GCE

MATHEMATICS

UNIT 1: PURE MATHEMATICS A

SAMPLE ASSESSMENT MATERIALS

(2 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. The circle C has centre A and equation

$$x^2 + y^2 - 2x + 6y - 15 = 0.$$

(*a*) Find the coordinates of A and the radius of C. [3]

(*b*) The point P has coordinates $(4, -7)$ and lies on C. Find the equation of the tangent to C at P. [4]

2. Find all values of θ between 0° and 360° satisfying

$$7\sin^2\theta + 1 = 3\cos^2\theta - \sin\theta. \quad [6]$$

3. Given that $y = x^3$, find $\dfrac{dy}{dx}$ from first principles. [6]

4. The cubic polynomial $f(x)$ is given by $f(x) = 2x^3 + ax^2 + bx + c$, where a, b, c are constants. The graph of $f(x)$ intersects the x-axis at the points with coordinates $(-3, 0)$, $(2{\cdot}5, 0)$ and $(4, 0)$. Find the coordinates of the point where the graph of $f(x)$ intersects the y-axis. [5]

5. The points $A(0, 2)$, $B(-2, 8)$, $C(20, 12)$ are the vertices of the triangle ABC. The point D is the mid-point of AB.

(*a*) Show that CD is perpendicular to AB. [6]

(*b*) Find the exact value of $\tan C\hat{A}B$. [5]

(*c*) Write down the geometrical name for the triangle ABC. [1]

6. In each of the two statements below, c and d are real numbers. One of the statements is true while the other is false.

A Given that $(2c + 1)^2 = (2d + 1)^2$, then $c = d$.
B Given that $(2c + 1)^3 = (2d + 1)^3$, then $c = d$.

(*a*) Identify the statement which is false. Find a counter example to show that this statement is in fact false.

(*b*) Identify the statement which is true. Give a proof to show that this statement is in fact true. [5]

7. Figure 1 shows a sketch of the graph of $y = f(x)$. The graph has a minimum point at $(-3, -4)$ and intersects the x-axis at the points $(-8, 0)$ and $(2, 0)$.

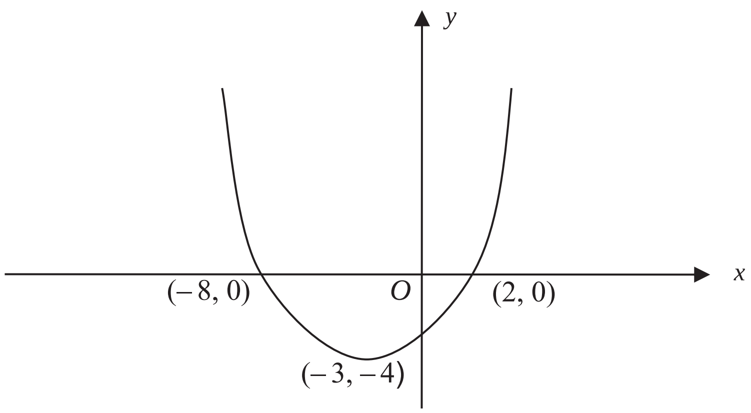


Figure 1

(*a*) Sketch the graph of $y = f(x + 3)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(*b*) Figure 2 shows a sketch of the graph having **one** of the following equations with an appropriate value of either p, q or r.

$y = f(px)$, where p is a constant
$y = f(x) + q$, where q is a constant
$y = rf(x)$, where r is a constant

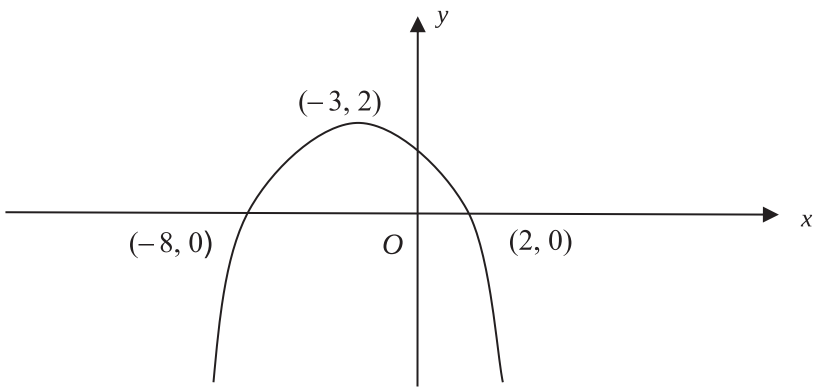


Figure 2

Write down the equation of the graph sketched in Figure 2, together with the value of the corresponding constant. [2]

8. The circle C has radius 5 and its centre is the origin.
The point T has coordinates (11, 0).
The tangents from T to the circle C touch C at the points R and S.

(*a*) Write down the geometrical name for the quadrilateral *ORTS*. [1]

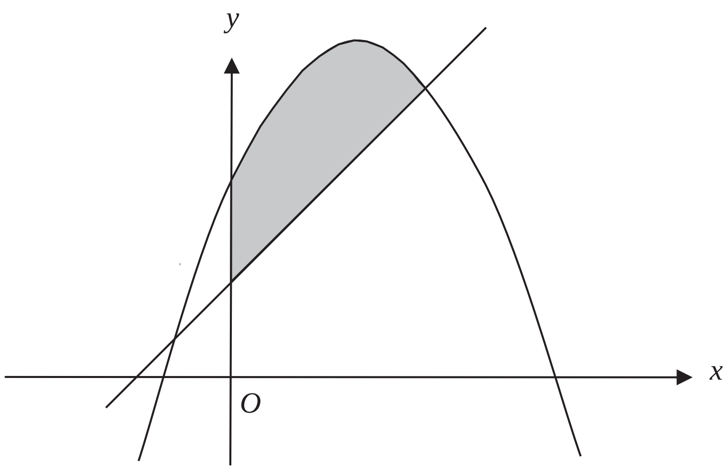
(*b*) Find the exact value of the area of the quadrilateral *ORTS*. Give your answer in its simplest form. [5]

9. The quadratic equation $4x^2 - 12x + m = 0$, where m is a positive constant, has **two distinct** real roots.
Show that the quadratic equation $3x^2 + mx + 7 = 0$ has **no** real roots. [7]

10. (*a*) **Use the binomial theorem** to express $\left(\sqrt{3} - \sqrt{2}\right)^5$ in the form $a\sqrt{3} + b\sqrt{2}$, where a, b are integers whose values are to be found. [5]

(*b*) Given that $\left(\sqrt{3} - \sqrt{2}\right)^5 \approx 0$, use your answer to part (*a*) to find an approximate value for $\sqrt{6}$ in the form $\frac{c}{d}$, where c and d are positive integers whose values are to be found. [3]

11.



The diagram shows a sketch of the curve $y = 6 + 4x - x^2$ and the line $y = x + 2$. The point P has coordinates (a, b). Write down the three inequalities involving a and b which are such that the point P will be strictly contained within the shaded area above, if and only if, all three inequalities are satisfied. [3]

12. Prove that

$$\log_7 a \times \log_a 19 = \log_7 19$$

whatever the value of the positive constant a. [3]

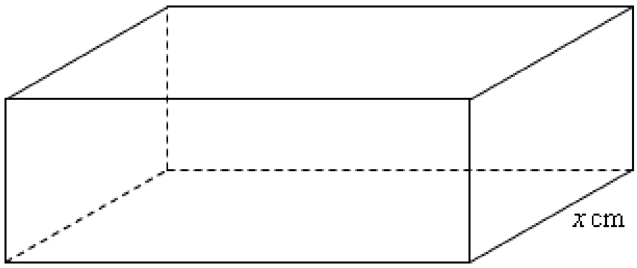
13. In triangle *ABC*, *BC* = 12 cm and $\cos A\hat{B}C = \frac{2}{3}$.

The length of *AC* is 2 cm greater than the length of *AB*.

(*a*) Find the lengths of *AB* and *AC*. [4]

(*b*) Find the exact value of $\sin B\hat{A}C$. Give your answer in its simplest form. [3]

14. The diagram below shows a closed box in the form of a cuboid, which is such that the length of its base is twice the width of its base. The volume of the box is 9000 cm^3. The total surface area of the box is denoted by S cm^2.



(*a*) Show that $S = 4x^2 + \frac{27000}{x}$, where x cm denotes the width of the base. [3]

(*b*) Find the minimum value of S, showing that the value you have found is a minimum value. [5]

15. The size N of the population of a small island at time t years may be modelled by $N = Ae^{kt}$, where A and k are constants. It is known that N = 100 when t = 2 and that N = 160 when t = 12.

(*a*) Interpret the constant A in the context of the question. [1]

(*b*) Show that k = 0·047, correct to three decimal places. [4]

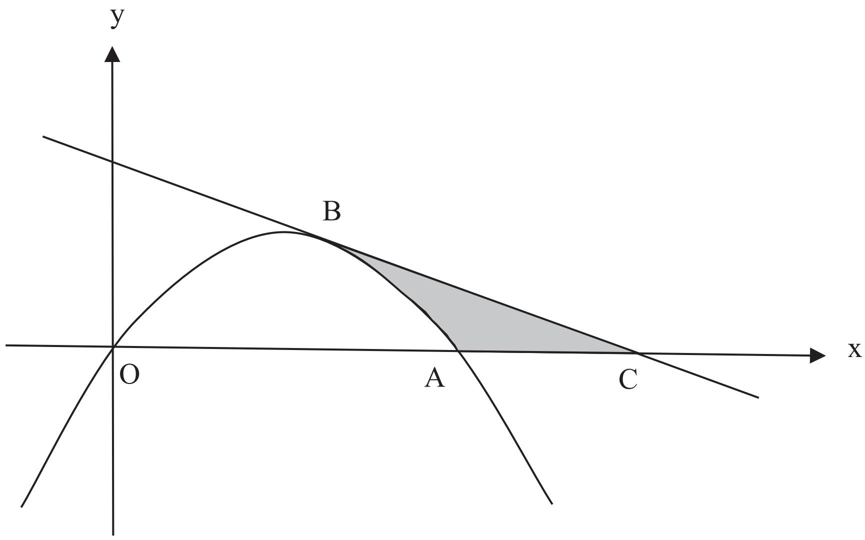
(*c*) Find the size of the population when t = 20. [3]

16. Find the range of values of x for which the function

$$f(x) = x^3 - 5x^2 - 8x + 13$$

is an increasing function. [5]

17.



The diagram above shows a sketch of the curve $y = 3x - x^2$. The curve intersects the x-axis at the origin and at the point A. The tangent to the curve at the point $B(2, 2)$ intersects the x-axis at the point C.

(*a*) Find the equation of the tangent to the curve at B. [4]

(*b*) Find the area of the shaded region. [8]

18. (*a*) The vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$.

(i) Find the vector $4\mathbf{u} - 3\mathbf{v}$.

(ii) The vectors $\mathbf{u}$ and $\mathbf{v}$ are the position vectors of the points U and V, respectively. Find the length of the line UV. [4]

(*b*) Two villages A and B are 40 km apart on a long straight road passing through a desert. The position vectors of A and B are denoted by $\mathbf{a}$ and $\mathbf{b}$, respectively.

(i) Village C lies on the road between A and B at a distance 4 km from B. Find the position vector of C in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Village D has position vector $\frac{2}{9}\mathbf{a} + \frac{5}{9}\mathbf{b}$. Explain why village D cannot possibly be on the straight road passing through A and B. [3]