Logarithm Proofs

(1) $\log_a(xy) = \log_a(x) + \log_a(y)$

Let $m = \log_a(x)$, $n = \log_a(y)$.
Then $a^m = x$, $a^n = y$.
Therefore $xy = a^m \times a^n$.
$xy = a^{m+n}$ (rules of indices).
$\log_a(xy) = m + n$.
$\log_a(xy) = \log_a(x) + \log_a(y)$.

(2) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

Let $m = \log_a(x)$, $n = \log_a(y)$.
Then $a^m = x$, $a^n = y$.
Therefore $\frac{x}{y} = \frac{a^m}{a^n}$.
$\frac{x}{y} = a^{m-n}$ (rules of indices).
$\log_a\left(\frac{x}{y}\right) = m - n$.
$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$.

(3) $\log_a\left(x^k\right) = k\log_a(x)$

Let $y = \log_a(x)$.
Then $a^y = x$.
Therefore $(a^y)^k = x^k$.
$a^{yk} = x^k$ (rules of indices).
$yk = \log_a(x^k)$.
$ky = \log_a(x^k)$.
Therefore $k\log_a(x) = \log_a(x^k)$
or $\log_a\left(x^k\right) = k\log_a(x)$.