


Trigonometreg Rhan 2

Trigonometry Part 2



 @mathemateg

 /adolygumathemateg

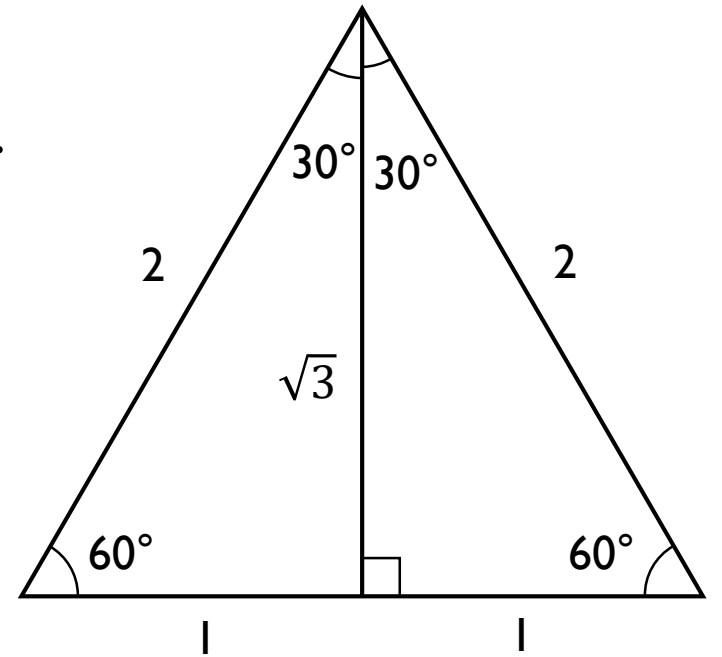
Trigonometreg: Union Werthoedd

Trigonometry: Exact Values

Ystyriwch driongl hafalochrog ble mae hyd yr ochrau yn 2 uned. Trwy hollti'r triongl yma yn ei hanner, mae'n bosib cyfrifo union werthoedd sin, cos a tan ar gyfer yr onglau 30° a 60° .

Consider an equilateral triangle having sides of 2 units in length. By halving this triangle, it is possible to calculate, for the angles 30° and 60° , the exact values of sin, cos and tan.

$$\begin{aligned}\sin(30^\circ) &= \frac{1}{2} & \cos(30^\circ) &= \frac{\sqrt{3}}{2} & \tan(30^\circ) &= \frac{1}{\sqrt{3}} \\ \sin(60^\circ) &= \frac{\sqrt{3}}{2} & \cos(60^\circ) &= \frac{1}{2} & \tan(60^\circ) &= \sqrt{3}\end{aligned}$$



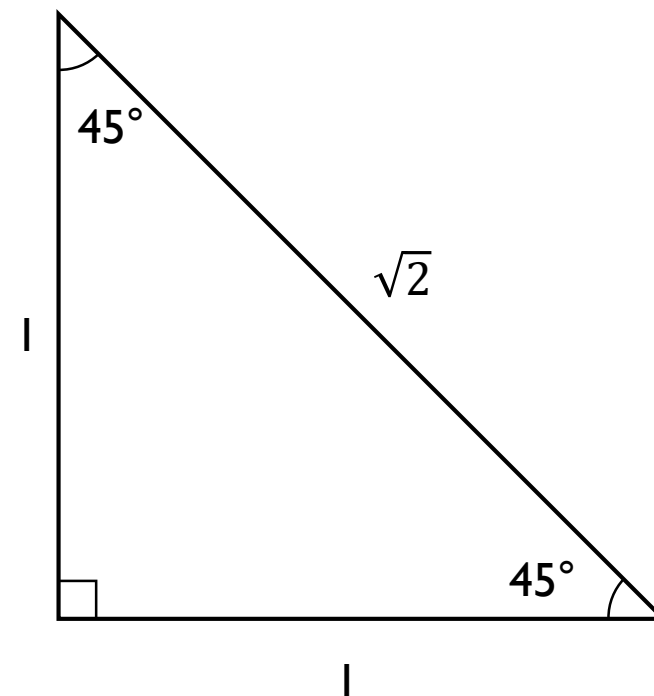
Trigonometreg: Union Werthoedd

Trigonometry: Exact Values

Ystyriwch driongl ongl sgwâr ble mae hyd yr ochrau byrraf yn 1 uned. Mae'n bosib defnyddio'r triongl yma i gyfrifo union werthoedd sin, cos a tan ar gyfer yr ongl 45° .

Consider a right-angled triangle whose shortest sides are 1 unit in length. It is possible to use this triangle to calculate the exact values of sin, cos and tan for the angle 45° .

$$\sin(45^\circ) = \frac{1}{\sqrt{2}} \quad \cos(45^\circ) = \frac{1}{\sqrt{2}} \quad \tan(45^\circ) = 1$$

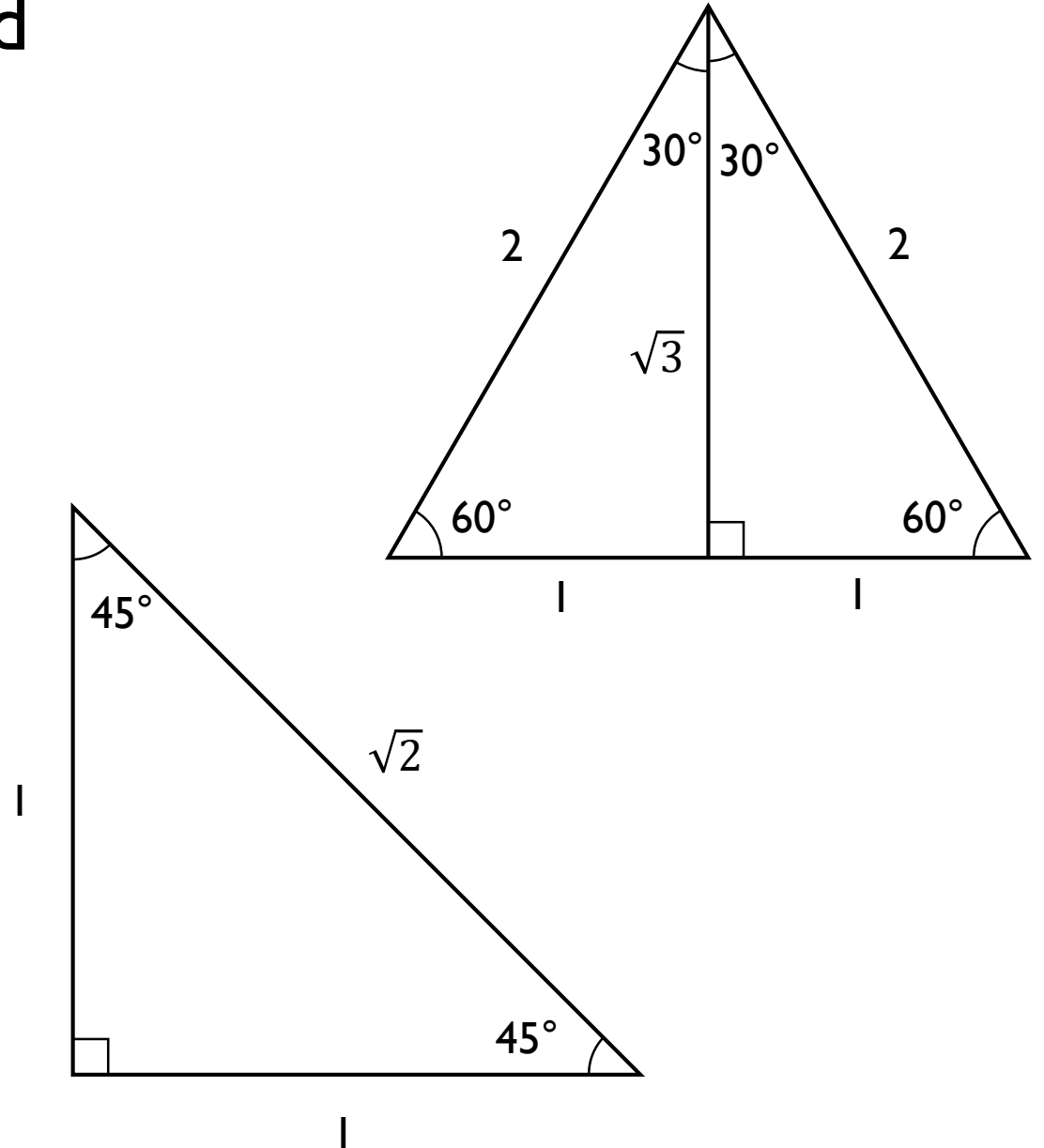


Trigonometreg: Union Werthoedd

Trigonometry: Exact Values

Crynodeb / Summary

	sin	cos	tan
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	Heb ei ddiffinio / Not defined



Rheol Sin

The Sine Rule

Yn defnyddio'r triongl BCD / Using the triangle BCD

$$h = a \sin B$$

Yn defnyddio'r triongl ACD / Using the triangle ACD

$$h = b \sin A$$

Felly / Therefore

$$a \sin B = b \sin A$$

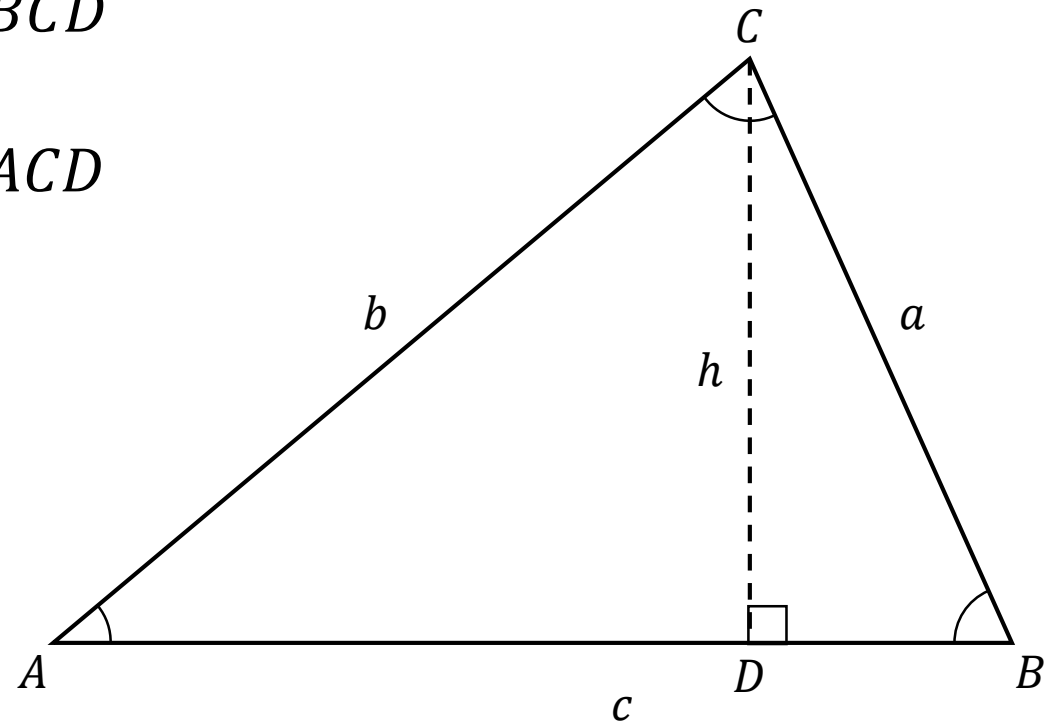
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Trwy gymryd y cilydd / By taking the reciprocal

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

I ffeindio ochr
/ To find a side

I ffeindio ongl
/ To find an angle



Rheol Cosin / *The Cosine Rule*

Yn defnyddio Theorem Pythagoras / Using Pythagoras' Theorem

$$x^2 + h^2 = b^2 \quad \text{and} \quad (c - x)^2 + h^2 = a^2$$

Yn tynnu er mwyn cael gwared â h^2 / Subtracting to eliminate h^2

$$x^2 - (c - x)^2 = b^2 - a^2$$

$$x^2 - (c - x)(c - x) = b^2 - a^2$$

$$x^2 - (c^2 - cx - cx + x^2) = b^2 - a^2$$

$$2cx - c^2 = b^2 - a^2$$

$$a^2 = b^2 + c^2 - 2cx$$

Ond / But $x = b \cos A \quad \therefore$

$$a^2 = b^2 + c^2 - 2c(b \cos A)$$

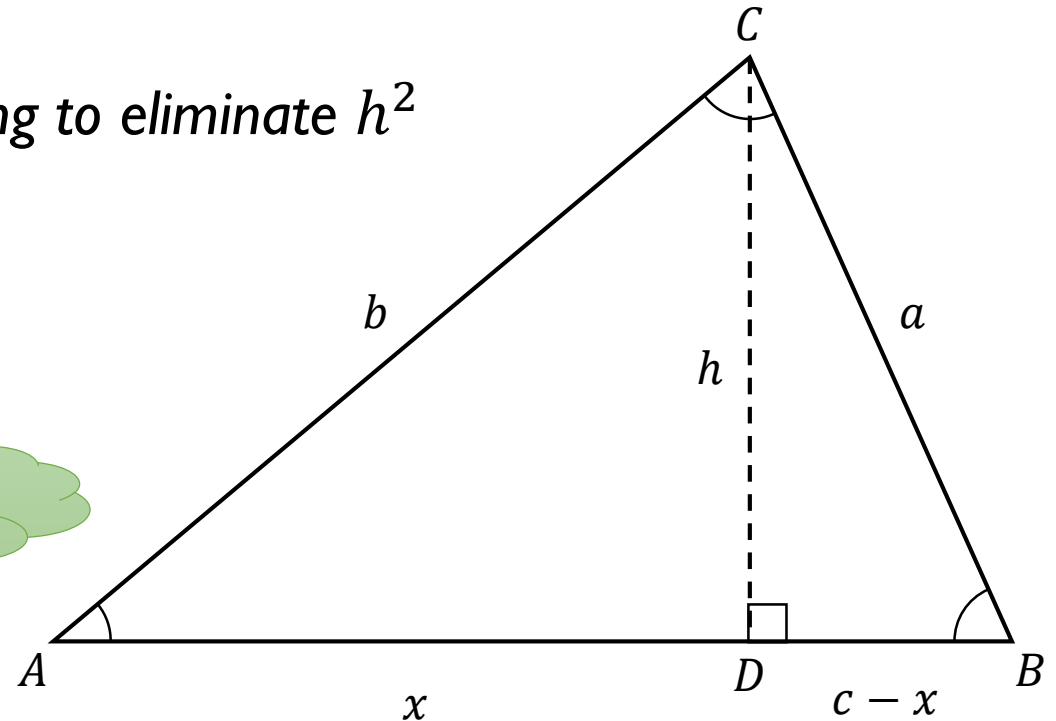
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Gellir ail-drefnu hwn i roi / This can be re-arranged to give

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

I ffeindio ochr
/ To find a side

I ffeindio ongl
/ To find an angle



Arwynebedd Triongl *Area of a Triangle*

Arwynebedd triongl = $\frac{1}{2} \times \text{sail} \times \text{uchder}$

Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

Arwynebedd / Area = $\frac{1}{2} \times c \times h$

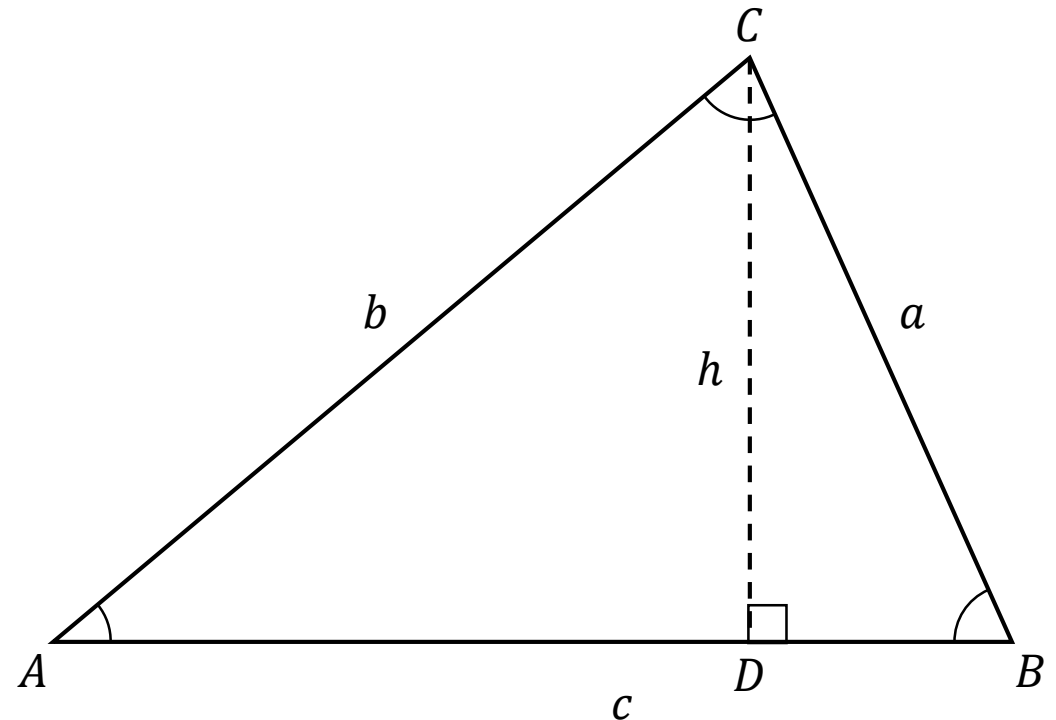
Ond / But $h = b \sin A \quad \therefore$

Arwynebedd / Area = $\frac{1}{2} \times c \times b \sin A$

Arwynebedd / Area = $\frac{1}{2} bc \sin A$

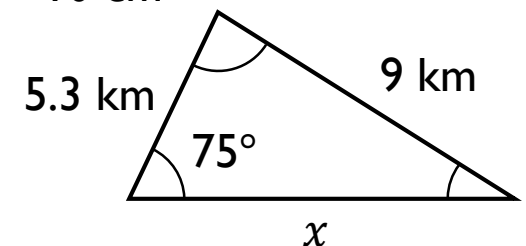
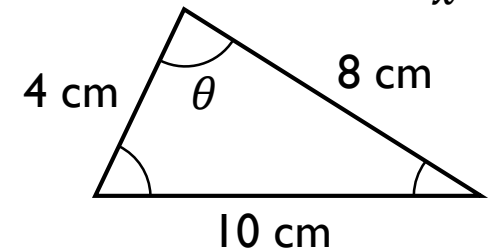
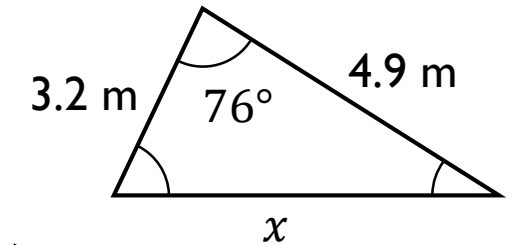
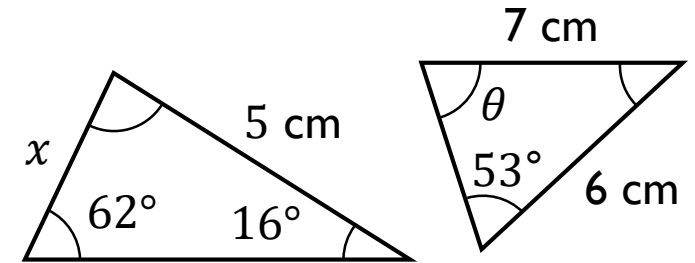
Gellir ail-labelu'r triongl i roi'r fersiwn
cyfarwydd / *The triangle can be re-labelled
to give the familiar version*

Arwynebedd / Area = $\frac{1}{2} ab \sin C$



Rheol Sin neu Cosin? Sine or Cosine Rule?

Yn gwybod: <i>You know:</i>	Eisiau ffeindio: <i>You need to find:</i>	Rheol Sin neu Cosin? <i>Sine or Cosine Rule?</i>
Tri o'r gwerthoedd mewn dau bâr o ochrau/onglau cyferbyn. <i>Three values in two pairs of opposite angles/sides.</i>	Yr ongl neu'r ochr coll mewn un pâr. <i>The missing angle or side in one pair.</i>	Rheol Sin. <i>The Sine Rule.</i>
Dwy ochr a'r ongl rhwng yr ochrau. <i>Two sides and the angle between the sides.</i>	Yr ochr gyferbyn yr ongl rydych yn gwybod. <i>The side opposite the known angle.</i>	Rheol Cosin. <i>The Cosine Rule.</i>
Yr ochrau i gyd. <i>All three sides.</i>	Unrhyw ongl. <i>Any angle.</i>	Rheol Cosin. <i>The Cosine Rule.</i>
Dwy ochr ag un ongl nad yw rhwng yr ochrau. <i>Two sides and an angle that is not between the sides.</i>	Yr ochr arall. <i>The other side.</i>	Rheol Sin dwywaith, neu'r Rheol Cosin. <i>The Sine Rule twice, or the Cosine Rule.</i>



Rheol Sin: Yr achos amwys

The Sine Rule: The ambiguous case

O wybod bod $AB = 5$ cm, $BC = 4$ cm, $\hat{BAC} = 38^\circ$,
beth yw gwerthoedd posib \hat{ACB} ?

*Given that $AB = 5$ cm, $BC = 4$ cm, $\hat{BAC} = 38^\circ$,
what are the possible values of \hat{ACB} ?*

Mae C yn gorwedd rhywle ar y llinell lorweddol.
C lies somewhere on the horizontal line.

Yn defnyddio Rheol Sin / *Using the Sine Rule*

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = 5 \times \frac{\sin 38^\circ}{4}$$

$$C = \sin^{-1} \left(5 \times \frac{\sin 38^\circ}{4} \right)$$

Naill ai / *either $C = 50.32^\circ$ neu / or $C = 129.68^\circ$ (2 l.d. / 2 d.p.)*

