2.1 AS UNIT 1

Unit 1: Further Pure Mathematics A
Written examination: 1 hour 30 minutes
$13\frac{1}{3}$% of A level qualification ($33\frac{1}{3}$% of AS qualification)
70 marks

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in AS Mathematics.
Where specific content requires knowledge of concepts or results from A2 Mathematics, this will be made explicit in the Guidance section of the content.

Topics	Guidance
2.1.1 Proof	
Construct proofs using mathematical induction. Contexts include sums of series, powers of matrices and divisibility.	Including application to the proof of the binomial theorem for a positive integral power. eg. the proof of the divisibility of $5^{2n}-1$ by 24. *Knowledge of the Σ notation is assumed.*
2.1.2 Complex Numbers	
Solve any quadratic equation with real coefficients. Solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics).	
Add, subtract, multiply and divide complex numbers in the form $x + \mathrm{i}y$, with x and y real. Understand and use the terms 'real part' and 'imaginary part'.	

Topics	Guidance
Understand and use the complex conjugate. Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs	The complex conjugate of z will be denoted by $\overline{z}$.
Equate the real and imaginary parts of a complex number.	Including the solution of equations such as $z+2\overline{z}=\frac{1+2\text{i}}{1-\text{i}}$.
Use and interpret Argand diagrams	Includes representing complex numbers by points in an Argand diagram.
Understand and use the Cartesian (algebraic) and modulus-argument (trigonometric) forms of a complex number. Convert between the Cartesian form and modulus-argument form of a complex number.	$z=x+\text{i}y$ and $z=r(\cos\theta+\text{i}\sin\theta)$ where $\theta=\arg(z)$ may be taken to be in either $[0, 2\pi)$ or $(-\pi, \pi]$ or $[0, 360^{\circ})$ or $(-180^{\circ}, 180^{\circ}]$. *Knowledge of radians is assumed.*
Multiply and divide complex numbers in modulus-argument form.	*Knowledge of radians and compound angle formulae is assumed.*
Construct and interpret simple loci in an Argand diagram, such as $\|z-a\|>r$ and $\arg(z-a)=\theta$.	For example, $\|z-1\|=2\|z+\text{i}\|$. *Knowledge of radians is assumed.*
Simple cases of transformations of lines and curves defined by $w=f(z)$.	For example, the image of the line $x+y=1$ under the transformation defined by $w=z^2$.

Topics	Guidance
2.1.3 Matrices	
Add, subtract and multiply conformable matrices. Multiply a matrix by a scalar.	
Understand and use zero and identity matrices. Understand and use the transpose of a 2 x 2 matrix.	
Use matrices to represent • linear and non-linear transformations in 2-D, involving 2 x 2 and 3 x 3 matrices, • successive transformations, • single transformations in 3-D (3-D transformations confined to reflection in one of x = 0, y = 0, z = 0 or rotation about one of the coordinate axes).	Transformations to only include translation, rotation and reflection, using 2 x 2 and/or 3 x 3 matrices. Knowledge that the transformation represented by **AB** is the transformation represented by **B** followed by the transformation represented by **A**. *Knowledge of 3-D vectors is assumed.*
Find invariant points and lines for linear and non-linear transformations.	
Calculate determinants of 2 x 2 matrices.	Use and understand the notation $\lvert\mathbf{M}\rvert$ or det **M** or $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ or Δ.
Understand and use singular and non-singular matrices. Understand and use properties of inverse matrices. Calculate and use the inverse of non-singular 2 x 2 matrices.	
2.1.4 Further Algebra and Functions	
Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations.	
Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).	

Topics	Guidance
Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series. Understand and use the method of differences for summation of series, including the use of partial fractions.	Summation of a finite series. Use of formulae for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$. Including mathematical induction (see section on Proof) and difference methods. Summation of series such as $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ and $\sum_{r=1}^{n} (2r+1)^3$. *Knowledge of the* Σ *notation and partial fractions is assumed.*
2.1.5 Further Vectors	
Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D.	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and $\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3}$ *Knowledge of 3-D vectors is assumed.*
Understand and use the vector and Cartesian forms of the equation of a plane.	
Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane.	$\mathbf{a}.\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ The form $\mathbf{r}.\mathbf{n} = k$ for a plane.
Use the scalar product to check whether vectors are perpendicular.	
Find the intersection of a line and a plane.	
Calculate the perpendicular distance between two lines, from a point to a line and a point to a plane.	