# Logarithmau 1

*Logarithms 1*

YSGOL Y CREUDDYN
DAWN DYSG DAIONI

@mathemateg

YouTube /adolygumathemateg

# Logarithmau / *Logarithms*

Mae logarithm yn gwrthdroi cymryd **esbonydd rhif**.
*A logarithm is the inverse of taking the* ***exponent of a number***.

Er enghraifft / *For example,*

$$2^3 = 2 \times 2 \times 2$$
$$= 8$$

Felly / *Therefore* $\log_2(8) = 3$

$$4^5 = 4 \times 4 \times 4 \times 4 \times 4$$
$$= 1024$$
$$\log_4(1024) = 5$$

**Diffiniad / Definition**

Os yw $b^y = x$ yna $y = \log_b(x)$.

$b$ yw bôn y logarithm.

*If* $b^y = x$ *then* $y = \log_b(x)$.

*$b$ is the base of the logarithm.*

**Logarithmau pwysig / Important logarithms**

Bôn / *base* 10: $\log(x)$.

E.g. $\log(1000) = 3$

Bôn / *base* $e$: $\ln(x)$.

$e = 2.71828\cdots$ yw'r rhif fel bod $\frac{d}{dx}(e^x) = e^x$.

# Logarithmau / *Logarithms*

**Ymarfer / Exercise 1**

Cyfrifwch / *Calculate*

(a) $\log_2(16)$

(b) $\log_3(9)$

(c) $\log(100)$

(ch) $\log_3(81)$

(d) $\log_4(16)$

(dd) $\log_{16}(16)$

(e) $\log_5(1)$

(f) $\log_3\left(\frac{1}{9}\right)$

(ff) $\log_4(2)$

(g) $\log_2\left(\frac{1}{64}\right)$

# Logarithmau / *Logarithms*

**Ymarfer / Exercise 1**

Cyfrifwch / *Calculate*

(a) $\log_2(16) = 4$

(b) $\log_3(9) = 2$

(c) $\log(100) = 2$

(ch) $\log_3(81) = 4$

(d) $\log_4(16) = 2$

(dd) $\log_{16}(16) = 1$

(e) $\log_5(1) = 0$

(f) $\log_3\left(\frac{1}{9}\right) = -2$

(ff) $\log_4(2) = \frac{1}{2}$

(g) $\log_2\left(\frac{1}{64}\right) = -6$

# Rheolau Logarithmau / *Logarithm Rules*

## (1) $\log_a(xy) = \log_a(x) + \log_a(y)$

Gadewch i $m = log_a(x),\ n = log_a(y)$.
Yna $a^m = x, a^n = y$.
Felly $xy = a^m \times a^n$.
$xy = a^{m+n}$ (rheolau indecsau).
$log_a(xy) = m + n$.
$log_a(xy) = log_a(x) + log_a(y)$.

Let $m = log_a(x), n = log_a(y)$.
Then $a^m = x, a^n = y$.
Therefore $xy = a^m \times a^n$.
$xy = a^{m+n}$ (rules of indices).
$log_a(xy) = m + n$.
$log_a(xy) = log_a(x) + log_a(y)$.

# Rheolau Logarithmau / *Logarithm Rules*

## (2) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

Gadewch i $m = log_a(x), n = log_a(y)$.

Yna $a^m = x, a^n = y$.

Felly $\frac{x}{y} = \frac{a^m}{a^n}$.

$\frac{x}{y} = a^{m-n}$ (rheolau indecsau).

$log_a\left(\frac{x}{y}\right) = m - n$.

$log_a\left(\frac{x}{y}\right) = log_a(x) - log_a(y)$.

Let $m = log_a(x), n = log_a(y)$.

Then $a^m = x, a^n = y$.

Therefore $\frac{x}{y} = \frac{a^m}{a^n}$.

$\frac{x}{y} = a^{m-n}$ (rules of indices).

$log_a\left(\frac{x}{y}\right) = m - n$.

$log_a\left(\frac{x}{y}\right) = log_a(x) - log_a(y)$.

# Rheolau Logarithmau / *Logarithm Rules*

## (3) $\log_a(x^k) = k\log_a(x)$

Gadewch i $y = log_a(x)$.
Yna $a^y = x$.
Felly $(a^y)^k = x^k$.
$a^{yk} = x^k$ (rheolau indecsau).
$yk = log_a(x^k)$.
$ky = log_a(x^k)$.
Felly $k\, log_a(x) = log_a(x^k)$
neu $log_a(x^k) = k\, log_a(x)$.

Let $y = log_a(x)$.
Then $a^y = x$.
Therefore $(a^y)^k = x^k$.
$a^{yk} = x^k$ (rules of indices).
$yk = log_a(x^k)$.
$ky = log_a(x^k)$.
Therefore $k\, log_a(x) = log_a(x^k)$
or $log_a(x^k) = k\, log_a(x)$.