Hafaliad Cylch

Equation of a Circle

@mathemateg

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Hafaliad Cylch
Equation of a Circle

Mae dwy ffordd o ysgrifennu hafaliad cylch. *There are two ways of writing the equation of a circle.*

1) $(x-a)^2+(y-b)^2=r^2$

Hafaliad cylch efo radiws r a chanol (a,b).
Mae hwn yn achos arbennig o Theorem Pythagoras.
The equation of a circle with radius r and centre (a,b).
This is a special case of Pythagoras' Theorem.

https://ggbm.at/RARM4esQ

2) $x^2+y^2+2gx+2fy+c=0$

Hafaliad cyffredinol cylch efo canol $(-g,-f)$ a radiws $\sqrt{g^2+f^2-c}$.
The general equation of a circle with centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$.

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Ymarfer 1 / *Exercise 1*

Darganfyddwch ganol a radiws y cylchoedd canlynol.
Find the radius and centre of the following circles.

(a) $(x-3)^2+(y+7)^2=25$

(b) $(x+8)^2+(y-4)^2=20$

(c) $(x-2)^2+(y+11)^2=49$

(ch) $x^2+y^2=144$

(d) $x^2+y^2+6x+2y-6=0$

(dd) $x^2+y^2+14x-4y-5=0$

(e) $2x^2+2y^2-24x+16y+6=0$

(f) $x^2+y^2+7y+3.25=0$

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Ymarfer 1 / *Exercise 1*

Darganfyddwch ganol a radiws y cylchoedd canlynol. *Find the radius and centre of the following circles.*	Canol *Centre*	Radiws *Radius*
(a) $(x-3)^2+(y+7)^2=25$	(3, –7)	5
(b) $(x+8)^2+(y-4)^2=20$	(–8, 4)	$\sqrt{20}=2\sqrt{5}$
(c) $(x-2)^2+(y+11)^2=49$	(2, –11)	7
(ch) $x^2+y^2=144$	(0, 0)	12
(d) $x^2+y^2+6x+2y-6=0$	(–3, –1)	4
(dd) $x^2+y^2+14x-4y-5=0$	(–7, 2)	$\sqrt{58}$
(e) $2x^2+2y^2-24x+16y+6=0$	(6, –4)	7
(f) $x^2+y^2+7y+3.25=0$	(0, –3.25)	3

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Dangos bod y ddau ddull yn gyfwerth. *Showing that the two methods are equivalent.*

$x^2 + y^2 + 2gx + 2fy + c = 0$

$(x + g)^2 - g^2 + (y + f)^2 - f^2 + c = 0$ Cwblhau'r sgwâr / *Completing the square*

$(x + g)^2 + (y + f)^2 = g^2 + f^2 - c$ Ail-drefnu / *Re-arranging*

Yn cymharu efo / *Comparing with* $(x - a)^2 + (y - b)^2 = r^2$:

$g = -a$ $f = -b$ $g^2 + f^2 - c = r^2$

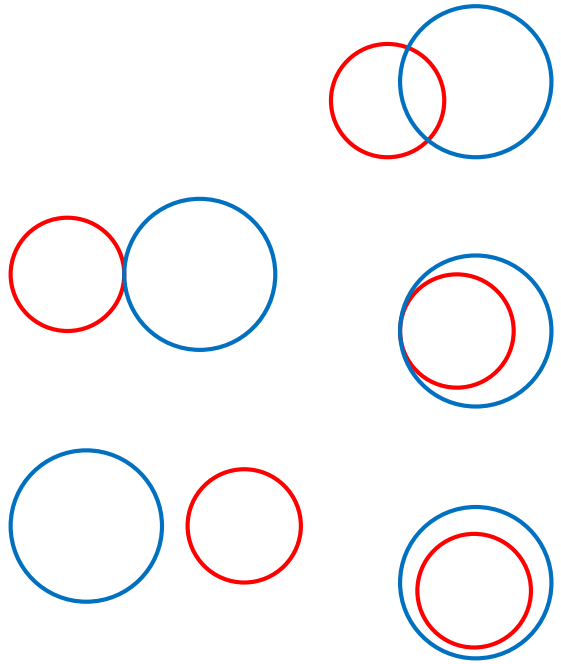
$-a = g$ $-b = f$ $r^2 = g^2 + f^2 - c$

$a = -g$ $b = -f$ $r = \sqrt{g^2 + f^2 - c}$

Cylchoedd yn Croestorri
Intersecting Circles

Ystyriwch ddau gylch C_1 ag C_2, efo radiysau r_1 ag r_2, yn ôl eu trefn.
Gadewch i p gynrychioli'r pellter rhwng canol C_1 a chanol C_2.
Consider two circles C_1 and C_2, with respective radii r_1 and r_2.
Let d represent the distance between the centres of the two circles.

- Os yw C_1 ag C_2 yn croestorri mewn dau bwynt, yna $p < r_1 + r_2$.
  If C_1 and C_2 intersect at two points, then $d < r_1 + r_2$.
- Os yw C_1 ag C_2 yn cyfarfod mewn un pwynt yn unig, yna naill ai $p = r_1 + r_2$ (cyffwrdd yn allanol) neu $p = |r_1 - r_2|$ (cyffwrdd yn fewnol).
  If C_1 and C_2 meet at exactly one point, then either $d = r_1 + r_2$ (touch externally) or $d = |r_1 - r_2|$ (touch internally).
- Os nad yw C_1 ag C_2 yn croestorri, yna naill ai $p > r_1 + r_2$ (cylchoedd ar wahân) neu $p < |r_1 - r_2|$ (un cylch o fewn y llall).
  If C_1 and C_2 do not intersect, then either $d > r_1 + r_2$ (circles are separate) or $d < |r_1 - r_2|$ (one circle appears inside the other).



https://ggbm.at/gpPhx6B3

Theoremau Cylchoedd
Circle Theorems

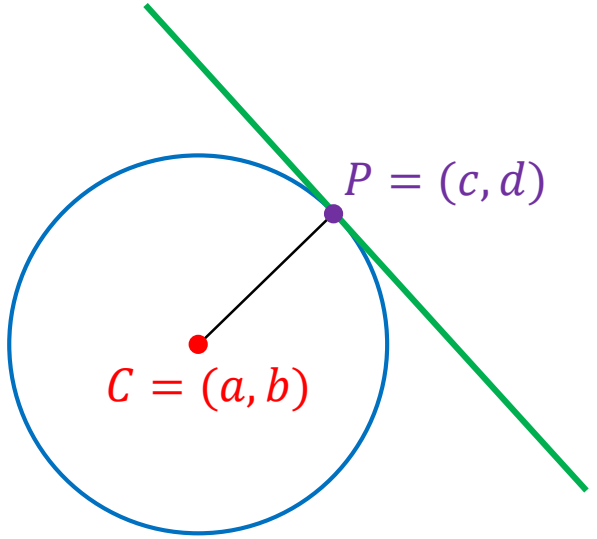
Mae angen bod yn gyfarwydd â'r theoremau cylchoedd canlynol.
You need to be aware of the following circle theorems.

https://ggbm.at/aBB3qZC6

- Mae'r ongl mewn hanner cylch yn ongl sgwâr.
  The angle in a semicircle is a right angle.
- Mae'r perpendicwlar o'r canol i gord yn haneru'r cord.
  The perpendicular from the centre to a chord bisects the chord.
- Mae radiws cylch ar bwynt penodol ar ei gylchyn yn berpendicwlar i'r tangiad i'r cylch ar y pwynt hwnnw.
  The radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.

Hafaliad y Tangiad
Equation of the Tangent

Gadewch i $P = (c, d)$ fod yn bwynt ar gylch efo canol $C = (a, b)$.
Let $P = (c, d)$ be a point on the circle with centre $C = (a, b)$.



- Graddiant y radiws PC yw $\frac{b-d}{a-c}$.
  The gradient of the radius PC is $\frac{b-d}{a-c}$.
- Mae radiws a tangiad yn cyfarfod ar ongl sgwâr,
  felly graddiant y tangiad i'r cylch yn y pwynt P yw $-\frac{a-c}{b-d}$.
  Radius and tangent meet at a right angle,
  so the gradient of the tangent to the circle at the point P is $-\frac{a-c}{b-d}$.
- Hafaliad y tangiad i'r cylch yn y pwynt P yw $y - d = -\frac{a-c}{b-d}(x - c)$.
  The equation of the tangent to the circle at the point P is $y - d = -\frac{a-c}{b-d}(x - c)$.

https://ggbm.at/W7Pby5sd