respectively. The points *C* and *D* have position vectors $4\mathbf{a} - \mathbf{b}$ and $-10\mathbf{a} + 5\mathbf{b}$ respectively. The point *E* lies on *CD* and is such that *CE* : *ED* = $1:3$.

(i) Find and simplify an expression for the position vector of the point *E* in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Interpret your result geometrically. [4]

10. Prove by contradiction the following proposition.

When x is real and positive,

$$25x + \frac{4}{x} \geqslant 20.$$

The first line of the proof is given below.

Assume that there is a real and positive value of x such that

$$25x + \frac{4}{x} < 20.$$ [3]

END OF PAPER