Logarithmau 2

Logarithms 2

@mathemateg

YouTube /adolygumathemateg

Ffwythiannau Esbonyddol / *Exponential Functions*

Ystyriwch y dilyniant rhif canlynol. / *Consider the following number sequence.*

$$4, 7, 10, 13, 16, 19, \ldots$$

Gwelwn ein bod yn **adio tri** i gael y rhif nesaf. / *We see that we need to **add three** to obtain the next number.*

Yr $n$fed term yw / *The nth term is*

$$3n + 1$$

Nawr ystyriwch y dilyniant rhif canlynol. / *Now consider the following number sequence.*

$$4, 12, 36, 108, 324, 972, \ldots$$

Gwelwn ein bod yn **lluosi efo tri** i gael y rhif nesaf. / *We see that we need to **multiply by three** to obtain the next number.*

Yr $n$fed term yw / *The nth term is*

$$4 \times 3^{n-1}$$

Mae hwn yn enghraifft o **ffwythiant esbonyddol**. Rydym yn defnyddio'r rhain mewn dilyniannau ble rydym yn lluosi efo'r un rhif bob tro. / *This is an example of an **exponential function**. We use these in sequences where we multiply by the same amount each time.*

Adlog / *Compound Interest*

Ystyriwch fuddsoddi £1,000 mewn banc sy'n talu adlog blynyddol ar gyfradd 3%. / *Consider investing £1,000 in a bank which offers yearly compound interest at a rate of 3%.*

Ar ôl blwyddyn / *After a year:*

$$£1{,}000 \times 1.03 = £1{,}030$$

Ar ôl dwy flynedd / *After two years:*

$$£1{,}000 \times 1.03^2 = £1{,}060.90$$

Ar ôl n mlynedd / *After n years:*

$$£1{,}000 \times 1.03^n$$

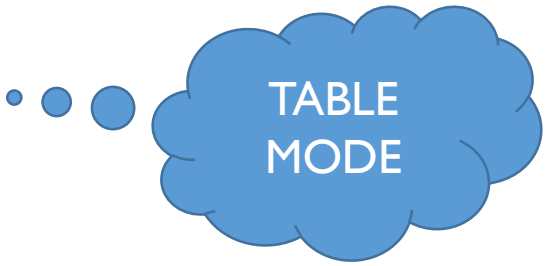
Mae hwn yn enghraifft o **ffwythiant esbonyddol**. / *This is an example of an* ***exponential function***.

Ffwythiannau Esbonyddol / *Exponential Functions*

Rhaid i chi wybod sut i blotio graff o'r ffwythiant esbonyddol $y = a^x$, ble mae a yn bositif. /
You need to know how to plot a graph of the exponential function $y = a^x$, where a is positive.

Enghraifft / *Example*

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$y = 2^x$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32
$y = \left(\frac{1}{2}\right)^x$	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$



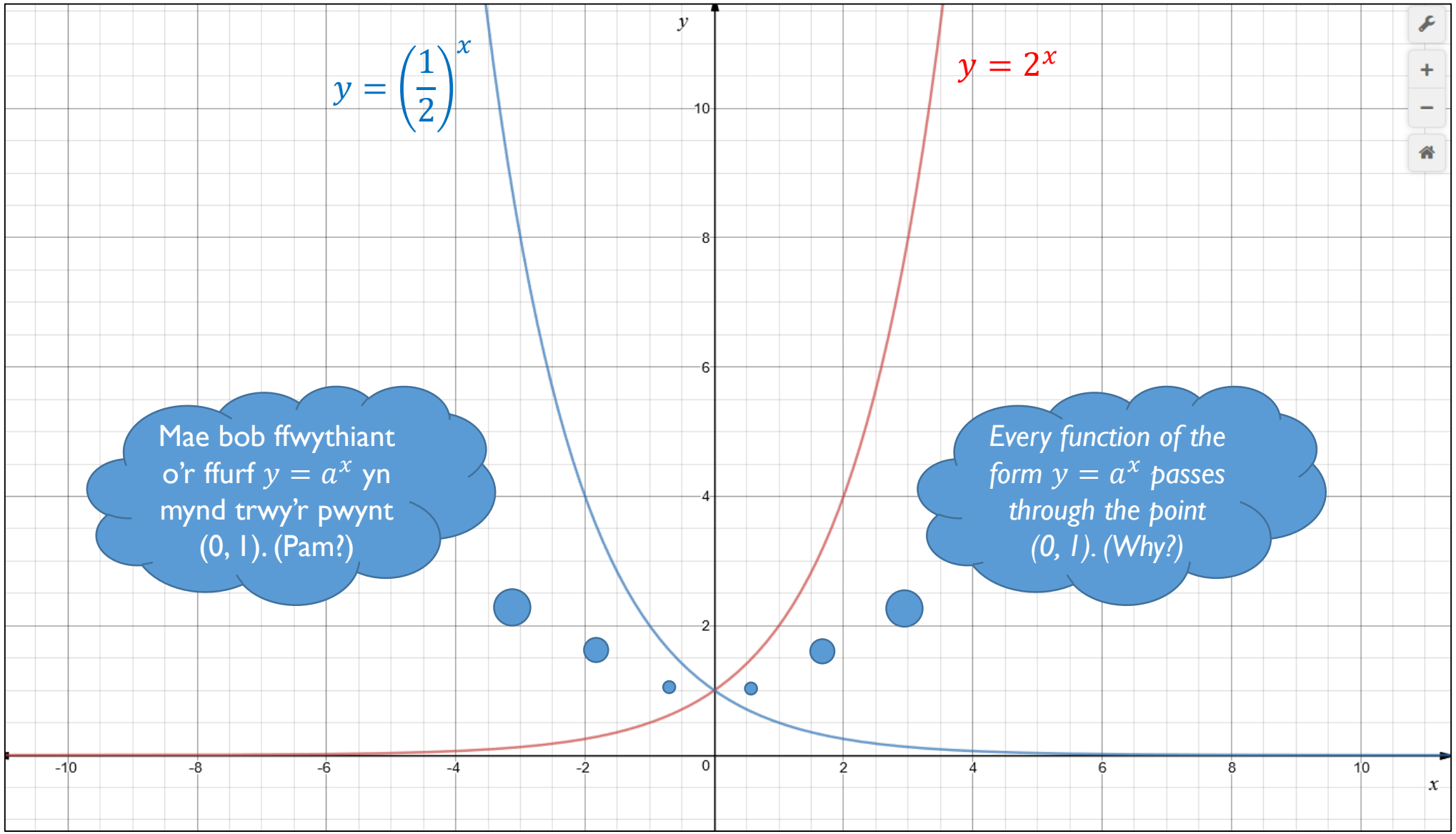
Sylwch bod $y = 2^x$ yn dyblu bob tro, tra bod $y = \left(\frac{1}{2}\right)^x$ yn haneru bob tro. /

Note that $y = 2^x$ doubles each time, whilst $y = \left(\frac{1}{2}\right)^x$ halves each time.

Nodyn / *Note:* $\left(\frac{1}{2}\right)^x = \frac{1^x}{2^x} = \frac{1}{2^x} = 2^{-x}$. Yn gyffredinol / *In general,* $a^{-x} = \left(\frac{1}{a}\right)^x$.

Ffwythiannau Esbonyddol / *Exponential Functions*

www.desmos.com/calculator



Ffwythiannau Esbonyddol / *Exponential Functions*

Ymarfer 1 / *Exercise 1*

Ar bapur graff addas, plotiwch y ffwythiannau $y = 3^x$ a $y = \left(\frac{1}{3}\right)^x$.

On suitable graph paper, plot the functions $y = 3^x$ *and* $y = \left(\frac{1}{3}\right)^x$.

Ffwythiannau Esbonyddol / *Exponential Functions*

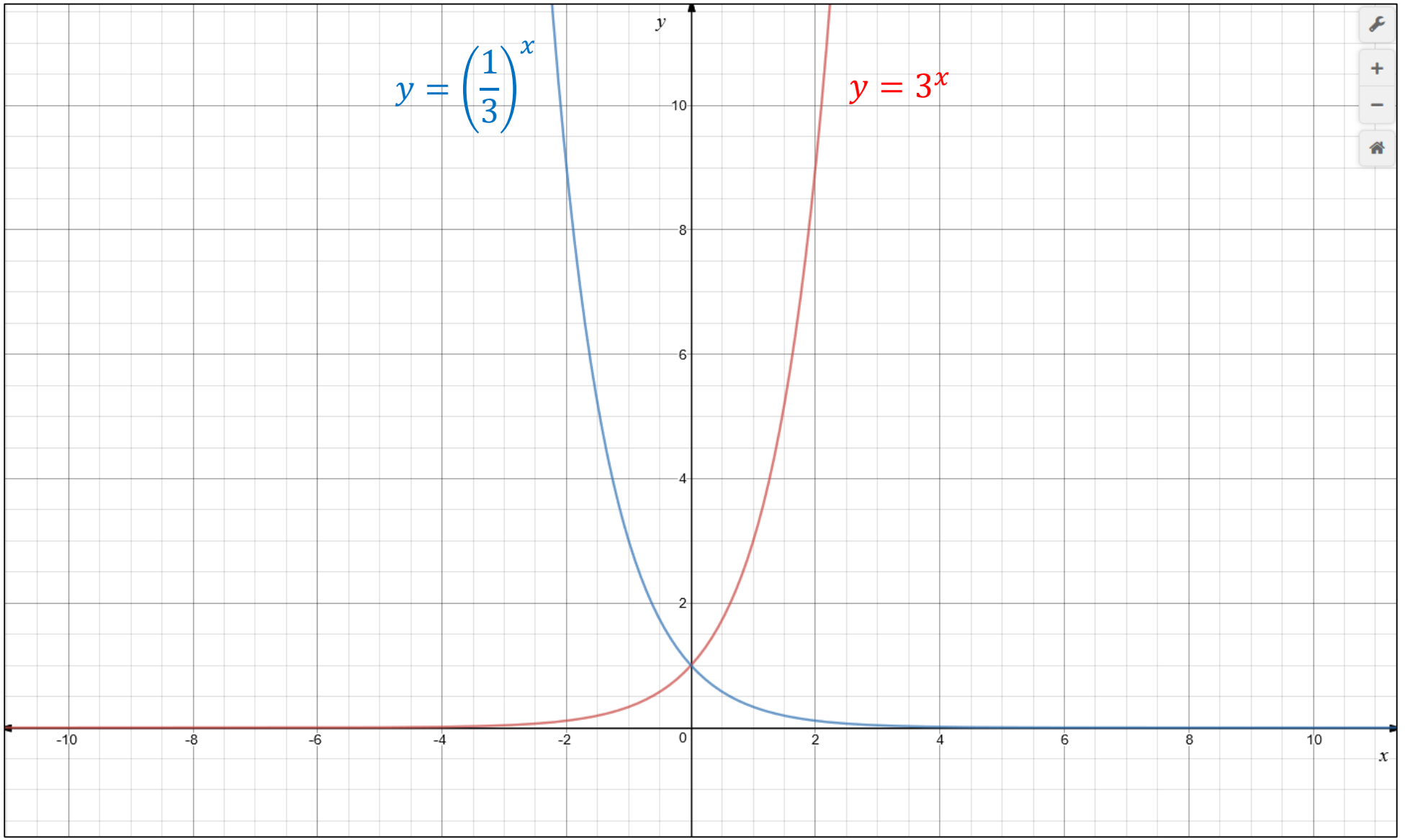
Ymarfer 1 / *Exercise 1*

Ar bapur graff addas, plotiwch y ffwythiannau $y = 3^x$ a $y = \left(\frac{1}{3}\right)^x$.

On suitable graph paper, plot the functions $y = 3^x$ *and* $y = \left(\frac{1}{3}\right)^x$.

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$y = 3^x$	$\frac{1}{243}$	$\frac{1}{81}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81	243
$y = \left(\frac{1}{3}\right)^x$	243	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

Ffwythiannau Esbonyddol / *Exponential Functions*



www.desmos.com/calculator

Ffwythiannau Esbonyddol / *Exponential Functions*

Ymarfer 2 / *Exercise 2*

Arbrofwch efo www.desmos.com/calculator i ymchwilio i siâp $y = a^x$ ar gyfer gwerthoedd gwahanol o a. Beth sy'n digwydd ar gyfer $a = 1$?

Experiment with www.desmos.com/calculator to explore the shape of $y = a^x$ for different values of a. What happens for $a = 1$?

Rhif Euler / *Euler's Number*

Ystyriwch fuddsoddi £1 mewn banc sy'n talu adlog ar gyfradd 100%. /
Consider investing £1 in a bank which offers compound interest at a rate of 100%.

Faint o arian sydd gennych ar ôl blwyddyn? / *How much money do you have after a year?*

Mae hyn yn dibynnu ar y nifer o randaliadau yn ystod y flwyddyn. / *This depends on the number of instalments during the year.*

Rhandaliadau / *Instalments*	Arian ar ôl blwyddyn / *Money after a year*
1 (100% ar ôl blwyddyn / *after a year*)	$1 \times 2^1 = £2$
2 (50% bob chwe mis / *every six months*)	$1 \times 1.5^2 = £2.25$
3 ($33.\dot{3}$% bob pedwar mis / *every four months*)	$1 \times 1.\dot{3}^3 = £2.37$
4 (25% bob tri mis / *every three months*)	$1 \times 1.25^4 = £2.44$
n	$\left(1 + \frac{1}{n}\right)^n$

Rhif Euler / *Euler's Number*

Fel mae n yn cynyddu, mae'r arian yn cynyddu, ond nid am byth. Wrth fodelu efo taenlen Excel, gwelwn bod yr arian yn setlo lawr i'r gwerth £2.71... Mae'r rhif yma'n arbennig mewn mathemateg, ac yn cael ei adnabod fel rhif Euler, e. I bum lle degol, $e = 2.71828$.

As n increases, the amount of money increases, but not indefinitely. Modelling using an Excel spreadsheet, we see that the money settles down to the value £2.71...This is a special number in mathematics, known as Euler's number, e. To five decimal places, $e = 2.71828$.

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$$

Mae'r ffwythiant esbonyddol $y = e^x$ yn arbennig: mae graddiant y graff yn hafal i werth e^x ar unrhyw bwynt ar y graff. Hynny yw, mae $\frac{d}{dx}(e^x) = e^x$.

The exponential function $y = e^x$ is special: the gradient of the graph is equal to the value of e^x at any point on the graph. In other words, $\frac{d}{dx}(e^x) = e^x$.

Ffwythiannau Esbonyddol / *Exponential Functions*

Ymarfer 3 / *Exercise 3*

Ar bapur graff addas, plotiwch y ffwythiant $y = e^x$.

On suitable graph paper, plot the function $y = e^x$.

Ffwythiannau Esbonyddol / *Exponential Functions*

Ymarfer 3 / *Exercise 3*

Ar bapur graff addas, plotiwch y ffwythiant $y = e^x$.

On suitable graph paper, plot the function $y = e^x$.

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$y = \mathrm{e}^x$	0.0067	0.0182	0.0498	0.1353	0.3679	1	2.7183	7.3891	20.0855	54.5982	148.4132

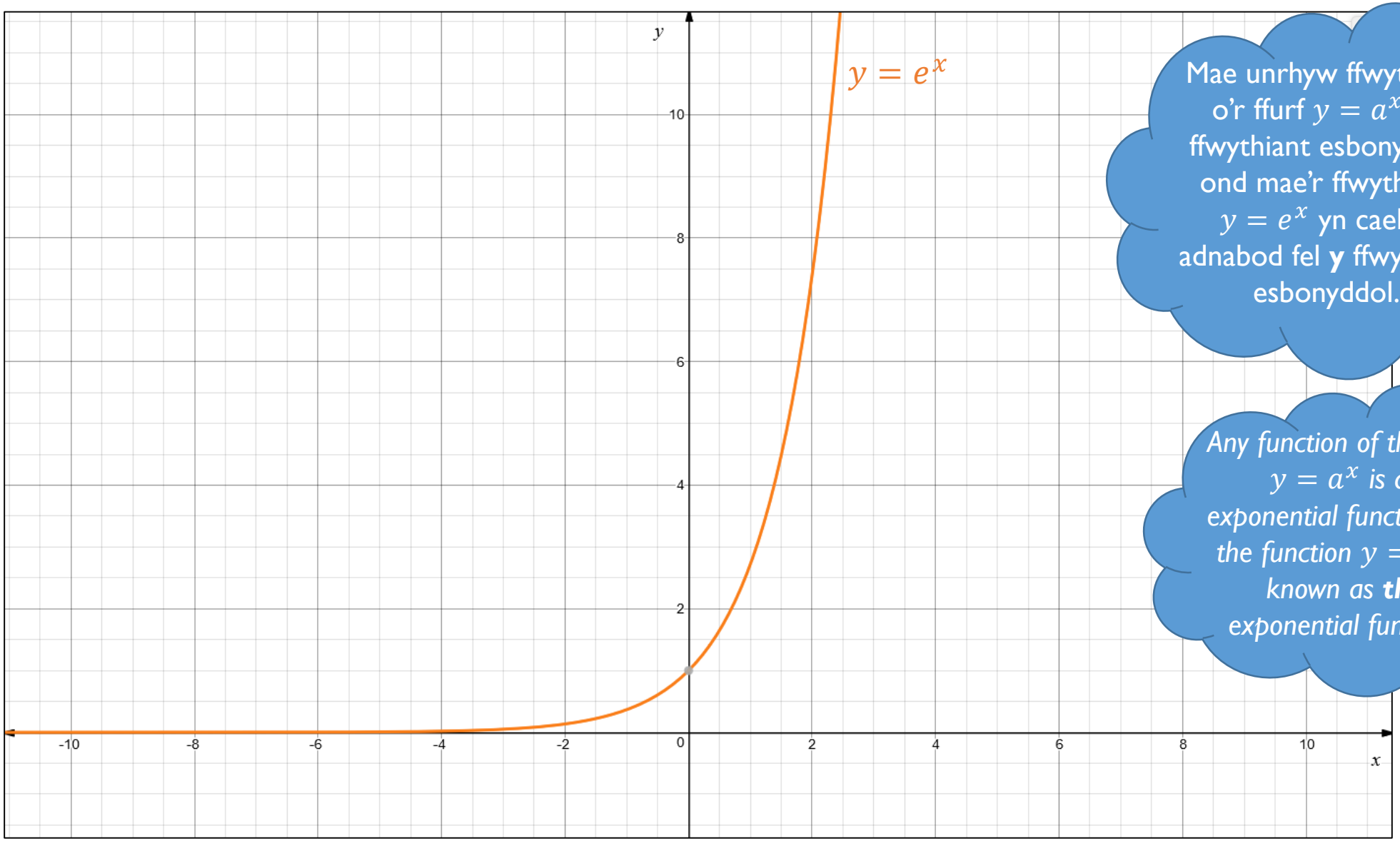
TABLE MODE

Estyniad / *Extension:*

O wybod bod $\frac{d}{dx}(a^x) = \ln(a) \times a^x$, arbrofwch efo plotio'r graffiau $y = a^x$ a $y = \ln(a) \times a^x$ ar gyfer gwerthoedd gwahanol o a.

Given that $\frac{d}{dx}(a^x) = \ln(a) \times a^x$, *experiment with plotting the graphs of* $y = a^x$ *and* $y = \ln(a) \times a^x$ *for different values of* a.

Ffwythiannau Esbonyddol / *Exponential Functions*



Mae unrhyw ffwythiant o'r ffurf $y = a^x$ yn ffwythiant esbonyddol, ond mae'r ffwythiant $y = e^x$ yn cael ei adnabod fel **y** ffwythiant esbonyddol.

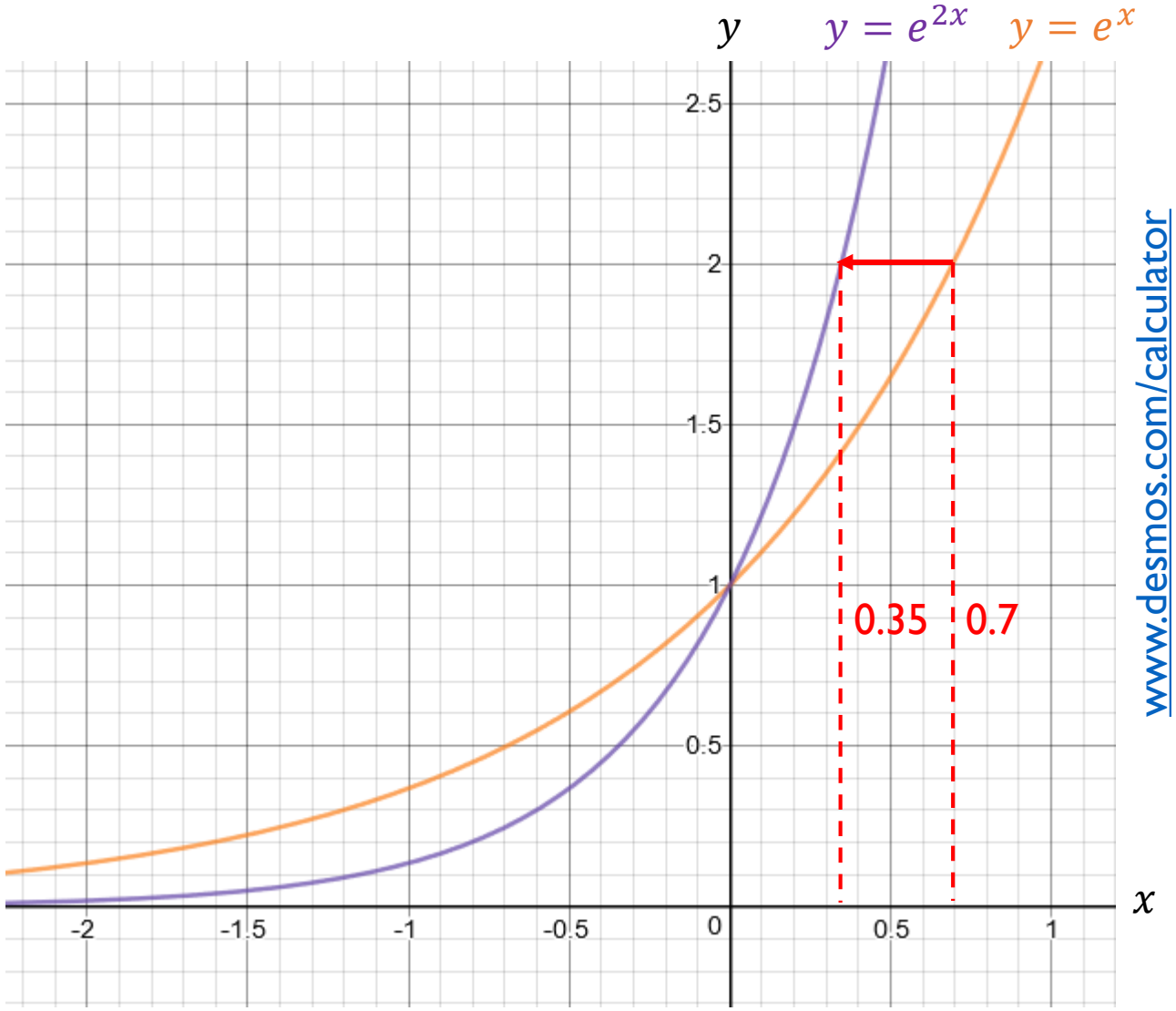
*Any function of the form $y = a^x$ is an exponential function, but the function $y = e^x$ is known as **the** exponential function.*

www.desmos.com/calculator

Y ffwythiant esbonyddol / *The exponential function*

O gychwyn efo cromlin y ffwythiant $y = e^x$, mae'n bosib ei gywasgu ar hyd yr echelin–x, efo ffactor graddfa $\frac{1}{k}$, i roi cromlin y ffwythiant $y = e^{kx}$.

Starting with the curve of the function $y = e^x$, *it is possible to compress it along the* x*–axis, with scale factor* $\frac{1}{k}$, *to give the curve of the function* $y = e^{kx}$.



Ffwythiannau Esbonyddol / *Exponential Functions*

Graddiant y ffwythiant $y = e^{kx}$ yw ke^{kx}. Hynny yw, $\frac{dy}{dx} = ke^{kx}$ neu $\frac{dy}{dx} \propto e^{kx}$.
Mae hyn yn fodel da ar gyfer digwyddiadau yn y byd naturiol fel dadfeiliad ymbelydrol neu twf poblogaeth.

Er enghraifft, os yw cyfradd cynnydd poblogaeth o facteria mewn cyfrannedd union â'r nifer o facteria, y, yna $\frac{dy}{dt} = ky$. Mae cyfradd newid $y = e^{kt}$ mewn cyfrannedd union ag y, felly mae ffwythiant esbonyddol yn fodel da i'r sefyllfa yma.

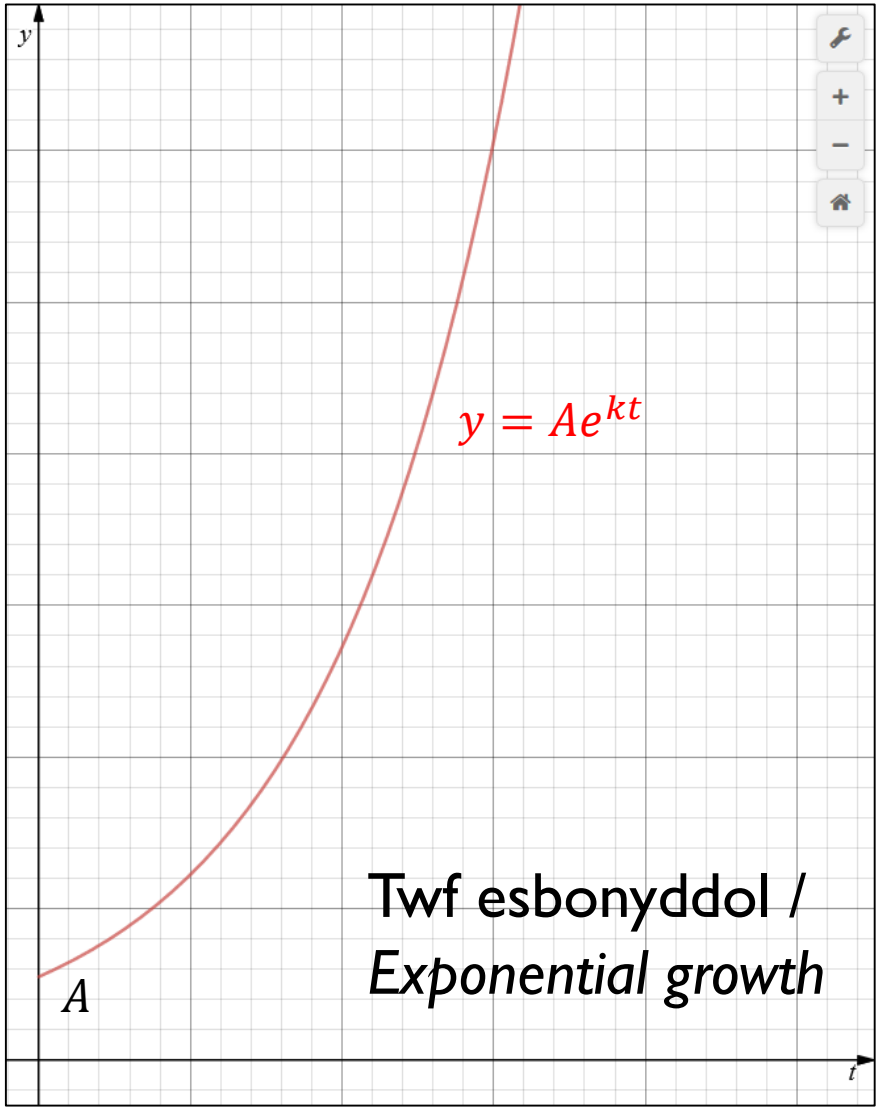
Mae hafaliad o'r ffurf $y = Ae^{kt}$ yn rhoi model esbonyddol, ble mae A a k yn gysonion.

The gradient of the function $y = e^{kx}$ is ke^{kx}. Therefore $\frac{dy}{dx} = ke^{kx}$ or $\frac{dy}{dx} \propto e^{kx}$.
This is a good model for events in the natural world such as radioactive decay or population growth.

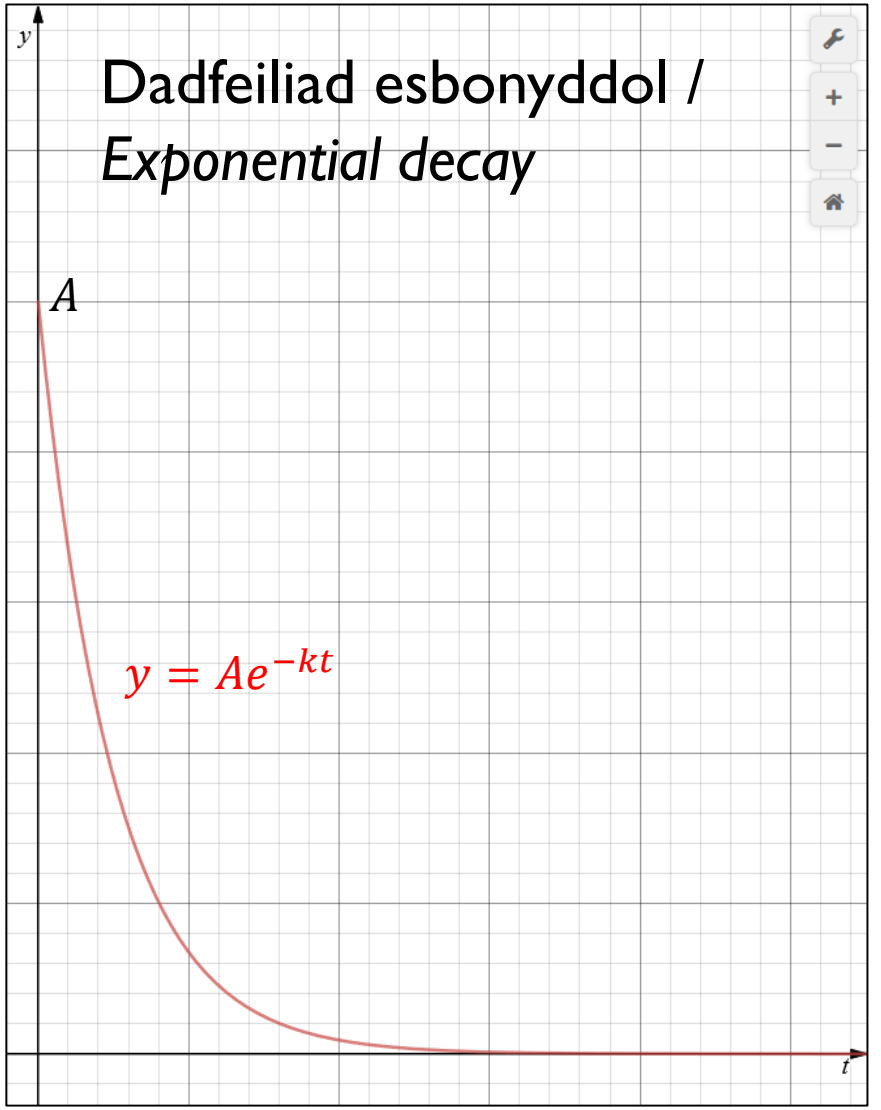
For example, if the rate of increase of a population of bacteria is directly proportional to the number of bacteria, y, then $\frac{dy}{dt} = ky$.The rate of change of $y = e^{kt}$ is directly proportional to y, so an exponential function is a good model for this situation.

An equation of the form $y = Ae^{kt}$ gives an exponential model where A and k are constants.

Twf a dadfeiliad esbonyddol / *Exponential growth and decay*



www.desmos.com/calculator



www.desmos.com/calculator

Ffwythiannau Logarithmic / *Logarithmic Functions*

Mae gwrthdro ffwythiant esbonyddol yn ffwythiant logarithmic. / *The inverse of an exponential function is a logarithmic function.*

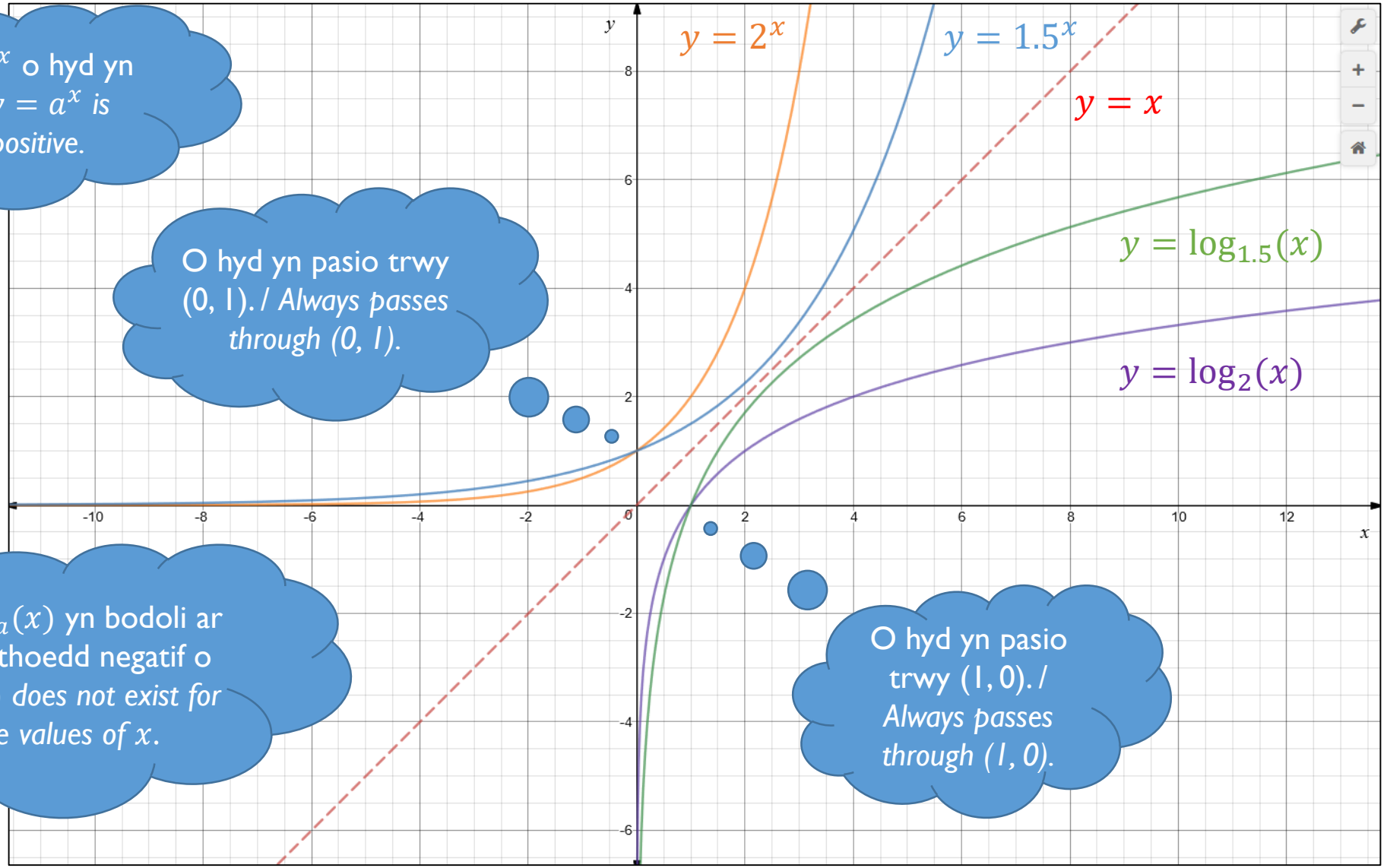
$y = a^x$

$\log_a(y) = \log_a(a^x)$	Cymryd logarithm bôn a o bob ochr. / *Taking logarithms (base a) of each side.*
$\log_a(y) = x \log_a(a)$	Rheolau logarithm. / *Rules of logarithms.*
$\log_a(y) = x$	Diffiniad logarithm. / *Definition of a logarithm.*

Gwrthdro y ffwythiant esbonyddol $y = a^x$ yw'r ffwythiant logarithmic $y = \log_a(x)$.

The inverse of the exponential function $y = a^x$ is the logarithmic function $y = \log_a(x)$.

Ffwythiannau Logarithmic / *Logarithmic Functions*



Mae $y = a^x$ o hyd yn bositif. / $y = a^x$ *is always positive.*

O hyd yn pasio trwy (0, 1). / *Always passes through (0, 1).*

Nid yw $\log_a(x)$ yn bodoli ar gyfer gwerthoedd negatif o x. / $\log_a(x)$ *does not exist for negative values of* x.

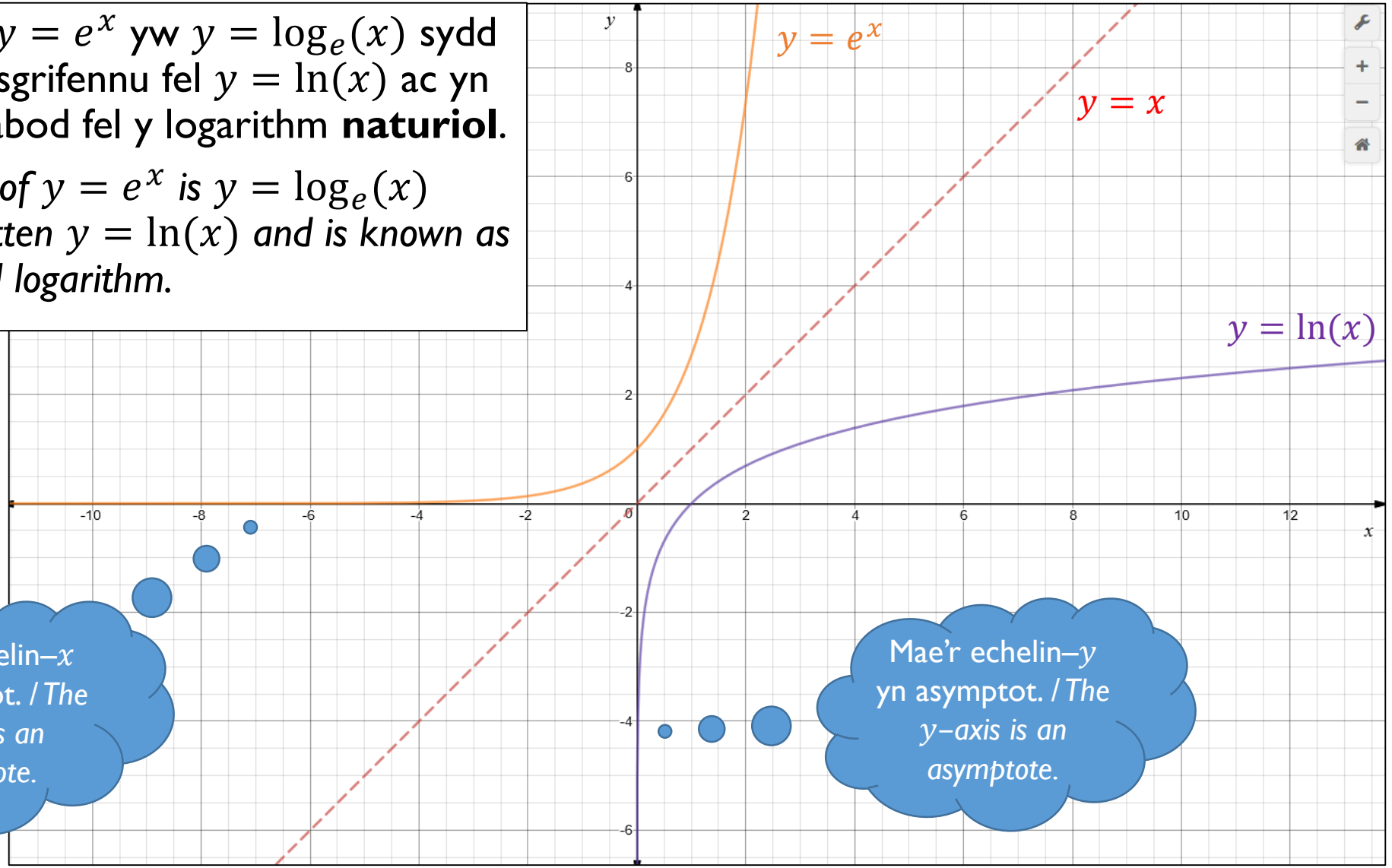
O hyd yn pasio trwy (1, 0). / *Always passes through (1, 0).*

www.desmos.com/calculator

Logarithm Naturiol / *Natural Logarithm*

Gwrthdro $y = e^x$ yw $y = \log_e(x)$ sydd yn cael ei ysgrifennu fel $y = \ln(x)$ ac yn cael ei adnabod fel y logarithm **naturiol**.

The inverse of $y = e^x$ *is* $y = \log_e(x)$ *which is written* $y = \ln(x)$ *and is known as the* ***natural*** *logarithm.*



Mae'r echelin–x yn asymptot. / *The x–axis is an asymptote.*

Mae'r echelin–y yn asymptot. / *The y–axis is an asymptote.*

www.desmos.com/calculator

Ffwythiannau Esbonyddol / *Exponential Functions*

Cwblhau DAE Cwestiwn 15. / *Complete SAMs Question 15.*

Cwblhau ymarferion o'r gwerslyfr. / *Complete exercises from the book.*