GCE AS MARKING SCHEME

SUMMER 2024

AS
MATHEMATICS
UNIT 1 PURE MATHEMATICS A
2300U10-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE AS MATHEMATICS

UNIT 1 PURE MATHEMATICS A

SUMMER 2024 MARK SCHEME

Q	Solution	Mark	Notes
1	$y = 12x^{\frac{1}{2}} - 27x^{-1} + 4$	B1	index form, si
	$\frac{dy}{dx} = 12 \times \frac{1}{2} \times x^{-\frac{1}{2}} - 27 \times (-1)x^{-2}$	B1	correct 1st term, ft fractional index.
		B1	correct 2nd term, ft -ve index.
	$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + 27x^{-2}$		
	When $x = 9$, $\frac{dy}{dx} = 2 - \left(-\frac{1}{3}\right) = \frac{7}{3}$	B1	cao, accept decimal answers correctly derived from $\frac{7}{3}$, isw

Q	Solution	Mark	Notes
2	$2\sin 2\theta = 1$		
	$\sin 2\theta = \frac{1}{2}$		
	$2\theta = 30°, 150°$	B1	either angle, si
	$\theta = 15°,$	B1	
	$\theta = 75°$	B1	

Notes:

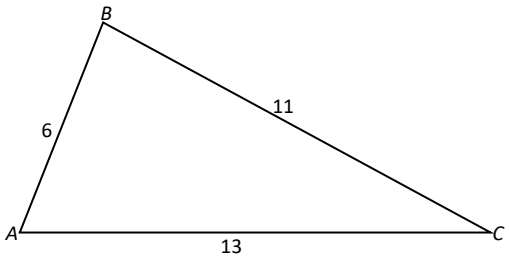
Ignore answers not in the range $0° < \theta < 180°$.

For an incorrect 3rd answer, -1 mark from the last 2 marks.

For an incorrect 4th answer, -1 mark from the last 2 marks.

Q	Solution	Mark	Notes
3	$\int(5x^{\frac{1}{4}} + 3x^{-2} - 2)\,\mathrm{d}x$		
	$= \frac{5}{5/4} \times x^{\frac{5}{4}} + \frac{3x^{-1}}{-1} - 2x + C$	B1	$\frac{5}{5/4} \times x^{\frac{5}{4}}$, oe, isw
		B1	$+\frac{3}{-1}x^{-1}$, oe, isw
		B1	$-2x$, isw
			–1 no C
	$= 4x^{\frac{5}{4}} - 3x^{-1} - 2x + C$		

Q	Solution		Mark	Notes
4	n	$n^2 - 2$		
	1	-1		
	2	2		
	3	7		
	4	14		
	5	23		
	6	34	M1	evaluating $n^2 - 2$ (for 1,…,6) at least twice
			A1	at least 4 values correct. Implied by correct conclusion.
	$n^2 - 2$ is not divisible by 3 for $1 \leq n \leq 6$		A1	all values correct, and conclusion.

Q	Solution	Mark	Notes
5			
	Cosine rule attempted	M1	allow one error Must have 2 and cos
	$11^2 = 6^2 + 13^2 - 2\times6\times13\cos\alpha$ $13^2 = 6^2 + 11^2 - 2\times6\times11\cos\beta$ $6^2 = 13^2 + 11^2 - 2\times13\times11\cos\gamma$	A1	any one
	$\cos\alpha = \frac{7}{13}$; $\cos\beta = -\frac{1}{11}$, $\cos\gamma = \frac{127}{143}$, ($\alpha = 57.421°$, $\beta = 95.216°$, $\gamma = 27.363°$)		si
	Area $= \frac{1}{2}\times 6\times 13\sin\left(\cos^{-1}\left(\frac{7}{13}\right)\right)$ Area $= \frac{1}{2}\times 6\times 11\sin\left(\cos^{-1}\left(-\frac{1}{11}\right)\right)$ Area $= \frac{1}{2}\times 11\times 13\sin\left(\cos^{-1}\left(\frac{127}{143}\right)\right)$	M1	any one, FT their angle
	Area = 32.86…(cm^2)	A1	Accept 32.7 to 33.0, cao

Q	Solution	Mark	Notes
6(a)	$7x^{\frac{3}{4}} = \sqrt{147}$	B1	collect x term, $x^{\frac{3}{4}}$.
	$7x^{\frac{3}{4}} = 7\sqrt{3}$		
	$x^{\frac{3}{4}} = 3^{\frac{1}{2}}$	B1	collect constant term, $3^{\frac{1}{2}}$.
	$x = \left(3^{\frac{1}{2}}\right)^{\frac{4}{3}}$ or $x^3 = 9$	B1	si by correct answer
	$x = 3^{\frac{2}{3}}$	B1	cao, oe $\sqrt[3]{3^2} = \sqrt[3]{9}$
	OR		
	$7x^{\frac{5}{4}} = \sqrt{147}x^{\frac{1}{2}}$		
	$49x^{\frac{5}{2}} = 147x$		
	$49x^{\frac{3}{2}} = 147$	(B1)	collect x term, $x^{\frac{3}{2}}$.
	$x^{\frac{3}{2}} = 3$	(B1)	collect constant term, 3.
	$x = 3^{\frac{2}{3}}$	(B1)(B1)	
	Note		
	Sight of 2.08… without working 0 marks		
	Sight of 2.08… from correct working B1 B1 B1 B0		

Q	Solution	Mark	Notes
6(b)	$\frac{(8x-18)}{(2\sqrt{x}-3)} = \frac{(8x-18)(2\sqrt{x}+3)}{(2\sqrt{x}-3)(2\sqrt{x}+3)}$	M1	
	$= \frac{(8x-18)(2\sqrt{x}+3)}{(4x-9)}$	A1	correct denominator simplified
	$= \frac{2(4x-9)(2\sqrt{x}+3)}{(4x-9)}$		
	$= 2(2\sqrt{x}+3)$	A1	convincing
	OR		
	$\frac{(8x-18)}{(2\sqrt{x}-3)} = \frac{2(4x-9)}{(2\sqrt{x}-3)}$	(B1)	
	$= \frac{2(2\sqrt{x}+3)(2\sqrt{x}-3)}{(2\sqrt{x}-3)}$	(M1)	
	$= 2(2\sqrt{x}+3)$	(A1)	convincing

Q	Solution	Mark	Notes
7(a)	Gradient $L_1 = \frac{4-0}{1-(-3)}$ (= 1)	B1	si
	Correct method for finding equation of line	M1	
	Equation of L_1 is $y - 0 = 1(x - (-3))$	A1	$y - 4 = 1(x - 1)$ isw
	$y = x + 3$		
7(b)(i)	$x + 3 = 3x - 3$	M1	FT part (a) for M1 A1 A1
	$x = 3$	A1	
	$y = 6$	A1	
7(b)(ii)	At D, $y = 0$, $0 = 3x - 3$		
	$x = 1$		
	D is the point (1, 0)	B1	allow verification
7(c)	area of $ACD = \frac{1}{2} \times 4 \times 6$	M1	oe, FT their coord of C
	= 12	A1	cao

Q	Solution	Mark	Notes
7(d)	angle $ACD = \tan^{-1}(6/6) - \tan^{-1}(2/6)$	M1	oe, FT their coord of C
	$= 45° - 18.4349°$		
	$= 26.57°$	A1	cao Accept 26.6°
OR	angle $ACD = \tan^{-1}(3) - \tan^{-1}(1)$	(M1)	oe, FT their coord of C
	$= 71.5651° - 45°$		
	$= 26.57°$	(A1)	cao Accept 26.6°
OR			
	$AC = \sqrt{(6-0)^2 + (3--3)^2} = \sqrt{72} = 6\sqrt{2}$		
	$CD = \sqrt{(6-0)^2 + (3-1)^2} = \sqrt{40} = 2\sqrt{10}$		
	Area $= \frac{1}{2} \times \sqrt{72} \times \sqrt{40} \sin ACD = 12$	(M1)	FT their coord of C
	$\sin ACD = \frac{1}{\sqrt{5}}$		
	angle $ACD = 26.57°$	(A1)	cao Accept 26.6°

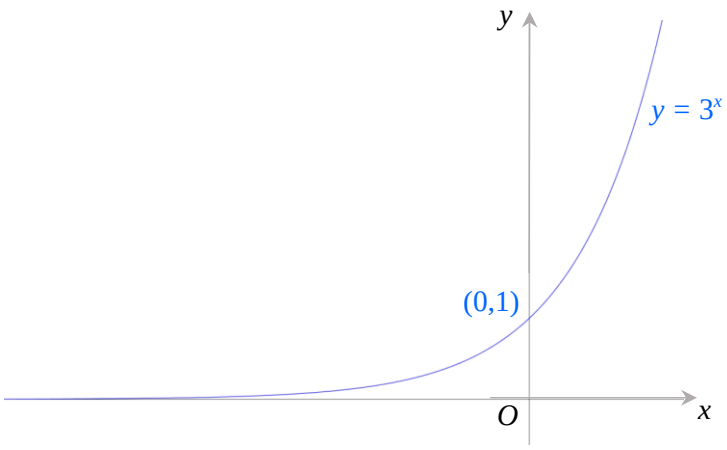
Q	Solution	Mark	Notes
8	$x - 10 < x^2 - 5x$		
	$x^2 - 6x + 10 > 0$	M1	For collecting terms on to one side
	$(x - 3)^2 + 1 > 0$	M1	oe or Showing minimum > 0 or Discriminant < 0 (and a point > 0) or Discriminant < 0 (and +ve quadratic) or correct sketch
		A1	all correct
	Valid explanation, e.g. minimum ≥ 1 This is true for all real values of x.	A1	Convincing

Q	Solution	Mark	Notes
9(a)	$(2 - x)^6$		
	$= 2^6 + 6 \times 2^5 \times (-x) + \frac{6 \times 5}{2 \times 1} \times 2^4 \times (-x)^2 + \ldots$	B3	B1 for each term, isw
	$= 64 - 192x + 240x^2 - \ldots$		
9(b)	$(1 + ax)(2 - x)^6$		
	$= (1 + ax)(64 - 192x + 240x^2 - \ldots)$	M1	replace $(2 - x)^6$ by answer in (a) provided 3 terms.
	$= 64 + 64ax - 192x + 240x^2 - 192ax^2 + \ldots$	A1	allow one slip, ignore extra terms FT (a)
	Therefore $64 + (64a - 192)x + (240 - 192a)x^2$ $\equiv 64 + bx + 336x^2 + \ldots$		
			FT (a)
	$64a - 192 = b$	m1	equating coefficients
	$240 - 192a = 336$	A1	both equations correct
	$a = -\frac{1}{2}$	A1	cao
	$b = -224$	A1	cao
	OR $(1 + ax)(2 - x)^6$		
	$= (1 + ax)(64 - 192x + 240x^2 - \ldots)$	M1	FT their (a), implied by next line
	$(1 + ax)(64 - 192x + 240x^2 - \ldots) \equiv$ $64 + bx + 336x^2 + \ldots$	A1	si
			FT (a)
	$64a - 192 = b$	m1	equating coefficients
	$240 - 192a = 336$	A1	both equations correct
	$a = -\frac{1}{2}$	A1	cao
	$b = -224$	A1	cao

Q	Solution	Mark	Notes
10(a)	$t^2 - 14t + 49 = 25$	M1	
	$t^2 - 14t + 24 = 0$		
	$(t - 2)(t - 12) = 0$	A1	method to solve must be seen
	$t = 2, 12$		
	Required value is $t = 2$, (since $t \leq 7$).	A1	$t = 12$ rejected
10(b)	$\frac{dy}{dt} = 2t - 14$	M1	at least 1 correct term.
	$t = 3$, $\frac{dy}{dt} = 2 \times 3 - 14$	A1	substitute $t = 3$, si
	$\frac{dy}{dt} = -8$	A1	cao
	Rate of decrease is 8 (cms^{-1})		

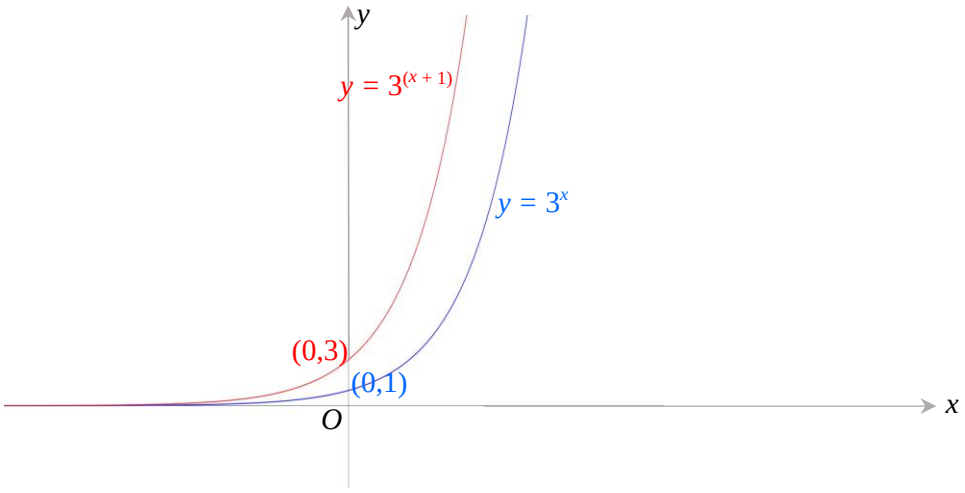
Q	Solution	Mark	Notes

11(a)



G1	graph of $y = 3^x$, -ve domain required. x-axis not crossed.
B1	(0,1) accept all correct methods.

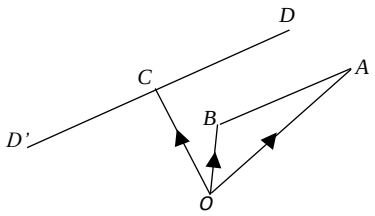
11(b)



B1	graph of $y = 3^{x+1}$, same shape as (a), graphs do not intersect, ft (a) provided G1 awarded.
B1	(0,3) accept all correct methods.

Q	Solution	Mark	Notes
12(a)	$\frac{dy}{dx} = -3x^2 + 12$	B1	
	$\frac{dy}{dx} = -3x^2 + 12 = 0$	M1	si
	$x = 2$, $x = -2$	A1	cao any pair of correct values
	$y = -4$, $y = -36$	A1	cao all 4 values correct, no extra.
	$\frac{d^2y}{dx^2} = -6x$	M1	oe, ft quadratic $\frac{dy}{dx}$, si Eg -ve cubic has min before max. Correct sketch of negative cubic.
	FT only if x-coordinate of min < x-coordinate of max.		
	($x = 2$, $\frac{d^2y}{dx^2} = -12 < 0$.)		
	(2, –4) is a maximum point	A1	ft their x value except $x = 0$.
	($x = -2$, $\frac{d^2y}{dx^2} = 12 > 0$.)		
	$(-2, -36)$ is a minimum point	A1	ft their x value provided different conclusion except $x = 0$.

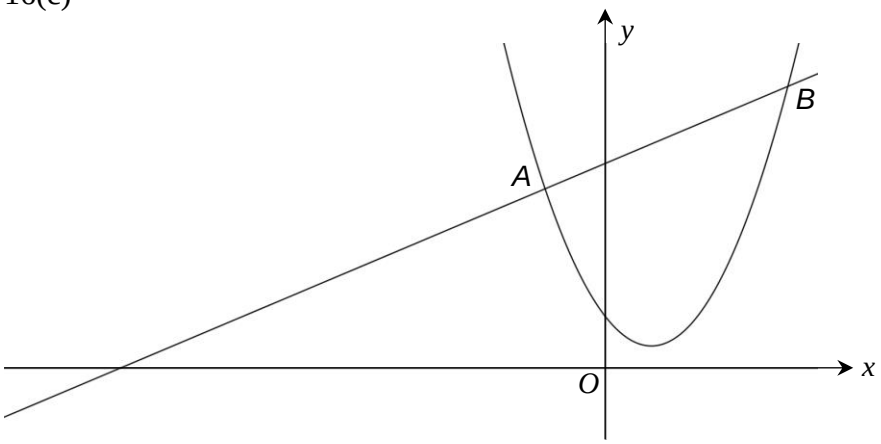
Q	Solution	Mark	Notes
12(b)	Curve is decreasing when		
	$\frac{dy}{dx} < 0$		FT their $\frac{dy}{dx}$
	$-3x^2 + 12 < 0$	M1	allow $\geq$ or $\leq$ throughout, oe
	$x^2 > 4$		
	$x < -2$ or $x > 2$	A1	A0 if 'and' instead of 'or'
	$(x \in) (-\infty, -2) \cup (2, \infty)$	B1	cao Allow –2] ∪ [2
	OR		
	$\{x : x < -2\} \cup \{x : x > 2\}$	(B1)	cao
	Alternative Solution		
	$x < -2$	B1	ft their -2, if used.
	or $x > 2$	B1	ft their 2, if used.
	$(x \in) (-\infty, -2) \cup (2, \infty)$	B1	cao Allow –2] ∪ [2
	OR		
	$\{x : x < -2\} \cup \{x : x > 2\}$	(B1)	cao

Q	Solution	Mark	Notes
13(a)	$\mathbf{AB} = (\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} + 7\mathbf{j})$	M1	allow $(4\mathbf{i} + 7\mathbf{j}) - (\mathbf{i} + 3\mathbf{j})$
	$\mathbf{AB} = -3\mathbf{i} - 4\mathbf{j}$	A1	cao
13(b)	Distance = $\sqrt{(-3)^2 + (-4)^2}$	M1	correct method for $\lvert a\mathbf{i} + b\mathbf{j}\rvert$, $a, b \neq 0$, si
	Distance = 5	A1	ft (a)
13(c)			
	$\mathbf{d} = \mathbf{c} - \mathbf{BA} = \mathbf{c} + \mathbf{AB}$	M1	si
	$\mathbf{d} = (-2\mathbf{i} + 5\mathbf{j}) + (-3\mathbf{i} - 4\mathbf{j})$		
	$\mathbf{d} = -5\mathbf{i} + \mathbf{j}$	A1	ft **AB**
	or		
	$\mathbf{d} = \mathbf{c} + \mathbf{BA} = \mathbf{c} - \mathbf{AB}$	M1	si $\mathbf{a} + \mathbf{BC}$, $\mathbf{BC} = -3\mathbf{i} + 2\mathbf{j}$
	$\mathbf{d} = (-2\mathbf{i} + 5\mathbf{j}) - (-3\mathbf{i} - 4\mathbf{j})$		$4\mathbf{i} + 7\mathbf{j} - 3\mathbf{i} + 2\mathbf{j}$
	$\mathbf{d} = \mathbf{i} + 9\mathbf{j}$	A1	ft **AB**

Q	Solution	Mark	Notes
14(a)	A is the point (–2, 0)	B1	
	B is the point (0, 2)	B1	
14(b)	$I = \int_{-2}^{0}(2 - 3x - 2x^2)\,dx$	M1	attempt to integrate y wrt x Limits not required. At least one power of x increased
	$= \left[2x - \frac{3}{2}x^2 - \frac{2}{3}x^3\right]_{-2}^{0}$	A1	correct integration
	$= \left[0 - \left(-4 - 6 + \frac{16}{3}\right)\right]$	m1	correct use of candidate's limits,
	$= \frac{14}{3}$	A1	cao, allow one decimal place correctly derived from $\frac{14}{3}$.
	Area of triangle $\left(= \frac{1}{2} \times 2 \times 2\right) = 2$	B1	oe, ft (a)
	Required area $\left(= \frac{14}{3} - 2\right) = \frac{8}{3}$	A1	cao, allow one decimal place correctly derived from $\frac{8}{3}$.
	Note: Must be supported by workings. If M0, award SC1 for sight of $\frac{14}{3}$, OR SC2 for $\frac{8}{3}$		

Q	Solution	Mark	Notes
	Alternative solution		
14(b)	$I = \int_{-2}^{0}(2 - 3x - 2x^2 - x - 2)\,dx$	(M1)	attempt to integrate *y* wrt *x* Limits not required. At least one power of *x* increased
		(A1)	attempt to subtract integrand
	$= \int_{-2}^{0}(-4x - 2x^2)\,dx$		
	$= \left[-2x^2 - \frac{2x^3}{3}\right]_{-2}^{0}$	(A1)	correct integration
	$= \left[0 - \left(-8 + \frac{16}{3}\right)\right]$	(m1)	correct use of candidate's limits,
	$= \frac{8}{3}$	(A2)	cao
	Note: Must be supported by workings. If M0, SC2 for $\frac{8}{3}$		

Q	Solution	Mark	Notes
15	$2\sin x + 3\cos^2 x - 3 = 0$	M1	
	$2\sin x + 3(1 - \sin^2 x) - 3 = 0$	M1	$\sin^2 x + \cos^2 x = 1$
	$3\sin^2 x - 2\sin x = 0$	A1	si by $\sin x = 0$ AND $\sin x = \frac{2}{3}$
	$\sin x(3\sin x - 2) = 0$		
	$\sin x = \frac{2}{3}$ or $\sin x = 0$	A1	cao both roots (no extra roots)
			FT their $\sin x = k$ for $0 < k \leq 1$.
	(At *A*,) $x = 41.81°$	B1	
	(At *B*,) $x = 138.19°$	B1	
	(At *C*,) $x = 180°$	B1	
	Note:		
	Do not follow through for trig functions other than sine.		
	Ignore angles greater than 180°		

Q	Solution	Mark	Notes
16(a)	Discriminant = $(-k)^2 - 4\times1\times4$	B1	si
	$= k^2 - 16$		
	If no real roots, discriminant < 0	M1	condone ≤, si by correct answer.
	$k^2 - 16 < 0$		
	Critical values –4, 4	B1	si
	$-4 < k < 4$	A1	cao, oe, e.g. $k > -4$ **and** $k < 4$ Mark final answer
16(b)	$x^2 - 3x + 4 = x + 16$	M1	
	$x^2 - 4x - 12 = 0$	m1	write as quadratic equation
	$(x + 2)(x - 6) = 0$		
	$x = -2$, $x = 6$	A1	cao one correct pair
	$y = 14$, $y = 22$	A1	cao all correct
	Points of intersection are (–2, 14) and (6, 22).		
16(c)			
		G1	+ve quadratic, above *x*-axis
		G1	straight line, +ve gradient 1 point of intersection in 1st quadrant, 1 point of intersection in 2nd quadrant

Q	Solution	Mark	Notes
17(a)	$1 = \log_{10}(2 - c)$	M1	si
	$2 - c = 10$		
	$c = -8$	A1	
17(b)	$\log_{10}(5 - \alpha) - \log_{10}(2 - \alpha) = 1.2$	M1	Condone $\log_{10}(2 - \alpha) - \log_{10}(5 - \alpha) = 1.2$
	$\log_{10}\left(\frac{5 - \alpha}{2 - \alpha}\right) = 1.2$	B1	subtraction law
	$\frac{5 - \alpha}{2 - \alpha} = 10^{1.2}$ (= 15.8489…)	m1	logs removed correctly
	$5 - \alpha = 10^{1.2}(2 - \alpha)$	A1	removal of denominator
	$5 - \alpha = 2\times10^{1.2} - 10^{1.2}\times\alpha$		
	$\alpha(10^{1.2} - 1) = 2\times10^{1.2} - 5$		
	$\alpha = 1.798$	A1	cao

Q	Solution	Mark	Notes
18(a)	$(x - -3)^2 + (y - - 1)^2 = \left(\sqrt{5}\right)^2$	M1	condone $\sqrt{5}$ on RHS
	$(x + 3)^2 + (y + 1)^2 = 5$		
	$x^2 + 6x + 9 + y^2 + 2y + 1 = 5$		
	$x^2 + y^2 + 6x + 2y + 5 = 0$	A1	convincing
	OR		
	Equation of circle radius r is		
	$x^2 + y^2 + 2fx + 2gy + c = 0$		
	Centre $(-f, -g)$, $c = f^2 + g^2 - r^2$	(M1)	used
	$f = 3$, $g = 1$, $c = 3^2 + 1^2 - \sqrt{5}^2 = 5$	(A1)	
	OR		
	$x^2 + y^2 + 6x + 2y + 5 = 0$		
	$(x + 3)^2 + (y + 1)^2 - 9 - 1 + 5 = 0$	(M1)	M0 if no working shown.
	$(x + 3)^2 + (y + 1)^2 = 5$		
	Hence centre = $(-3, -1)$, radius = $\sqrt{5}$	(A1)	

Q	Solution	Mark	Notes
18(b)(i)	Tangents have equations $y = mx$	B1	
	Touches circle when		
	$x^2 + (mx)^2 + 6x + 2mx + 5 = 0$	M1	
	$(1 + m^2)x^2 + 2(3 + m)x + 5 = 0$	A1	si
	Discriminant $= 4(3 + m)^2 - 4(1 + m^2)\times 5$	B1	ft 1 slip in quadratic
	If tangent, discriminant = 0	M1	used
	$9 + 6m + m^2 - 5 - 5m^2 = 0$		
	$2m^2 - 3m - 2 = 0$		
	$(2m + 1)(m - 2) = 0$		
	$m = -\frac{1}{2}$ $m = 2$	A1	cao, both values
	$y = -\frac{1}{2}x$ $y = 2x$		

Special case 1

Candidates who substitute $y = mx + c$ can only earn method marks, B0 M1 A0 B0 M1 A0

Special case 2

Candidates who obtain the correct answer using any method, award as follows:

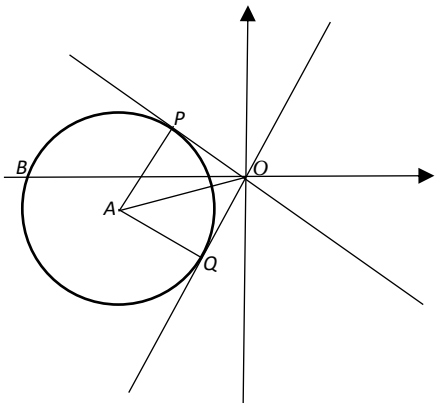
$y = -\frac{1}{2}x$ $y = 2x$ SC1

Q	Solution	Mark	Notes
18(b)(ii)	$m = -\frac{1}{2}$, $\frac{5}{4}x^2 + 5x + 5 = 0$	M1	FT their derived m provided $y = mx$
	$x^2 + 4x + 4 = 0$		
	$(x + 2)^2 = 0$	m1	must be a perfect square, si
	$x = -2$, $y = 1$ $(-2, 1)$	A1	cao
	$m = 2$, $5x^2 + 10x + 5 = 0$	(M1)	award if previous M1 not awarded
	$x^2 + 2x + 1 = 0$		
	$(x + 1)^2 = 0$	(m1)	must be a perfect square, si
	$x = -1$, $y = -2$ $(-1, -2)$	A1	cao
	OR		
	$x = \frac{-b}{2a} = \frac{-2(3+m)}{2(1+m^2)} = \frac{-(3+m)}{(1+m^2)}$	(M1)	
	At $m = -\frac{1}{2}$, $x = \frac{-\left(3-\frac{1}{2}\right)}{\left(1+\frac{1}{4}\right)} = -2$	(m1)	
	$y = -\frac{1}{2} \times (-2) = 1$ $(-2, 1)$	(A1)	
	At $m = 2$, $x = \frac{-(3+2)}{(1+4)} = -1$		
	$y = 2 \times (-1) = -2$ $(-1, -2)$	(A1)	
	Special case		
	Candidates who obtained the correct answer using any method, award as follows:		
	$(-2, 1)$ $(-1, -2)$	SC1	

Q	Solution	Mark	Notes

Alternative solution

18(b)(i)



tan(*BOA*) = $\frac{1}{3}$

BOA = 18.43494882° B1 *BOP* = 26.56505118°

OA = $\sqrt{(3)^2 + (1)^2} = \sqrt{10}$ B1

OP = *OQ* = $\sqrt{\left(\sqrt{10}\right)^2 - \left(\sqrt{5}\right)^2} = \sqrt{5}$ B1

Triangle *POA* and *QOA* are isosceles right angled

Angles *POA* = 45° and *QOA* = 45° B1

Gradient *OQ* = tan(45 + 18.43494882) = 2 B1 tan(243. 43494882) = 2

y = 2*x*

Gradient *OP* = $-\frac{1}{2}$ B1 tan(26.56505118) = $\frac{1}{2}$

tan(116.56505118) = $-\frac{1}{2}$

$y = -\frac{1}{2}x$

GCE AS MARKING SCHEME

SUMMER 2024

AS
MATHEMATICS
UNIT 2 APPLIED MATHEMATICS A
2300U20-1

About this marking scheme

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WJEC GCE AS MATHEMATICS

UNIT 2 APPLIED MATHEMATICS A

SUMMER 2024 MARK SCHEME

SECTION A – Statistics

Qu	Solution	Mark	Notes
1 (a)	The members of the gym.	B1 **[1]**	Condone any of the following: People who go to the gym All people that use the gym The participants of the gym B0 for "The members of the gym that arrive early." B0 for "The 30 members of the gym."
(b)	Opportunity/convenience sampling.	B1 **[1]**	
(c)	Valid improvement. E.g. • Ask people at different times of the day. • Use random sampling. • Ask people on different days. • Ask more people.	B1 **[1]**	
	Total for Question 1	**3**	

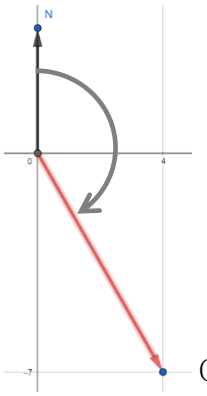
Qu	Solution	Mark	Notes
2(a)	Valid assumptions. e.g. • Cakes sold per hour must be at a constant **average** rate. • Cakes must be sold independently / randomly. • Cakes must be sold individually / singly.	E1 **[1]**	E1 for at least two valid assumptions. E0 for the following: Cakes are sold at a constant rate. Cakes are discrete. It involves rate per hour. There is a mean given for cakes sold. An average rate of cakes sold is given. Constant flow of buyers.
(b)	(Let the rv$W$ be the number of cakes sold in a 1-hour period.) $\mathrm{P}(W = 2) = \dfrac{3.5^2 \times \mathrm{e}^{-3.5}}{2!}$ $= 0.1850$	M1 A1 **[2]**	M0 if no calculation shown using the formula for the Poisson distribution. 3sf or better. M0A0 for calculator entries shown.
(c)	(Let the rv$X$ be the number of cakes sold in a 3-hour period.) $X \sim \mathrm{Po}(10.5)$ $\mathrm{P}(X > 10) = 1 - \mathrm{P}(X \leq 10)$ $= 1 - 0.5207$ $= 0.4793$	B1 M1 A1 **[3]**	si 3sf or better. M0A0 for $\mathrm{P}(X \geq 10)$
(d)	(Let the rv$Y$ be the number of cakes sold in a half hour period.) $\mathrm{Y} \sim \mathrm{Po}(1.75)$ Probability that she sells the next cake before 10:00 is $\mathrm{P}(Y \geq 1)$ $\mathrm{P}(Y \geq 1) = 1 - \mathrm{P}(Y = 0)$ $= 1 - 0.1738$ $= 0.8262$	B1 M1 A1 **[3]**	si (for realising that next cake sold before 10am means $Y \geq 1$) written or used (M1 for $1 - \mathrm{P}(Y \leq 0)$) 3sf or better. SC1 for $\mathrm{P}(Y = 1) = 0.3041$ with $\mathrm{Po}(1.75)$
(e)	Valid comment on reasonableness. e.g. e.g. Not valid because some people might buy lots of cakes. e.g. Not valid because there might be busier periods than others. e.g. Valid. It's unlikely that people would buy multiple birthday cakes. e.g. Independent assumption reasonable as at birthday parties there is only one cake. e.g. Valid. There's no reason to think that cakes wouldn't be sold at a constant rate during the working day.	E1 **[1]**	**Must comment on reasonableness.** e.g. E0 for "Cakes might be sold more than one at a time", but E1 for "cakes might be sold more than one at a time so not reasonable to assume sold individually".
	Total for Question 2	**10**	

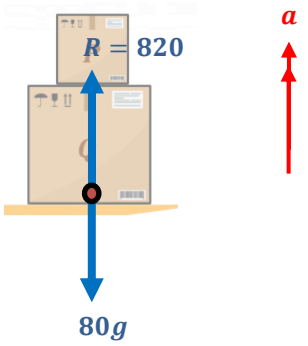
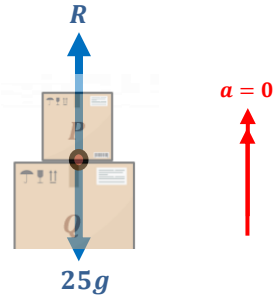
Qu	Solution	Mark	Notes
3(a)	$\left(\frac{20+x}{100}\right)\times\left(\frac{50+x}{100}\right)=\left(\frac{8+x}{100}\right)$	M1	Attempt to use $\mathrm{P}(A\cap C)=\mathrm{P}(A)\times\mathrm{P}(C)$ in terms of x to form an equation in x. Equation must involve at least one fraction. oe A1 for arriving at correct quadratic.
	$x^2-30x+200=0$	A1	
	$x=10$ or $x=20$ *ag is 10	m1	Solving their 3-term quadratic to obtain real solutions. FT their quadratic provided M1 awarded Must follow from $x=10$ or $x=20$
	Reject $x=20$ and reason, e.g. • because $x=20$ would lead to a total greater than 100 • y would be negative Therefore, $x=10$. ***ag**	A1	
	$y=8$	B1	Not dependent on any of the above marks SC2 for verification that $x=10$ [note: can still earn B1] e.g. $\mathrm{P}(A)\times\mathrm{P}(C)=\frac{30}{100}\times\frac{60}{100}=\frac{18}{100}$ and $\mathrm{P}(A\cap C)=\frac{18}{100}$, so $x=10$.
		[5]	
(b)	P(first Athletics and second only climbing) $=\frac{30}{100}\times\frac{15}{99}$	M2	M2 for correct method. M1 for either correct fraction in a product of 2 fractions Allow 0.152 or better in place of $15/99$ for M2 cao
	$=\frac{1}{22}$ or $0.0\dot{4}\dot{5}$	A1	Condone 0.045 or better from correct working. Condone 0.0454. NOTE: the exact answer of 0.045 is obtained from $\frac{30}{100}\times\frac{15}{100}$, which earns M1 only. Sight of $\frac{30}{100}$ and $\frac{15}{99}$ not in a product earns M0.
		[3]	
	Total for Question 3	**8**	

Qu	Solution	Mark	Notes
4(a)	$H_0: p = 0.136 \quad H_1: p < 0.136$	B1 **[1]**	Allow use of θ Condone other letters if defined as proportion/prob. Allow $H_0: \mathrm{P}(\text{blue}) = 0.136$, $H_1: \mathrm{P}(\text{blue}) < 0.136$ Allow 13.6% (B0 for $p = 13.6$) B0 for non-strict inequality in H_1 B0 for omission of p (oe) [e.g. $H_0 = 0.136$] B0 for $H_0: x = 0.136$ or $H_0: \mathrm{P}(X = 0.136)$ B0 for hypothesis labels omitted Allow worded hypotheses (must refer to proportion/prob.). Do not award retrospectively from (b).
(b)(i)	Under $H_0, X \sim \mathrm{B}(80, 0.136)$ $\mathrm{P}(X \leq 5) = 0.0309$ and/or $\mathrm{P}(X \leq 6) = 0.0690$ CR $(0 \leq) X \leq 5$	B1 M1 A1	si M0 for $\mathrm{P}(X = 5)$ or $\mathrm{P}(X = 6)$. Do not accept as probability statement, i.e. $\mathrm{P}(X \leq 5)$ is the CR.
(ii)	6 is not in the critical region so there is insufficient evidence to reject H_0. There is insufficient evidence to suggest that the proportion of blue sweets is less than the company's claim.	M1 A1 **[5]**	FT their CR for M1 (provided a region) Accept $\mathrm{P}(X \leq 6) = 0.06902 > 0.05$ as justification to not reject H_0. M0 for $\mathrm{P}(X = 6)$. CSO (all previous marks in (b) awarded)
(c)	$\mathrm{P}(\text{reject } H_0) = \mathrm{P}(X \leq 5) = 0.0309$ Expected number of occasions $= 20 \times 0.0309$ $= 0.618$	 M1 A1 **[2]**	 FT the probability of their CR for M1A1 provided a region A0 for sight of 1 only without seeing 0.618 SC1 for $20 \times 0.05 = 1$
(d)	Prob (Type II error) = $\mathrm{P}(X \geq 6 \mid X \sim \mathrm{B}(80, 0.07))$ $= 0.49183$ Valid interpretation. e.g. There is almost a 50% chance of failing to detect that the proportion of blue sweets is less than the company's claim (when in fact it is 7%). e.g. The probability of failing to reject the hypothesis that the proportion of blue sweets is not less than the company's claim when, in fact, it should have been rejected is almost 50%. e.g. There is a 49.183% chance of supporting the company's claim when in fact it is false. e.g. There is a 49.183% chance of concluding the proportion of blue sweets has not reduced, when in fact it has.	M1 A1 E1 **[3]**	si FT their CR provided a region for M1A1 3sf or better FT their Type II error (even if M0 awarded). E0 for "You would make the wrong decision almost 50% of the time." E0 for "There is almost a 50% chance that the company makes a false negative." E0 for "This error probability is too large" or "this is not a good test".
	Total for Question 4	**11**	

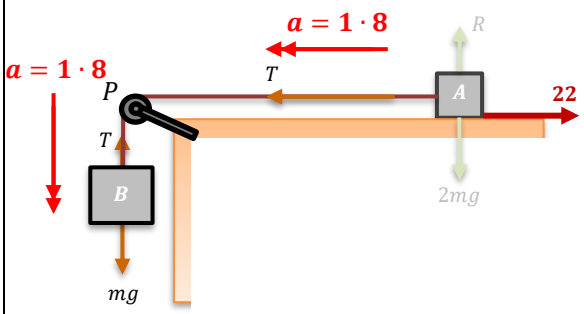
Qu	Solution	Mark	Notes
5(a)	Valid explanation, e.g., • It implies that the numbers of vaccines administered falling is a bad thing rather than an inevitable part of more of the population being vaccinated. • Seeing the graphs tailing off could cause alarm.	E1 **[1]**	E0 for "Vaccines increased from March to May 2021" or "There are sections of the graph where number of vaccines increase." E1 "less people need a second dose because they've had it already." No requirement to reference the quote directly.
(b)(i)	$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{21150}{61}$ $= 346.721$ Mean number of vaccines $= 346\,721$ $\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$ $= \sqrt{\frac{8272500}{61} - \left(\frac{21150}{61}\right)^2}$ $= 124.093$ Standard deviation $= 124093$	M1 A1 M1 A1	Correct method for Σfx, allow one slip. Allow embedded for M1. M1A1 for use of calculator Must be labelled as mean or $\bar{x}$. Accept correctly rounded answers. -1 if ×1000 is omitted. Penalise once only. Allow 346.721 thousands. Allow 347(1000s). FT their $\bar{x}$ for M1 only M1A1 for use of calculator Accept correctly rounded answers. -1 if ×1000 is omitted. Penalise once only. Allow 124.093 thousands. Allow 124.093(1000s). Accept calculation of $s = \sqrt{\frac{1}{60}\left(8272500 - \frac{21150^2}{61}\right)} = 125.12289$ leading to 125123, which can earn M1A1 SC1 for sight of any of 15399.08627, 1.5399×10^{10}, 15655.7377, 1.5656×10^{10} (in place of final two marks) M0 for sight of method for variance with no numerical answer. Alternative method: $\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{n}} = \sqrt{\frac{939344.2623}{61}} = 124.093$
(b)(ii)	Data is negatively skewed.	B1 **[5]**	Accept left skew / skewed to the left. Ignore additional non-contradictory comments such as "but only very slightly". NOTE: mean>median/mode is a contradictory comment.
(c)	Valid reason for the pattern, e.g., • There seem to be lower bars on a weekly cycle. This could be a Sunday when some vaccine centres are closed. • People are more likely to get vaccinated on the weekend when not working. • More likely to get vaccines on certain days. • More workers available on days with higher peaks.	E1 **[1]**	E1 for comments about the weekly pattern. E0 for vaccines may have been done in batches. E0 for "Affected by number of vaccines available." E0 for "Dips are where the last of the age are group getting the vaccine." E0 for a comment relating to the overall increasing trend of the graph.
(d)	Valid reason, e.g., • It may not be incorrect because it could be a bank holiday weekend. • It may be incorrect because it doesn't match the pattern of the rest of the graph. • It may not be incorrect as there may have been an event on these dates that many people are participating in.	E1 **[1]**	Must comment on correctness. Need to see decision with a reason. Must have reason for saying anomaly / anomalous result – reason may include, for example, the data for these days does not fit with the rest of the data / other days were higher, etc. E1 for unable to tell if the data is correct just from the graph. E1 for a comment relating to supplies of the vaccine. Allow E1 for correct because they are government figures. Allow E1 for incorrect because they are government figures.
	Total for Question 5	**8**	

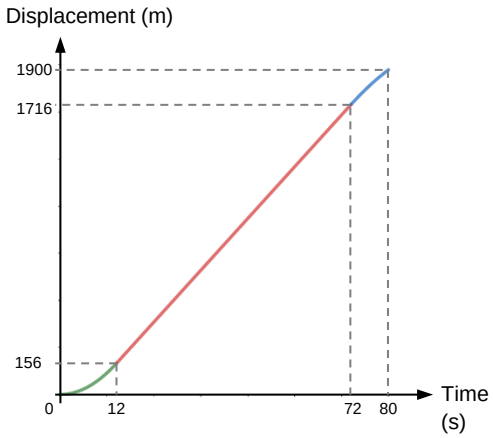
SECTION B – Mechanics

Q6	Solution	Mark	Notes
	 $(4\,\mathbf{i} - 7\,\mathbf{j})\ \text{ms}^{-1}$		
	Speed $= \sqrt{(4)^2 + (\mp 7)^2}$	M1	
	$= \sqrt{65} = 8 \cdot 06(225 \ldots)\ \ (\text{ms}^{-1})$	A1	
	Direction		
	$\theta = \begin{cases} \tan^{-1}\left(\frac{\pm 7}{4}\right) = (\pm 60 \cdot 255^{\circ} \ldots) \\ \text{OR} \\ \tan^{-1}\left(\frac{\pm 4}{7}\right) = (\pm 29 \cdot 744^{\circ} \ldots) \end{cases}$	M1	oe
	Bearing $90 + 60 \cdot 255^{\circ} \ldots$ OR $180 - 29 \cdot 744^{\circ} \ldots$		
	Bearing is $150°$	A1	
		[4]	
	Total for Question 6	**4**	

Q7	Solution	Mark	Notes
a)			
	N2L applied to **BOTH** boxes, upwards positive	M1	Dimensionally correct equation $R/820$ and $80g$ opposing
	$R - 80g = 80a$ $\quad (R - 55g - 25g = (55 + 25)a)$ $820 - 80g = 80a$ $\quad (820 - 539 - 245 = (55 + 25)a)$ $820 - 784 = 80a$ $36 = 80a$	A1	Correct equation
	$a = 0 \cdot 45$ or $\frac{9}{20}$ (ms^{-2})	A1 **[3]**	
b)			
	At constant speed $R = 25g = 245$ (N)	B1 **[1]**	cao, any form
	Total for Question 7	**4**	

Q8	Solution	Mark	Notes
a)	Using N2L, $4a = 12\sqrt{t} - 32$ $a = 3\sqrt{t} - 8$ When $t = 9$, $a = 3\sqrt{9} - 8$ $a = 1 \quad (\text{ms}^{-2})$	M1 A1 **[2]**	Used
OR	When $t = 9$, $F = 12\sqrt{9} - 32$ $F = 4$ Using N2L, $4a = 4$ $a = 1 \quad (\text{ms}^{-2})$	 (M1) (A1) **([2])**	 Used, FT F
b)	$v = \int (3\sqrt{t} - 8)\,\text{d}t \quad = \int \left(3t^{\frac{1}{2}} - 8\right)\text{d}t$ $v = \frac{3}{3/2}t^{\frac{3}{2}} - 8t\ (+C) \quad \left[= 2t^{\frac{3}{2}} - 8t\ (+C)\right]$ When $t = 4, v = -1$ $C = 15$ $v = 2t^{\frac{3}{2}} - 8t + 15$	M1 A1 A1 **[3]**	Attempt to integrate a
c)	When $t = 9$, $v = 2(9)^{\frac{3}{2}} - 8(9) + 15$ $(v = -3)$ When $t = 9, a > 0$ (from part a)) and $v < 0$ $\therefore v$ is increasing $\Longrightarrow$ speed is **decreasing**	M1 A1 **[2]**	FT derived v
	Total for Question 8	**7**	

Q9	Solution	Mark	Notes
a)	$v^2 = u^2 + 2as$, with $s = 0\cdot4, u = 0, v = 1\cdot2$ $(1\cdot2)^2 = (0)^2 + 2a(0\cdot4)$ $\quad (1\cdot44 = 0\cdot8a)$ $a = 1\cdot8$ $\quad (\text{ms}^{-2})$	M1 A1 **[2]**	Used, oe $a = \frac{v^2-u^2}{2s} = \frac{(1\cdot2)^2-(0)^2}{2(0\cdot4)}$ Convincing
b)	 Apply N2L to **both** A and B separately A: $T - 22 = 2ma$ OR $T - 22 = 3\cdot6m$ B: $mg - T = ma$ OR $9\cdot8m - T = 1\cdot8m$ $T = 8m$ Eliminating T $mg - 22 = 3ma$ OR $9\cdot8m - 22 = 5\cdot4m$ $m = 5$ $T = 40$ $\quad (\text{N})$	M1 B1 A1 m1 A1 A1 **[6]**	At least one dimensionally correct equation T and $mg/22$ opposing 1st correct equation 2nd correct equation Attempted
c)	Inextensible string enables me to assume that (the magnitude of) the **acceleration** of objects A and B is **equal.**	E1 **[1]**	
	Total for Question 9	**9**	

Q10	Solution	Mark	Notes
a)	Distance travelled during acceleration $s = \frac{1}{2}(u+v)t,\ u = 0, v = 26, t = 12$ $s = 156 \quad (\text{m})$	 M1 A1 **[2]**	 Used Convincing
b)	**(i)** Distance covered at constant speed $60 \times 26 = 1560\ (\text{m})$	 B1	
	(ii) Distance travelled whilst decelerating $1900 - 1560 - 156 = 184 \quad (\text{m})$ Time for deceleration $s = ut + \frac{1}{2}at^2,\ s = 184,\ u = 26, a = (\pm)0 \cdot 75$ $184 = 26t + \frac{1}{2}(-0 \cdot 75)t^2$ $0 \cdot 75t^2 - 52t + 368 = 0$ Solving the quadratic $\left(t = 8 \quad \text{or} \quad t = \frac{184}{3} = 61 \cdot 33 \ldots.\right)$ $t = 8 \quad (\text{s})$	 B1 M1 A1 m1 A1 **[6]**	 FT answer to (i) FT distance $t = \frac{184}{3}$ clearly discounted.
c)	Sketch  	 B1 B1 B1 **[3]**	B1 for any 1 below B2 for any 2 below **Concave up** from $(0,0)$ to $(12,156)$ OR **Straight line** from $(12,156)$ to $(72,156+1560)$ (FT '1560') OR **Concave down** to $(\mathbf{72+8}, 1900)$ (FT '72' and '8') All correct
	Total for Question 10	**11**	

GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1

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WJEC GCE A LEVEL MATHEMATICS

UNIT 3 PURE MATHEMATICS B

SUMMER 2024 MARK SCHEME

Q	Solution	Mark	Notes
1(a)	$\frac{25x+32}{(2x-5)(x+1)(x+2)} = \frac{A}{(2x-5)} + \frac{B}{(x+1)} + \frac{C}{(x+2)}$	M1	
	$25x + 32 = A(x+1)(x+2) + B(2x-5)(x+2) + C(2x-5)(x+1)$	m1	correct removal of denom, oe, si
	Put $x = -1$, $-25 + 32 = B(-7)(1)$		
	$B = -1$	A1	one constant correct
	Put $x = -2$, $-50 + 32 = C(-9)(-1)$		
	$C = -2$		
	Coefficient x^2 (or put $x = 2.5$),		oe
	$0 = A + 2B + 2C = A - 2 - 4$		
	$A = 6$	A1	all three constants correct
	$f(x) = \frac{6}{(2x-5)} - \frac{1}{(x+1)} - \frac{2}{(x+2)}$		
1(b)	$\int_1^2 \left(\frac{6}{(2x-5)} - \frac{1}{(x+1)} - \frac{2}{(x+2)}\right) dx$	M1	FT at least 2 of their non-zero A, B, C, limits not required, si
	$= \left[\frac{6}{2}\ln\lvert 2x-5\rvert - \ln\lvert x+1\rvert - 2\ln\lvert x+2\rvert\right]_1^2$	A1	for one correct term, FT A, B, C.
		A1	all 3 correct, FT A, B, C condone brackets instead of ||.
	$= [3\ln\lvert -1\rvert - \ln 3 - 2\ln 4] -$		
	$[3\ln\lvert -3\rvert - \ln 2 - 2\ln 3]$	m1	correct use of limits, si
	$= -2\ln 3 - 3\ln 2$		
	$= -\ln 72$	A1	cao, $P = 72$

Q	Solution	Mark	Notes
1(c)	$f(2) = -\frac{82}{12} < 0$		
	$f(3) = \frac{107}{20} > 0$	B1	
	Denominator = 0 when $x = 2.5$.	E1	oe Eg (vertical) asymptote at $x = 2.5$. Function discontinuous when $x = 2.5$.

Q	Solution	Mark	Notes
2(a)	$3\cot\theta + 4(1 + \cot^2\theta) = 5$	M1	Use of $\text{cosec}^2\theta = 1 + \cot^2\theta$
	$4\cot^2\theta + 3\cot\theta - 1 = 0$		
	$(4\cot\theta - 1)(\cot\theta + 1) = 0$	m1	coeff $\cot\theta$ multiply to 4, constant terms multiply to -1
	$\cot\theta = -1, \frac{1}{4}$	A1	cao, both values
	$\tan\theta = -1$		
	$x = 135°, 315°$	B1	ft only if $\tan\theta$ is correct from $\cot\theta$. or allow one correct value from each branch $2.326^c, 5.498^c$
	$\tan\theta = 4,$		
	$x = 75.96°, 255.96°$	B1	ft if sign different $1.326^c, 4.467^c$

Note

Ignore all roots outside range.

For each branch, award B0 if extra root(s) in range present.

2 +ve roots ft for B1, 2 -ve roots ft for B1.

Only award for $\tan\theta$.

If radians used, units are required.

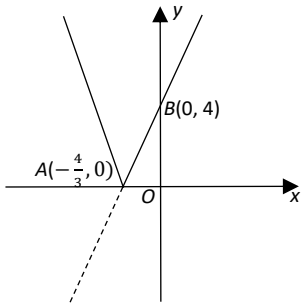
Q	Solution	Mark	Notes
2(a)	OR		
	$3\cot\theta + 4(1 + \cot^2\theta) = 5$	(M1)	Use of $\text{cosec}^2\theta = 1 + \cot^2\theta$
	$4\cot^2\theta + 3\cot\theta - 1 = 0$		
	multiplying by $\tan^2\theta$ $4 + 3\tan\theta - \tan^2\theta = 0$, $\tan^2\theta - 3\tan\theta - 4 = 0$		
	$(\tan\theta - 4)(\tan\theta + 1) = 0$	(m1)	coeff $\tan\theta$ multiply to 1, constant terms multiply to –4
	$\tan\theta = -1, 4$	(A1)	cao
	$\tan\theta = -1$,		
	$\theta = 135°, 315°$	(B1)	ft or allow one correct value from each branch $2.326^c, 5.498^c$
	$\tan\theta = 4$,		
	$\theta = 75.96°, 255.96°$	(B1)	ft if sign different, $1.326^c, 4.467^c$
	Note		
	Ignore all roots outside range.		
	For each branch, award B0 if extra root(s) in range present.		
	2 +ve roots ft for B1, 2 -ve roots ft for B1.		
	Only award for $\tan\theta$.		
	If radians used, units are required.		

Q	Solution	Mark	Notes
2(b)	$24\cos x - 7\sin x = R\cos(x + \alpha)$		
	$= R\cos x\cos\alpha - R\sin x\sin\alpha$		
	$R\cos\alpha = 24$, $R\sin\alpha = 7$	M1	implied by $\alpha = 16.26°$
	$R = \sqrt{24^2 + 7^2} = 25$	B1	no working required
	$\alpha = \tan^{-1}\left(\frac{7}{24}\right)$		$\sin^{-1}\left(\frac{7}{25}\right)$, $\cos^{-1}\left(\frac{24}{25}\right)$
	$\alpha = 16.26°$	A1	cao
	$25\cos(x + 16.26°) = 16$	M1	ft R, α
	$\cos(x + 16.26°) = \frac{16}{25} = 0.64$		
	$x + 16.26° = 50.21°, 309.79°$	A1	any one value, ft R, α, si
	$x = 33.95°, 293.53°$	A1	ft R, α, both values. A0 if 3 or more values between $0° < x < 360°$. Accept whole numbers. Ignore values outside the range.

Q	Solution	Mark	Notes
3(a)	Area of sector = $\frac{1}{2}r^2\theta$	B1	used si
	$\frac{1}{2}(5 + r)^2 \times \frac{\pi}{5} - \frac{1}{2}r^2 \times \frac{\pi}{5} = \frac{13\pi}{2}$	M1	difference between two sectors Condone wrong way round.
		A1	correct equation
	$r^2 + 10r + 25 - r^2 = 65$		
	$10r + 25 = 65$		
	$2r + 5 = 13$		
	$r = 4$	A1	cao
3(b)	length $CD = \frac{(5+r)\pi}{5} = \frac{9\pi}{5}$		FT their part (a)
	length $AB = \frac{r\pi}{5} = \frac{4\pi}{5}$	B1	si, either correct with r substituted
	Perimeter = $5 + 5 + CD + AB$	M1	
	Perimeter = $5 + 5 + \frac{4\pi}{5} + \frac{9\pi}{5}$		
	Perimeter = $10 + \frac{13\pi}{5}$ (= 18.168…)	A1	cao, isw

Q	Solution	Mark	Notes

4(a)

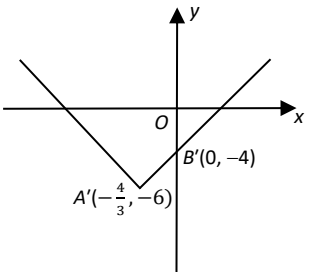


G1 shape, above the *x*-axis and vertex on *x*-axis

B1 $\left(-\frac{4}{3}, 0\right)$, any correct method

B1 (0, 4), any correct method

4(b)



G1 move downwards, ft (a)

B1 $\left(-\frac{4}{3}, -6\right)$, ft their $\left(-\frac{4}{3}, 0\right)$, any correct method

B1 (0, –4) , any correct method

Q	Solution	Mark	Notes
5	(Assume that there is a real and positive value of x such that)		
	$x + \frac{81}{x} < 18.$	M1	condone $\leq$
	Then		
	$x^2 + 81 < 18x$	A1	
	$x^2 - 18x + 81 < 0$		
	$(x - 9)^2 < 0$	A1	
	which is impossible, hence contradiction		
	Therefore $x + \frac{81}{x} \geq 18.$	A1	cso

Q	Solution	Mark	Notes
6(a)	$y = \cos x$		
	$\frac{dy}{dx} = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$	M1	
	$= \lim_{h\to 0} \frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$	B1	$\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$
	As $h\to 0$, $\sin h \cong h$ and $\cos h \cong 1 - \frac{h^2}{2}$		
	$\frac{dy}{dx} = \lim_{h\to 0} \frac{\cos x\left(1 - \frac{h^2}{2}\right) - h\sin x - \cos x}{h}$	M1	substitution
	$\frac{dy}{dx} = \lim_{h\to 0} \frac{\cos x\left(-\frac{h^2}{2}\right) - h\sin x}{h}$	A1	
	$\frac{dy}{dx} = \lim_{h\to 0}(-\frac{h}{2}\cos x - \sin x)$		
	$\frac{dy}{dx} = -\sin x$	A1	Everything correct & convincing

Q	Solution	Mark	Notes
	OR		
6(a)	$y = \cos x$		
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\cos(x + \delta x) - \cos x}{\delta x}$	(M1)	
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\cos x \cos \delta x - \sin x \sin \delta x - \cos x}{\delta x}$	(B1)	$\frac{\cos x \cos \delta x - \sin x \sin \delta x - \cos x}{\delta x}$
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \left[\frac{\cos x(\cos \delta x - 1)}{\delta x} - \frac{\sin x \sin \delta x}{\delta x}\right]$	(M1)	factorisation
	$\frac{dy}{dx} = \cos x \lim_{\delta x \to 0} \left[\frac{\cos \delta x - 1}{\delta x}\right]$ $- \sin x \lim_{\delta x \to 0} \left[\frac{\sin \delta x}{\delta x}\right]$		
	But $\lim_{\delta x \to 0} \left[\frac{\cos \delta x - 1}{\delta x}\right] = 0$		
	and $\lim_{\delta x \to 0} \left[\frac{\sin \delta x}{\delta x}\right] = 1$	(A1)	both limits used
	Hence $\frac{dy}{dx} = \cos x \times 0 - \sin x \times 1$		
	$\frac{dy}{dx} = -\sin x$	(A1)	Everything correct & convincing

Q	Solution	Mark	Notes
6(b)	$y = e^{3x}\sin 4x$		
	$\frac{dy}{dx} = e^{3x} \times 4\cos 4x + 3e^{3x} \times \sin 4x$	M1	$e^{3x}f(x) + g(x)\sin 4x$, $f(x)$, $g(x) \neq 0, 1$.
		A1	$f(x) = 4\cos 4x$, isw
		A1	$g(x) = 3e^{3x}$, isw
	$\frac{dy}{dx} = e^{3x}(4\cos 4x + 3\sin 4x)$		

Q	Solution	Mark	Notes
6(c)	$\int x^2 \sin 2x \, dx$		
	$= \left[-\frac{1}{2}\cos 2x \times x^2\right] - \int -\frac{1}{2}\cos 2x \times 2x \, dx$	M1	$[f(x)x^2] - \int f(x)g(x)dx$, $g(x) \neq 0,1$, condone a sign error. $f(x) \neq \sin 2x$
		A1	$f(x) = -\frac{1}{2}\cos 2x$
		A1	$g(x) = 2x$
	$= \left[-\frac{1}{2}x^2\cos 2x\right] + \left[\frac{1}{2}\sin 2x \times x\right] - \int \frac{1}{2}\sin 2x \times 1 dx$		
		A1	correct second and third terms
	$= \left[-\frac{1}{2}x^2\cos 2x\right] + \left[\frac{1}{2}x\sin 2x\right]$		
	$+ \left[\frac{1}{4}\cos 2x\right]$ + C	A1	third term correct and + C. isw

Q	Solution	Mark	Notes
7(a)	$\sum_{r=3}^{50}(4r+5)$		
	$= 17 + 21 + 25 + \ldots (+ 205)$	M1	oe AP recognised
	$= \frac{48}{2}[2 \times 17 + (48-1) \times 4]$	m1	use of formula, condone 1 error.
			$\frac{48}{2}[17 + 205]$
		A1	all correct
			Accept $a = 13$, $d = 4$, $n = 49$, Sum = 5341
			Accept $a = 9$, $d = 4$, $n = 50$, Sum = 5350
			Accept $a = 5$, $d = 4$, $n = 51$, Sum = 5355
	$= 5328$	A1	cao, from correct working
	Note: No workings, M0		

Q	Solution	Mark	Notes
7(b)	$\sum_{r=2}^{\infty}\left(540\times\left(\frac{1}{3}\right)^{r}\right)$		
	$=540\left(\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots\right)$	M1	GP recognised
	$=60+20+\frac{20}{3}+\ldots$		
	$=\frac{540\times\frac{1}{9}}{1-\frac{1}{3}}=\frac{60}{1-\frac{1}{3}}$	m1	correct use of formula, ratio = $\frac{1}{3}$
			Accept $\frac{540\times\frac{1}{3}}{1-\frac{1}{3}}=\frac{180}{1-\frac{1}{3}}$, Sum = 270
			Accept $\frac{540}{1-\frac{1}{3}}$, Sum = 810
	= 90	A1	cao
	Note: No workings, M0		

Q	Solution	Mark	Notes
8(a)	$f(0) = -1 < 0$		
	$f(1) = 1 > 0$		
	$f(x)$ changes sign in the interval [0, 1],		
	(and f is continuous / f is a cubic curve.)	B1	
	Hence, there is a root in the interval [0, 1]		
8(b)(i)	$f'(x) = 3x^2 + 8x - 3$	B1	si
	$x_{n+1} = x_n - \frac{x_n^3 + 4x_n^2 - 3x_n - 1}{3x_n^2 + 8x_n - 3}$	M1	$x_{n+1} = \frac{2x_n^3 + 4x_n^2 + 1}{3x_n^2 + 8x_n - 3}$, si
	$x_0 = 0.8$		
	$x_1 = 0.861\ 654\ 135\ 3\ldots$	A1	cao
8(b)(ii)	$x_2 = 0.857\ 641\ 074\ 1\ldots$		
	$x_3 = 0.857\ 623\ 607\ 4\ldots$		
	$x_4 = 0.857\ 623\ 607\ldots$		
	Root = 0.857 624 (correct to 6 dp)	A1	cao

Q	Solution	Mark	Notes
8(c)	$f'(x) = 3x^2 + 8x - 3$		
	$f'\left(\frac{1}{3}\right) = 3 \times \frac{1}{9} + 8 \times \frac{1}{3} - 3 = 0$	B1	$f'(x) = 3x^2 + 8x - 3 = (3x - 1)(x + 3)$ When $x = \frac{1}{3}$, $f'(x) = 0$
	Therefore N-R requires division by 0.	E1	oe eg tangent at $x = \frac{1}{3}$ is horizontal, hence not intersecting the x-axis for the next iteration.

Q	Solution	Mark	Notes
9	Area under $C_1 = \int_0^{\pi}(-x^2 + \pi x + 1)\mathrm{d}x$	M1	Attempt to integrate, index increased in at least one term.
	$= \left[-\frac{x^3}{3} + \frac{\pi x^2}{2} + x\right]_0^{\pi}$	B1	correct integration, limits not required
	$= -\frac{\pi^3}{3} + \frac{\pi^3}{2} + \pi$	m1	correct use of limits, si
	$= \pi + \frac{\pi^3}{6}$	A1	cao, si, allow $-\frac{\pi^3}{3} + \frac{\pi^3}{2} + \pi$
	Area under C_2		
	$= \int_0^{\frac{\pi}{4}}(\cos 2x)\mathrm{d}x + \int_{\frac{3\pi}{4}}^{\pi}(\cos 2x)\mathrm{d}x$	M1	oe, either integral
	$= 2\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$	B1	correct integration of $\cos 2x$, limits not required
	$= 2\left(\frac{1}{2} - 0\right)$	(m1)	si correct use of limits
	$= 1$	A1	cao, si or each integral = $\frac{1}{2}$, both required.
	Note: Award the m1 once for correct substitution of limits seen in either integration.		
	Required area		
	= Area under C_1 – Both Areas under C_2	m1	used
	Required area = $\pi + \frac{\pi^3}{6} - 1$	A1	cao, exact value required
	Note: SC3 for answer only of 7.3093. SC2 for answer only of 8.3093 AND ½ or 1. SC1 for answer only of 8.3093 OR ½ or 1		

Q	Solution	Mark	Notes
9	Alternative solution		
	Area between curves		
	$= \int_0^{\pi} ((-x^2 + \pi x + 1) - \cos 2x) \mathrm{d}x$	M1	
	$= \left[-\frac{x^3}{3} + \frac{\pi x^2}{2} + x - \frac{1}{2}\sin 2x\right]_0^{\pi}$	B1	correct integration of quadratic
		B1	correct integration of $\cos 2x$
	$= \left(-\frac{\pi^3}{3} + \frac{\pi^3}{2} + \pi - \frac{1}{2}\sin 2\pi\right) - 0$	m1	correct use of limits, si
	$= \pi + \frac{\pi^3}{6}$	A1	cao, si
	Area below x-axis		
	$= \left\| \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\cos 2x) \mathrm{d}x \right\|$	M1	
	$= \left\| \left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \right\|$	(B1)	correct integration of $\cos 2x$
	$= \left\| \left[\frac{1}{2}\sin\frac{3\pi}{2}\right] - \left[\frac{1}{2}\sin\frac{\pi}{2}\right] \right\|$	(m1)	si
	$= \left\| -\frac{1}{2} - \frac{1}{2} \right\|$		
	$= 1$	A1	cao, si, condone -1 if used correctly below
	Note: Award the m1 once for correct substitution of limits seen in either integration.		
	Required area		
	= Area between curves − Area below x-axis	m1	
	Required area = $\pi + \frac{\pi^3}{6} - 1$	A1	cao, exact value required
	Note: SC3 for answer only of 7.3093.		
	SC2 for answer only of 8.3093 AND (–)1.		
	SC1 for answer only of 8.3093 OR (–)1.		

Q	Solution	Mark	Notes
10(a)	$\frac{2(3x+1)}{x^2-2x-3}+\frac{x}{x+1}=\frac{2(3x+1)}{(x-3)(x+1)}+\frac{x}{x+1}$		
	$=\frac{2(3x+1)+x(x-3)}{(x-3)(x+1)}$	B1	common denominator
	$=\frac{x^2+3x+2}{(x-3)(x+1)}$	B1	simplify numerator
	$=\frac{(x+1)(x+2)}{(x-3)(x+1)}=\frac{x+2}{x-3}$	B1	factorise and cancel ($x \neq -1$). AG
10(b)	$f(x)=\frac{x+2}{x-3}=1+\frac{5}{x-3}$		
	Range is (1, 6]	B1	(1, oe
		B1	6] oe
			SC1 [6, 1)
10(c)	$y=\frac{x+2}{x-3}$	M1	x and y may be interchanged at start
	$xy-3y=x+2$		
	$xy-x=3y+2$	m1	attempt to isolate x (or y)
	$x(y-1)=3y+2$		
	$x=\frac{3y+2}{y-1}$		
	$f^{-1}(x)=\frac{3x+2}{x-1}$	A1	oe
	Domain (1, 6], range [4, ∞)	B1	FT part (b) only for domain, oe
10(d)	$\frac{x+2}{x-3}=\frac{3x+2}{x-1}$	M1	$x=\frac{3x+2}{x-1}$; $x=\frac{x+2}{x-3}$; $\frac{\frac{x+2}{x-3}+2}{\frac{x+2}{x-3}-3}=x$ ft (c)
	$2x^2-8x-4=0$	A1	$x^2-4x-2=0$, ft (c) only if similar expression for $f^{-1}(x)$
	$x=2\pm\sqrt{6}$	A1	cao
	Since $x \geq 4$, $x=2+\sqrt{6}=4.449\ldots$	A1	cao
	Note: Answer only M1 A0 m0 A0		

Q	Solution	Mark	Notes
11(a)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M1	used, si
	$\frac{dx}{d\theta} = 2 + 2\cos 2\theta$	B1	
	$\frac{dy}{d\theta} = -2\sin 2\theta$	B1	
	$\frac{dy}{dx} = \frac{-2\sin 2\theta}{2(1+\cos 2\theta)}$		
	$\frac{dy}{dx} = \frac{-2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	m1	$\sin 2\theta = 2\sin\theta\cos\theta$
		m1	$\cos 2\theta = 2\cos^2\theta - 1$
	$\frac{dy}{dx} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$	A1	convincing, AG
11(b)	Where $\theta = \frac{\pi}{4}$, point is $P\left(\left(\frac{\pi}{2}+1\right), 1\right)$	B1	si, $P(2.57, 1)$
	Gradient of tangent $= -\tan\frac{\pi}{4} = -1$	B1	si
	Equation of tangent is		
	$y - (1 + \cos 2\theta) = -\tan\theta(x - (2\theta + \sin 2\theta))$	M1	oe, method for equation, si e.g. $y = -1x + c$
	$y - 1 = -1\left(x - \left(\frac{\pi}{2}+1\right)\right)$	A1	oe, all correct, isw
	$y + x = 2 + \frac{\pi}{2}$ (= 3.57)		

Q	Solution	Mark	Notes
12(a)	For small θ, $\sin\theta \approx \theta$; $\cos\theta \approx 1 - \frac{\theta^2}{2}$.	M1	used
	$2\cos\theta + \sin\theta - 1$		
	$= 2\left(1 - \frac{\theta^2}{2}\right) + \theta - 1$		
	$= 2 - \theta^2 + \theta - 1$		
	$= 1 + \theta - \theta^2$	A1	AG
12(b)	$[1 + \theta - \theta^2]^{-1}$	M1	
	$= [1 + (\theta - \theta^2)]^{-1}$		
	$= 1 + (-1)(\theta - \theta^2) + \frac{(-1)(-2)}{2}(\theta - \theta^2)^2 + \cdots$	A1	$1 + (-1)(\theta - \theta^2)$
		A1	$\frac{(-1)(-2)}{2}(\theta - \theta^2)^2$
	$= 1 - \theta + \theta^2 + \theta^2 + \cdots$		
	$= 1 - \theta + 2\theta^2 + \cdots$	A1	cao
	($a = -1$, $b = 2$)		

Q	Solution	Mark	Notes
13(a)	$f(x)$ is decreasing, so $f'(x) < 0$		
	$f(x)$ is convex, so $f''(x) > 0$,		
	$f'(x)$ is increasing.	E1	oe
	Required interval is $(x_3, 0)$	B1	condone $[x_3, 0]$, oe
13(b)	(Points of inflection occur when there is		
	a change of concavity,		
	ie change of sign in $f''(x)$.)		
	x-coordinates are x_3 or x_4	B1	either

Q	Solution	Mark	Notes
14(a)	$y = \frac{1+\ln x}{x}$		
	$\frac{dy}{dx} = \frac{xf(x) - (1+\ln x)g(x)}{x^2}$	M1	Allow omission of $g(x)$
	$= \frac{x\left(\frac{1}{x}\right) - (1+\ln x)1}{x^2}$		$f(x) = \frac{1}{x}$, $(g(x) = 1)$
	$\frac{dy}{dx} = \frac{-\ln x}{x^2}$	A1	convincing, AG
	Alternative solution 1		
	$y = \frac{1}{x} + \frac{\ln x}{x}$		
	$\frac{dy}{dx} = -\frac{1}{x^2} - \frac{1}{x^2}\ln x + \frac{1}{x} \times \frac{1}{x}$	(M1)	one correct term
	$\frac{dy}{dx} = \frac{-\ln x}{x^2}$	(A1)	convincing, AG
	Alternative solution 2		
	$y = (1 + \ln x)x^{-1}$		
	$\frac{dy}{dx} = x^{-1}f(x) + (1 + \ln x)g(x)$	(M1)	
	$= x^{-1}\frac{1}{x} + (1 + \ln x) \times -1x^{-2}$		$f(x) = \frac{1}{x}$, $g(x) = -1x^{-2}$
	$\frac{dy}{dx} = \frac{-\ln x}{x^2}$	(A1)	convincing, AG
	Alternative solution 3		
	$xy = 1 + \ln x$		
	$y + x\frac{dy}{dx} = \frac{1}{x}$	(M1)	
	$x\frac{dy}{dx} = \frac{1}{x} - y = \frac{1}{x} - \frac{1+\ln x}{x} = -\frac{\ln x}{x}$		
	$\frac{dy}{dx} = \frac{-\ln x}{x^2}$	(A1)	convincing, AG

Q	Solution	Mark	Notes
14(b)	$\int \frac{\ln x}{x^2}\,\mathrm{d}x = \int t\,\mathrm{d}t$	M1	variables separated, 1 error only.
	$-\frac{1+\ln x}{x} = \frac{t^2}{2}\ (+C)$	A1	$\frac{1+\ln x}{x}$
		A1	$\frac{t^2}{2}$
			–1 for incorrect signs if A1A1
	When $x = 1$, $t = 3$		
	$-1 = \frac{9}{2} + C$	m1	use of conditions
	$C = -\frac{11}{2}$		
	$t^2 = 11 - \frac{2(1+\ln x)}{x}$	A1	cao, oe, isw

Q	Solution	Mark	Notes
15	For £P invested, the return after n years is S.		
	Use of GP formula for kth term	M1	
	$S_A = P(1.01)^n$	B1	Allow $P = 1$
	$S_B = P(1.05)(1.006)^{n-1}$	B1	Allow $P = 1$
	We want minimum n st $S_A > S_B$		
	$P(1.01)^n > P(1.05)(1.006)^{n-1}$	m1	Allow $P = 1$. Condone ≥ or =
	$(1.006)(1.01)^n > (1.05)(1.006)^n$		
	$\left(\frac{1.01}{1.006}\right)^n > \left(\frac{1.05}{1.006}\right)$		
	$n\ln\left(\frac{1.01}{1.006}\right) > \ln\left(\frac{1.05}{1.006}\right)$		
	$n > 10.78(762515)$		
	min $n = 11$	A1	cao Mark final answer
	Note: Accept any valid method (eg trial and error).		
	Use of GP formula for kth term	M1	at least two iterations
	$S_A = P(1.01)^n$	B1	Allow $P = 1$, used for 2 values of n
	$S_B = P(1.05)(1.006)^{n-1}$	B1	Allow $P = 1$, used for 2 values of n
	Two iterations for $n = 9, 10, 11, 12, 13$	m1	
	min $n = 11$	A1	cao Mark final answer

	Bank *B*	**Bank *A***	
	$1.05 \times 1.006^{n-1}$	1.01^n	
$n = 9$	1.101 471 196…	1.093 685 272…	$B > A$
$n = 10$	1.108 080 023…	1.104 622 125…	$B > A$
$n = 11$	1.114 728 503…	1.115 668 346…	$B < A$
$n = 12$	1.121 416 874…	1.126 825 030…	$B < A$
$n = 13$	1.128 145 376…	1.138 093 280…	$B < A$

Therefore $n = 11$

GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
MATHEMATICS
UNIT 4 APPLIED MATHEMATICS B
1300U40-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL MATHEMATICS

UNIT 4 APPLIED MATHEMATICS B

SUMMER 2024 MARK SCHEME

SECTION A – Statistics

Qu	Solution	Mark	Notes
1	$\text{P(Both girls}\vert\text{1st in school or college)} = \frac{80}{162} \times \frac{87}{179}$ $\frac{40}{81}$	M2	M2 for correct method. M1 for either correct fraction in a product of 2 fractions.
	$= \frac{1160}{4833} = 0.2400\ldots$ oe	A1	Cao A1 for 3sf or greater if using decimals. 0.24 with no working scores 0 marks.
	Alt Method $\text{P(Both girls}\vert\text{1st Education)} = \frac{\text{P(girleducation} \cap \text{girl)}}{\text{P(1st Education)}}$ $= \frac{\frac{80}{180} \times \frac{87}{179}}{\frac{162}{180}}$	(M1) (M1)	M1 for sight of $\frac{80}{180} \times \frac{87}{179}$ as a numerator M1 for sight of $\frac{162}{180} (= 0.9)$ as a denominator
	$= \frac{1160}{4833} = 0.2400\ldots$ oe	(A1) **[3]**	cao A1 is 3sf or greater if using decimals. 0.24 with no working scores 0 marks.
	Total for Question 1	**3**	

Qu	Solution	Mark	Notes
2(a)	$\theta \sim \mathrm{U}[0,45]$	B1	Allow $\theta \sim \mathrm{U}(0,45)$ si by correct values, diagram, or calculation of mean (22.5), variance (675/4=168.75) or SD ($15\sqrt{3}/2$=12.99). Condone mislabelling of θ.
	(mean $=\frac{0+45}{2}=$) 22.5 oe (e.g. $\frac{45}{2}$)	B1	FT their $\mathrm{U}[a,b]$ provided $0 \leq a < 90$, $a < b$ and $0 < b \leq 90$ if stated, implied by a diagram or implied by calculations.
	(SD $= \sqrt{\frac{(45-0)^2}{12}} =) \frac{15\sqrt{3}}{2}$ oe (e.g. $\frac{45}{2\sqrt{3}}, 12.99$)	B1	FT their $\mathrm{U}[a,b]$ provided $0 \leq a < 90$, $a < b$ and $0 < b \leq 90$ if stated or implied by a diagram or implied by calculations. 3sf or better for SD
		[3]	
(b)	$X = 8\sin\theta$	B1	si
	$\mathrm{P}(X > 5) = \mathrm{P}(8\sin\theta > 5)$ $= \mathrm{P}(\sin\theta > \frac{5}{8})$ $= \mathrm{P}(\theta > 38.68\ldots)$ OR $1 - \mathrm{P}(\theta < 38.68\ldots)$	M1 A1	oe, e.g. $1 - \mathrm{P}(8\sin\theta \leq 5)$
	$= \frac{45-38.68\ldots}{45}$ OR $= 1 - \frac{38.68\ldots-0}{45}$	m1	FT their Uniform distribution for θ from (a) where possible (even if mislabelled). FT their 38.68 provided $38.68 < \text{their } b$. Do not award M1 if a uniform distribution used for the length X instead of the angle θ e.g.
	$= 0.1404$	A1	Cao (condone 0.14 from use of 38.7) Do not condone 0.133… from using 39
		[5]	
	Total for Question 2	**8**	

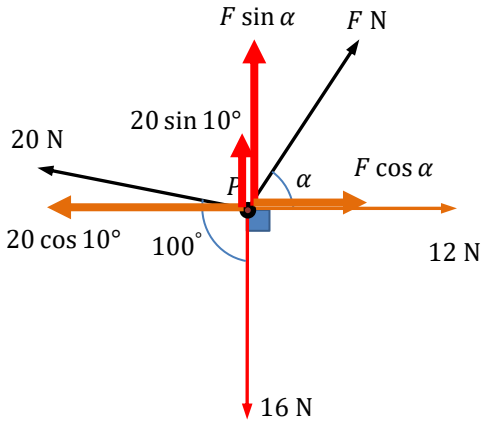
Qu	Solution	Mark	Notes
3(a)	Valid reason, e.g., • It is obvious that, in the general population, people with bigger biceps would have bigger forearms. • It is very unreasonable to think that people with bigger forearms have smaller biceps in general. • No need to check for negative correlation. • She knows there will be some kind of positive correlation.	E1 **[1]**	Condone "negative correlation is impossible." E0 for the diagram shows positive correlation. E0 for "it should be a two-tailed test." E0 for "specifically looking for a positive or negative correlation" (both positive and negative stated). E0 for implying a causal relationship, e.g. "increase in forearm girth will cause an increase in bicep girth".
(b)	(Let ρ denote the population correlation coefficient between forearm girth and bicep girth.) H_0: $\rho = 0$ $\quad$ H_1: $\rho > 0$ TS $\left(= \sqrt{0.9412}\right) = 0.97015\ldots$ CV = 0.7348 Since 0.970 > 0.7348 (there is sufficient evidence to) Reject H_0. Sufficient evidence to suggest there is a **positive** correlation between bicep girth and forearm girth.	B1 B1 B1 B1 E1 **[5]**	Allow other letters if defined. Allow worded hypotheses B0 for H_0: correlation = 0. Population must be stated or implied. B0 for omission of ρ or use of r B0 for a non-strict inequality in H_1 Labelled as TS or used in correct comparison with a critical value (not with significance level). B0 for $r = 0.97015$ without label as TS or used in comparison. B0 for $TS = \pm 0.97015$ unless the positive value correctly used later. B0 for 0.7545 from Spearman's rank. Condone 0.7348 stated as the CV even if using a two-tailed test. FT for using 0.9412 and their CV. (Comparison may be implied by a diagram.) cso Do not allow categorical statements E0 for omission of the word positive (unless positive implied by contextualised comment, e.g. E1 for sufficient evidence to suggest that the bigger the bicep girth the bigger the forearm girth.)
(c) (i)	Her n is not in the table.	E1	oe Condone "likely not in the table".
(ii)	Valid comment, e.g. • Use a software package to carry out the test. • Use a software package to find the CV. • Use a software package to find the p-value. • Realise that the CVs are lower than those for $n = 100$ in the tables. • Realise that there is no need to carry out a test for correlation when the scatter diagram clearly shows very strong correlation (and is bivariate normal).	E1 **[2]**	Condone "Use an online table that goes to 507." Allow "Use a calculator to find the CVs." E0 for "use the calculator" without reference to a possible use of the calculator (e.g. find the CV). E0 for "use a smaller sample size." E0 for "take a sample so that you can use tables." Allow "work out critical values using a t-distribution."
	Total for Question 3	**8**	

Qu	Solution	Mark	Notes
4 (a)	Let the random variable X be the mass in kg of a parcel. $X \sim \mathrm{N}(2.2, 0.3^2)$ $\mathrm{P}(X < 1.8) = 0.091211$	M1A1 **[2]**	3sf or better M1A1 for correct answer from calculator M1 for correctly standardising $z = \frac{1.8-2.2}{0.3}\left(= -\frac{4}{3} = -1.3333..\right)$ M1A1 for 0.09176 from tables using -1.33
(b)	$\mathrm{P}(X < m) = 0.8$ $m = 2.452$ Mass of parcel is 2.452 kg.	M1 A1 **[2]**	M1 implied by correct answer from calculator or for correctly standardising $0.842 = \frac{m-2.2}{0.3}$ (note: 0.8 on the LHS earns M0, allow 0.84 or better) A1 for 2.45 or better (A0 for 2.5) M1A1 for 2.4526 from tables.
(c)	$\mathrm{P}(X < 3\|\mathrm{car}) = \frac{\mathrm{P}(m<X<3)}{\mathrm{P}(\mathrm{car})}$ $= \frac{0.196(1697275)}{0.2}$ $= 0.981$	M1 M1 m1 A1 **[4]**	Recognising conditional probability leading to fraction M1 for $\mathrm{P}(m < X < 3)$ (not needed in a fraction), FT their (b) if $0 < m < 3$. Dependent on first M1 only. Correct denominator in a fraction cao 3sf or better A1 for use of 2.452, which gives 0.983. A0 for use of 2.45, which gives 0.993. Use of tables gives 0.983(3) from 0.19666/0.2.

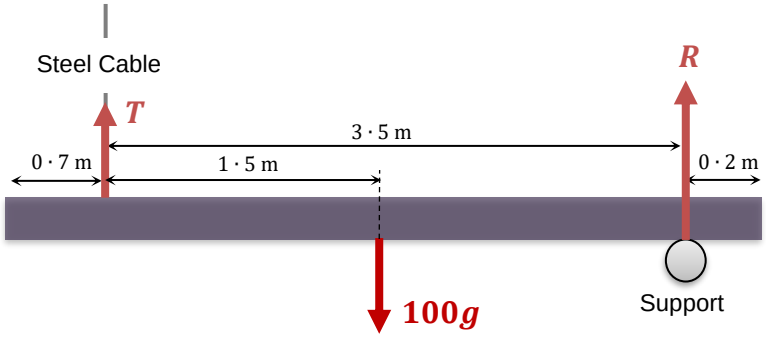
Qu	Solution	Mark	Notes
4 (d)	(Let μ be the mean mass of a parcel in kg) $H_0: \mu = 2.2$ $H_1: \mu > 2.2$	B1	Allow other letters if defined. Allow worded hypotheses (must refer to population). B0 for $H_0: \text{mean} = 2.2$, must imply/refer to population. B0 for omission of μ, or $\bar{x}$, or m, in place of μ. B0 for a non-strict inequality in H_1.
	$\bar{X} \sim \text{N}\left(2.2, \frac{0.3^2}{20}\right)$ under H_0	B1	Distribution of $\bar{X}$ si.
	Use of $\bar{x} = \frac{46}{20} = 2.3$.	B1	B1 for appropriate **use** of 2.3, not just calculating.
	METHOD 1 (*p*-value) $\text{P}(\bar{X} > 2.3 \mid H_0)$	M1	Must use $\sigma_{\bar{X}} = 0.3/\sqrt{20}$ for M1. M1 for $\text{P}\left(Z > \frac{2.3-2.2}{\frac{0.3}{\sqrt{20}}}\right) = \text{P}(Z > 1.49)$ 1.49 earns M1 only in a *p*-value method provided $\text{P}(Z > 1.49)$ considered. M0 for $\text{P}(Z < 1.49)$.
	$= 0.0680(18\ldots)$	A1	0.06811 from tables.
	Since $0.0680 > 0.05$ there is insufficient evidence to reject H_0.	m1	Dep on previous M1.
	METHOD 2 (Critical value) CV = 2.31 (CR is $\bar{X} > 2.31$)	(M1A1)	M1 implied by correct answer from calculator or for correctly standardising and equating to 1.645: $\frac{\text{CV}-2.2}{\frac{0.3}{\sqrt{20}}} = 1.645$. Must use $\sigma_{\bar{X}} = 0.3/\sqrt{20}$ for M1.
	Since $2.3 < 2.31$ there is insufficient evidence to reject H_0.	(m1)	Dep on previous M1.
	METHOD 3 (critical value with standardising): $\text{TS} = \frac{2.3-2.2}{\frac{0.3}{\sqrt{20}}}$	(M1)	Must use $\sigma_{\bar{X}} = 0.3/\sqrt{20}$ for M1.
	$= 1.49$ Since $1.49 < 1.645$ there is insufficient evidence to reject H_0.	(A1) (m1)	1.49 earns M1A1 if used as a TS. Dependent on previous M1. FT their TS. Correct comparison with 1.645 required.
	There is insufficient evidence to suggest that parcels are heavier in the run up to Christmas.	A1	cso Do not allow categorical statements
		[7]	

Qu	Solution	Mark	Notes
4(e)	Probability of ABC is $\frac{1}{2}\times\frac{1}{6}\times\frac{1}{3}\left(=\frac{1}{36}\right)$	M1	M1 for sight of correct product or correct value (1/36 or $0.02\dot{7}$)
	In any order is $\times 6 \therefore \frac{1}{6}$	A1	CAO (3sf or better if working in decimals)
		[2]	
(f)	$\mathrm{P}(AA)=\frac{1}{2}\times\frac{1}{2}(=\frac{1}{4}=0.25)$		
	$\mathrm{P}(BB)=\frac{1}{6}\times\frac{1}{6}(=\frac{1}{36}=0.02\dot{7})$		
	$\mathrm{P}(CC)=\frac{1}{3}\times\frac{1}{3}(=\frac{1}{9}=0.\dot{1})$	B2	B2 for all three products correct and no incorrect or additional products.
	$\mathrm{P}(AA \text{ or } BB \text{ or } CC)=\frac{1}{4}+\frac{1}{36}+\frac{1}{9}=\frac{14}{36}=\frac{7}{18}$		B1 for at least one correct product for either $\mathrm{P}(AA)$ or $\mathrm{P}(BB)$ or $\mathrm{P}(CC)$
	$\mathrm{P}(\text{More than one area})$ $=1-\mathrm{P}(\text{only one area})=1-\frac{7}{18}$	M1	M1 for $1-$ P(their only one area from sum of products for at least two correct terms).
	$\mathrm{P}(\text{More than one area})=\frac{11}{18}(=0.6\dot{1})$	A1	CAO 3sf or better if working in decimals
	Alternative solution 1		
	$\mathrm{P}(AA')=\frac{1}{2}\times\frac{1}{2}\left(=\frac{1}{4}\right)$ $\mathrm{P}(BB')=\frac{1}{6}\times\frac{5}{6}\left(=\frac{5}{36}\right)$ $\mathrm{P}(CC')=\frac{1}{3}\times\frac{2}{3}\left(=\frac{2}{9}\right)$	(B2)	B2 for all three products correct and no incorrect or additional products. B1 for at least one correct product for either $\mathrm{P}(AA')$, $\mathrm{P}(BB')$ or $\mathrm{P}(CC')$.
	$\mathrm{P}(\text{More than one area})=\frac{1}{4}+\frac{2}{9}+\frac{5}{36}$	(M1)	M1 for P(their >1 area from sum of products for at least two correct terms)
	$\mathrm{P}(\text{More than one area})=\frac{11}{18}(=0.6\dot{1})$	(A1)	CAO 3sf or better if working in decimals.
	Alternative solution 2		
	$\mathrm{P}(AB)=\frac{1}{2}\times\frac{1}{6}\left(=\frac{1}{12}\right)$ $\mathrm{P}(AC)=\frac{1}{2}\times\frac{1}{3}\left(=\frac{1}{6}\right)$ $\mathrm{P}(BC)=\frac{1}{6}\times\frac{1}{3}\left(=\frac{1}{18}\right)$	(B2)	B2 for all three products correct and no incorrect or additional products. $\frac{11}{36}$ earns B2. B1 for at least one correct product.
	$\mathrm{P}(\text{More than one area})=\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{18}\right)\times 2$	(M1)	M1 for sum of products for at least two correct terms and multiplying by 2 oe.
	$\mathrm{P}(\text{More than one area})=\frac{11}{18}(=0.6\dot{1})$	(A1)	CAO 3sf if working in decimals.
		[4]	
	Total for Question 4	**21**	

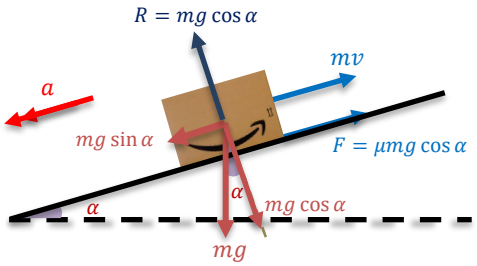
SECTION B – Differential Equations and Mechanics

Q5	Solution	Mark	Notes
			$20 \sin 10° = 3 \cdot 47296 \ldots$ $20 \cos 10° = 19 \cdot 69615 \ldots$ $20 \sin 100° = 19 \cdot 69615 \ldots$ $20 \cos 100° = -3 \cdot 47296 \ldots$
	Resolving horizontally **OR** vertically Parallel to 12 N (↔) $F \cos \alpha + 12 = 20 \cos 10$ $F \cos \alpha = 20 \cos 10 - 12 \quad (= 7 \cdot 696 \ldots)$ Parallel to 16 N (↕) $F \sin \alpha + 20 \sin 10 = 16$ $F \sin \alpha = 16 - 20 \sin 10 \quad (= 12 \cdot 527 \ldots)$ $\tan \alpha = \frac{16 - 20 \sin 10}{20 \cos 10 - 12} \quad (= 1 \cdot 6277 \ldots)$ $\alpha = 58 \cdot 4 (349\ldots) \quad (°)$ Correct method for calculating F $F = \begin{cases} \sqrt{(20 \cos 10 - 12)^2 + (16 - 20 \sin 10)^2} \\ \frac{16 - 20 \sin 10}{\sin \alpha} \\ \frac{20 \cos 10 - 12}{\cos \alpha} \end{cases}$ $= 14 \cdot 7 (0229386\ldots)$	M1 A1 A1 m1 A1 m1 A1 **[7]**	No missing or extra forces Correct equation, oe Correct equation, oe Attempt to eliminate F
	Total for Question 5	7	

Q6	Solution	Mark	Notes
a)	Working horizontally (→) $4w = \frac{60}{5}$ $w = 3$ Working vertically using $s = ut + \frac{1}{2}at^2$, with $u = \pm 21\ (\pm 7w), t = 5, a = \pm 9 \cdot 8$ $s = (\pm 21)\ (5) + \frac{1}{2}(\mp 9 \cdot 8)(5)^2$ $s = \mp 17 \cdot 5$ Height of tower is $17 \cdot 5$ (m) <u>Alternative solution for final 3 marks</u> $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2 + (\mathbf{s_0})$, with $\mathbf{u} = (12\mathbf{i} + 21\mathbf{j}), \mathbf{a} = \pm g\mathbf{j}, t = 5,$ $(\mathbf{s} = 60\mathbf{i})$ $\mathbf{s} = (12\mathbf{i} + 21\mathbf{j})(5) + \frac{1}{2}(-g\mathbf{j})(5)^2$ $+ (\mathbf{s_0} = h\mathbf{j})$ $\mathbf{s} = 60\mathbf{i} - 17 \cdot 5\mathbf{j}$ Height of tower is $17 \cdot 5$ (m)	M1 A1 M1 A1 A1 **[5]** (M1) (A1) (A1)	Using speed $= \frac{\text{distance}}{\text{time}}$ Numerically or in terms of w FT their '$7w$' 21 $(7w)$ opposing g Numerically or in terms of w FT their '$7w$' 21 $(7w)$ opposing g
b)	Working vertically using $v = u + at$, with $u = \pm 21\ (\pm 7w), v = 0,\ a = \pm g = \pm 9 \cdot 8,$ $0 = \mp 21 \pm 9 \cdot 8t$ or $0 = \mp 21 \pm gt$ $t = \frac{15}{7}$ (s) (Time to reach maximum height) Proportion of journey on way down $= \frac{5 - \frac{15}{7}}{5} = \frac{4}{7} = 0 \cdot 57(14 \ldots)$	M1 A1 A1 **[3]**	FT their '$7w$' 21 $(7w)$ opposing g
	Total for Question 6	**8**	

Q7	Solution	Mark	Notes
			
(a)	(i) Moments about base of cable $100g \times 1 \cdot 5 = R \times 3 \cdot 5$ $980 \times 1 \cdot 5 = R \times 3 \cdot 5$ $1470 = R \times 3 \cdot 5$ $R = \frac{300}{7}g = 420 \quad (\text{N})$ $420 \times 4 = 1680 < 2000 \quad (\text{N})$ OR $\frac{2000}{420} = 4 \cdot 7619 \ldots (> 4)$ Therefore, safety requirement satisfied.	M1 A1 A1	Dim. correct equation Allow incorrect distances with $R, 100g$ Correct equation, oe Convincing
	(ii) Resolve vertically $T + R = 100g$ $T = \frac{400}{7}g = 560 \quad (\text{N})$ Safety requirement satisfied: $560 \times 4 = 2240 \quad (\text{N})$ OR $\frac{2500/3000}{560} = 4 \cdot 46 \ldots / 5 \cdot 3 \ldots (> 4)$ Therefore, **Categories A** and/or **B**.	M1 A1 A1 **[6]**	Dim. correct eqn, allow 1 sign error (or 2nd moment equation) Correct equation, oe FT R Convincing
(b)	Examples, • **Weight** acts at the **centre** of the rod. • **Mass** is at the **centre** of the rod.	E1 **[1]**	
	Total for Question 7	**7**	

Q8	Solution	Mark	Notes
a)	$\mathbf{F_1}+\mathbf{F_2}+\mathbf{F_3}=(15c-12)\mathbf{i}+(9c)\mathbf{j}+(6c-12)\mathbf{k}$ $\mathbf{a}=\dfrac{\mathbf{F_1}+\mathbf{F_2}+\mathbf{F_3}}{3}=(5c-4)\mathbf{i}+(3c)\mathbf{j}+(2c-4)\mathbf{k}$ $\mathbf{a}$ is parallel to $(\mathbf{i}+\mathbf{j})$ $\Rightarrow$ $\mathbf{i}=\mathbf{j}$ component or $\mathbf{k}$ component $=\mathbf{0}$ $5c-4=3c$ or $2c-4=0$ $c=2$ Substitute $c=2$ into expression for $\mathbf{a}$ $\mathbf{a}=6\mathbf{i}+6\mathbf{j}$ (ms^{-2})	M1 m1 A1 A1 **[4]**	Sum attempted Convincing
b)	Use of $\mathbf{r}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ $(+\mathbf{r_0})$ with $\mathbf{u}=(-17\mathbf{i}+8\mathbf{j})$, $t=4$, $\mathbf{a}=6\mathbf{i}+6\mathbf{j}$ $\mathbf{r}=(-17\mathbf{i}+8\mathbf{j})(4)+\frac{1}{2}(6\mathbf{i}+6\mathbf{j})(4)^2$ $(+\mathbf{r_0})$ $\mathbf{r}=-20\mathbf{i}+80\mathbf{j}$ $(+\mathbf{r_0})$ $-13\mathbf{i}+84\mathbf{j}=-20\mathbf{i}+80\mathbf{j}+\mathbf{r_0}$ $\mathbf{r_0}=7\mathbf{i}+4\mathbf{j}$ (m)	M1 A1 A1 **[3]**	
	Total for Question 8	**7**	

Q9	Solution	Mark	Notes
			$\sin\alpha = \frac{7}{25} = 0\cdot 28$ $\cos\alpha = \frac{24}{25} = 0\cdot 96$ $\mu = \frac{1}{12}$
(a)	$R = mg\cos\alpha \qquad \left(= \frac{24}{25}mg\right)$	B1	si
	$F = \frac{1}{12}\times R \qquad \left(F = \frac{1}{12}\times mg\cos\alpha = \frac{1}{12}\times\frac{24}{25}mg = \frac{2}{25}mg\right)$	B1	si
	Apply N2L to parcel, downwards positive	M1	Dim. correct equation, all forces/terms
	$mg\sin\alpha - F - mv = ma$	A1	
	$\frac{7}{25}mg - \frac{2}{25}mg - mv = ma$		
	$5\frac{dv}{dt} = g - 5v$	A1	Convincing
		[5]	
(b)	$\int \frac{5}{g-5v}dv = \int dt \qquad \frac{5}{-5}\int \frac{1}{g-5v}dv = \int dt$	M1	Separating variables
	$-\frac{5}{5}\ln\lvert g-5v\rvert = t\ (+C)$	A1	Correct integration
	When $t = 0$, $v = 0$ $C = -\ln(g)$	m1	Used
	$-t = \ln\left\lvert\frac{g-5v}{g}\right\rvert$	m1	Attempted inversion,
	$e^{-t} = 1 - \frac{5}{g}v$		
	$v = \frac{g}{5}(1 - e^{-t})$	A1	oe (in terms of g) Examples, $v = \frac{g}{5}\left(1 - \frac{1}{e^t}\right) = \frac{ge^t - g}{5e^t}$
		[5]	
(c)	Parcel will **not exceed 2** with reason, e.g. • Limiting speed is $\frac{g}{5} = 1\cdot 96\ (< 2)$ • DE Valid for $v < \frac{g}{5} = 1\cdot 96\ \ (< 2)$	B1	Convincing
		[1]	
	Total for Question 9	**11**	

GCE AS MARKING SCHEME

SUMMER 2024

AS
FURTHER MATHEMATICS
UNIT 1 FURTHER PURE MATHEMATICS A
2305U10-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 1 FURTHER PURE MATHEMATICS A

SUMMER 2024 MARK SCHEME

Qu	Solution	Mark	Notes
1.	METHOD 1: $\frac{v}{w} = \frac{-16+11\mathrm{i}}{5+2\mathrm{i}} \times \frac{5-2\mathrm{i}}{5-2\mathrm{i}}$	M1	Multiplying by conjugate
	$\frac{v}{w} = \frac{-80+32\mathrm{i}+55\mathrm{i}-22\mathrm{i}^2}{25-10\mathrm{i}+10\mathrm{i}-4\mathrm{i}^2}$		
	$\frac{v}{w} = \frac{-58+87\mathrm{i}}{29} = -2+3\mathrm{i} = z$	A1	No workings M0A0
	$\lvert z\rvert = \sqrt{(-2)^2+3^2} = \sqrt{13}$	A1	FT z
	$\arg z = \tan^{-1}\left(\frac{3}{-2}\right) + 180^\circ = 123.69^\circ$	A1	FT z 2.16 radian Accept 124°
	$z = \sqrt{13}(\cos 123.69^\circ + \mathrm{i}\sin 123.69^\circ)$	B1	FT arg
	METHOD 2: $\lvert v\rvert = \sqrt{(-16)^2+11^2} = \sqrt{377}$ $\lvert w\rvert = \sqrt{5^2+2^2} = \sqrt{29}$	(B1)	Both v and w
	$\arg v = \tan^{-1}\left(\frac{11}{-16}\right) + 180^\circ = 145.49^\circ$ $\arg w = \tan^{-1}\left(\frac{2}{5}\right) = 21.80^\circ$	(B1)	Both v and w 2.54 and 0.38 rad
	$\lvert z\rvert = \frac{\sqrt{377}}{\sqrt{29}} = \sqrt{13}$	(M1)	FT using both mod and arg
	$\arg z = 145.49^\circ - 21.80^\circ = 123.69^\circ$	(A1)	FT both $\lvert z\rvert$ and $\arg(z)$ correct 2.16 radian Accept 124°
	$z = \sqrt{13}(\cos 123.69^\circ + \mathrm{i}\sin 123.69^\circ)$	(B1)	FT arg
		Total [5]	

Qu	Solution	Mark	Notes
2.	$x^3 + x^2 - 7x - 15 = (x-3)(x^2+4x+5)$ When $x - 3 = 0$, $x = 3$	B1	May be awarded at any point
	When $x^2 + 4x + 5 = 0$, Solving, e.g. $x = \frac{-4 \pm \sqrt{4^2-(4\times1\times5)}}{2}$ or $(x+2)^2 + 1 = 0$ $(x+2)^2 = -1$ $x + 2 = \pm\mathrm{i}$	M1	Complete method M0 no workings
	$x = -2 \pm \mathrm{i}$	A1	Both, cao
		Total [3]	

Qu	Solution	Mark	Notes
3.	METHOD 1:		
	Original quadratic with roots α $\alpha + \alpha = -p$ $\alpha \times \alpha = q$	B1	Both Allow use of other notation for p, q but must be α
	EITHER: New quadratic with roots $\frac{1}{\alpha}$ $\frac{1}{\alpha} + \frac{1}{\alpha} = -m$ $\frac{1}{\alpha} \times \frac{1}{\alpha} = m$	B1	Both si Allow use of other notation for m
	OR: New quadratic with roots $\frac{1}{\alpha}$ $\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\alpha}\right) = 0$ $x^2 - \frac{2}{\alpha}x + \frac{1}{\alpha^2} = 0$	(B1)	
	THEN: $-\left(\frac{1}{\alpha} + \frac{1}{\alpha}\right) = \frac{1}{\alpha} \times \frac{1}{\alpha}$ $\frac{-2}{\alpha} = \frac{1}{\alpha^2}$ $-2\alpha^2 = \alpha$ $2\alpha^2 + \alpha = 0$ $\alpha(2\alpha + 1) = 0$	M1	
	$\therefore \alpha = 0$ or $\alpha = -\frac{1}{2}$	A1	Either value
	As $\alpha \neq 0$, $\alpha = -\frac{1}{2}$.	A1	Must reject $\alpha = 0$
	Therefore, $-\frac{1}{2} + -\frac{1}{2} = -p \quad \rightarrow p = 1$ $-\frac{1}{2} \times -\frac{1}{2} = q \quad \rightarrow q = \frac{1}{4}$	A1	FT their α
	METHOD 2: Let $w = \frac{1}{x}$, then $x = \frac{1}{w}$	(M1)	
	leading to $\left(\frac{1}{w}\right)^2 + p\left(\frac{1}{w}\right) + q = 0$ $qw^2 + pw + 1 = 0.$ $w^2 + \frac{p}{q}w + \frac{1}{q} = 0$	(A1)	
	Hence, $\frac{p}{q} = m$ and $\frac{1}{q} = m$ therefore $\frac{p/q}{1/q} = \frac{m}{m}$ such that $p = 1$.	(A1) (A1)	
	Given than $\frac{1}{\alpha} + \frac{1}{\alpha} = -m$, $-\frac{2}{\alpha} = m \rightarrow \alpha = -\frac{2}{m}$ and $p = -2\alpha = \frac{4}{m} \rightarrow m = 4$	(B1)	
	Therefore, $\frac{1}{q} = 4 \rightarrow q = \frac{1}{4}$	(B1)	

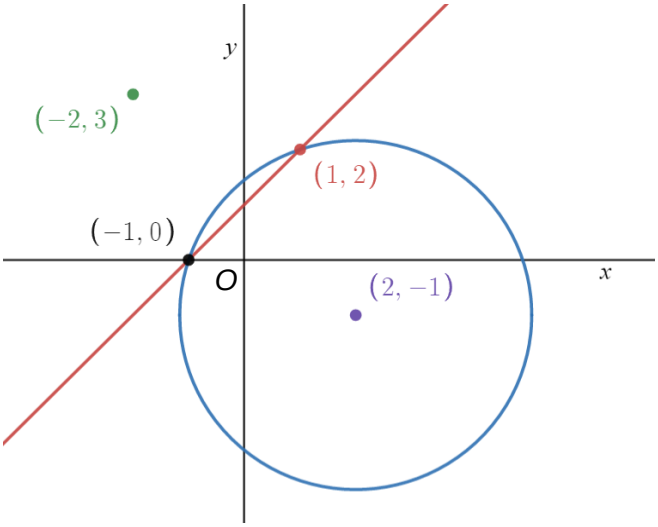
Qu	Solution	Mark	Notes
3.	METHOD 3: Given repeated root, $b^2 - 4ac = 0$ therefore $m^2 - 4m = 0$	(M1)	
	Solving, e.g. $m(m-4) = 0$ $m = 0$ or $m = 4$	(A1)	
	When $m = 0$, no possible roots so new equation is $x^2 + 4x + 4 = 0$	(A1)	Must reject $m = 0$
	Solving, e.g. $(x+2)^2 = 0$ $x = -2$	(A1)	
	Therefore, $-2 = \frac{1}{\alpha} \rightarrow \alpha = -\frac{1}{2}$	(B1)	
	Original equation: $\left(x + \frac{1}{2}\right)^2 = 0$ $x^2 + x + \frac{1}{4} = 0$ such that $p = 1, q = \frac{1}{4}$.	(B1)	
		Total [6]	

Qu	Solution	Mark	Notes
4. a)	$u+\mathrm{i}v=\dfrac{x+\mathrm{i}y}{1-x-\mathrm{i}y}$	M1	
	$u+\mathrm{i}v=\dfrac{x+\mathrm{i}y}{(1-x)-\mathrm{i}y}\times\dfrac{(1-x)+\mathrm{i}y}{(1-x)+\mathrm{i}y}$	m1	
	$u+\mathrm{i}v=\dfrac{(x+\mathrm{i}y)(1-x)+(x+\mathrm{i}y)(\mathrm{i}y)}{(1-x)^2-(\mathrm{i}y)^2}$ $u+\mathrm{i}v=\dfrac{x-x^2+\mathrm{i}y-\mathrm{i}xy+\mathrm{i}xy-y^2}{(1-x)^2+y^2}$ $u+\mathrm{i}v=\dfrac{(x-x^2-y^2)+\mathrm{i}y}{(1-x)^2+y^2}$	A1	
	Comparing coefficients Imaginary parts: $v=\frac{y}{(1-x)^2+y^2}$ (given) Real parts: $u=\frac{x-x^2-y^2}{(1-x)^2+y^2}$	m1 A1 [5]	Dependent on M1 Both correct If expanded, denominator must be correct, otherwise A0
b)	Putting $y=1-x$	M1	FT their u for method marks only
	$v=\dfrac{y}{y^2+y^2}\left(=\dfrac{y}{2y^2}=\dfrac{1}{2y}\right)$	A1	
	$u=\dfrac{(1-y)y-y^2}{y^2+y^2}=\dfrac{y-2y^2}{2y^2}$ $u=\dfrac{y}{2y^2}-1$	A1	
	Eliminating y, the equation of the locus Q is $u=v-1$	m1 A1	cao oe
	OR Putting $y=1-x$	(M1)	FT their u for method marks only
	$v=\dfrac{1-x}{(1-x)^2+(1-x)^2}\left(=\dfrac{1-x}{2(1-x)^2}=\dfrac{1}{2(1-x)}\right)$	(A1)	
	$u=\dfrac{x-x^2-(1-x)^2}{(1-x)^2+(1-x)^2}$ $u=\dfrac{x-x^2-1+2x-x^2}{2(1-x)^2}=\dfrac{-2x^2+3x-1}{2(1-x)^2}$ $u=\dfrac{(1-x)(2x-1)}{2(1-x)^2}=\dfrac{2x-1}{2(1-x)}=\dfrac{1-2(1-x)}{2(1-x)}$ $u=\dfrac{1}{2(1-x)}-1$	(A1)	
	Eliminating x, the equation of the locus Q is $u=v-1$	(m1) (A1) [5]	cao oe
		Total [10]	

Qu	Solution	Mark	Notes
5.	METHOD 1: $\sum_{r=k}^{76}(r-31)=\sum_{r=1}^{76}(r-31)-\sum_{r=1}^{k-1}(r-31)$	M1	Condone $\sum_{r=1}^{76}\square-\sum_{r=1}^{k}\square$
	$=\left[\left(\frac{1}{2}\times 76\times 77\right)-(31\times 76)\right]$ $-\left[\left(\frac{1}{2}\times(k-1)\times k\right)-\left(31\times(k-1)\right)\right]$	m1 A1	Use of $\sum\square$ formulae All correct
	METHOD 2: $\sum_{r=k}^{76}(r-31)=\sum_{r=1}^{76}(r-31)-\sum_{r=1}^{k-1}(r-31)$	(M1)	Condone $\sum_{r=1}^{76}\square-\sum_{r=1}^{k}\square$
	$\sum_{r=1}^{n}(r-31) = \sum_{r=1}^{n}r-\sum_{r=1}^{n}31$ $=\frac{n(n+1)}{2}-31n$ $=\frac{n(n-61)}{2}$	(m1)	Use of $\sum\square$ formulae
	$\sum_{r=k}^{76}(r-31)=\frac{76\times 15}{2}-\frac{(k-1)(k-62)}{2}$	(A1)	All correct
	THEN If $\sum_{r=k}^{76}(r-31)=980$, $570-\left[\left(\frac{1}{2}\times(k-1)\times k\right)-\left(31\times(k-1)\right)\right]=980$	M1	FT their expression provided at least quadratic
	$570-\frac{k^2}{2}+\frac{63k}{2}-31=980$ $\frac{k^2}{2}-\frac{63k}{2}+441=0$	A1	cao, oe
	Solving, $k^2-63k+882=0$ $(k-21)(k-42)=0$ $k=21$ or $k=42$ $\therefore k$ has two possible values.	m1 A1	m0 no working FT their quadratic for m1 cao
	If m0A0, then SC1 for: Discriminant = $(-63)^2-(4\times 1\times 882)$ Discriminant = $441>0$ $\therefore k$ has two possible values Note: Trial and improvement, 0 marks	**Total [7]**	

Qu	Solution	Mark	Notes
6. a)	METHOD 1:		
	L_1		
	$\|z-2+\mathrm{i}\| = \|z+2-3\mathrm{i}\|$		
	$\|x+\mathrm{i}y-2+\mathrm{i}\| = \|x+\mathrm{i}y+2-3\mathrm{i}\|$	M1	si
	$\|(x-2)+\mathrm{i}(y+1)\| = \|(x+2)+\mathrm{i}(y-3)\|$	A1	si
	$(x-2)^2+(y+1)^2 = (x+2)^2+(y-3)^2$	m1	
	$x^2-4x+4+y^2+2y+1 = x^2+4x+4+y^2-6y+9$		
	$-8x+8y-8=0$	A1	oe
	$y = x+1$		
	L_2		
	$\|z-2+\mathrm{i}\| = \sqrt{10}$		
	$\|x+\mathrm{i}y-2+\mathrm{i}\| = \sqrt{10}$		Award M1A1m1 here if not awarded for L_1
	$\|(x-2)+\mathrm{i}(y+1)\| = \sqrt{10}$		
	$(x-2)^2+(y+1)^2 = 10$	A1	
	$x^2-4x+4+y^2+2y+1 = 10$		
	$x^2+y^2-4x+2y-5=0$		
	Substituting from L_1 into L_2:	m1	FT their equation of a line
	$x^2+(x+1)^2-4x+2(x+1)-5=0$		
	$2x^2-2=0$	m1	Solving equation
	$x^2=1$		
	$\therefore\ x=1 \quad \text{or} \quad x=-1$	A1	cao
	OR		
	$(y-1)^2+y^2-4(y-1)+2y-5=0$		
	$2y^2-4y=0$		
	$2y(y-2)=0$		Award m1A1 for y if not awarded for x
	$\therefore\ y=0 \quad \text{or} \quad y=2$		
	Therefore points of intersection are $(1,2)$ and $(-1,0)$	B1	Must be paired correctly FT their x or y into their equation of a line provided real

Qu	Solution	Mark	Notes
6. a)	METHOD 2:		
	L_2		
	$\|z-2+\mathrm{i}\| = \sqrt{10}$		
	$\|x+\mathrm{i}y-2+\mathrm{i}\| = \sqrt{10}$	(M1)	si
	$\|(x-2)+\mathrm{i}(y+1)\| = \sqrt{10}$	(A1)	si
	$(x-2)^2+(y+1)^2 = 10$	(m1)	
	$x^2-4x+4+y^2+2y+1 = 10$		
	$x^2+y^2-4x+2y-5 = 0$	(A1)	
	Equating L_1 to $\sqrt{10}$:		
	$\|z+2-3\mathrm{i}\| = \sqrt{10}$		
	$\|x+\mathrm{i}y+2-3\mathrm{i}\| = \sqrt{10}$		Award M1A1m1 here if not awarded for L_2
	$\|(x+2)+\mathrm{i}(y-3)\| = \sqrt{10}$		
	$(x+2)^2+(y-3)^2 = 10$		
	$x^2+4x+4+y^2-6y+9 = 10$		
	$x^2+y^2+4x-6y+3 = 0$	(A1)	
	Finding the equation of L_1 and substituting into equation of circle:	(m1)	
	$(x^2+y^2+4x-6y+3)-(x^2+y^2-4x+2y-5) = 0-0$		
	$8x-8y+8 = 0$		oe
	$y = x+1$		
	$x^2+(x+1)^2-4x+2(x+1)-5 = 0$		
	$2x^2-2 = 0$	(m1)	Solving to find x or y
	$\therefore\ x = 1 \quad \text{or} \quad x = -1$	(A1)	Award m1A1 for any of these 4 options of solutions
	OR		
	$(y-1)^2+y^2-4(y-1)+2y-5 = 0$		
	$2y^2-4y = 0$		
	$2y(y-2) = 0$		
	$\therefore\ y = 0 \quad \text{or} \quad y = 2$		
	OR		
	$x^2+(x+1)^2+4x-6(x+1)+3 = 0$		
	$2x^2-2 = 0$		
	$\therefore\ x = 1 \quad \text{or} \quad x = -1$		
	OR		
	$(y-1)^2+y^2+4(y-1)-6y+3 = 0$		
	$2y^2-4y = 0$		
	$2y(y-2) = 0$		
	$\therefore\ y = 0 \quad \text{or} \quad y = 2$		
	Therefore points of intersection are $(1,2)$ and $(-1,0)$	(B1)	Must be paired correctly FT their x or y into their equation of a line provided real

Qu	Solution	Mark	Notes
6. a)	METHOD 3: (Geometric argument) L_1 is perp bisector of line of (2, -1) and (-2, 3) $m = \frac{3--1}{-2-2} = \frac{4}{-4} = -1 \rightarrow m(\text{perp}) = 1$ Midpoint: (0,1) Equation of L_1: $y = x + 1$ L_2 is a circle centred at (2, -1) with radius $\sqrt{10}$, such that $(x-2)^2 + (y+1)^2 = 10$ leading to $(x-2)^2 + (x+2)^2 = 10$ $2x^2 + 8 = 10$ $x^2 = 1$ $\therefore x = \pm 1$ When $x = 1, y = 2$ and when $x = -1, y = 0$	M1 A1 A1 m1 A1 m1 A1 B1 B1 [9]	 oe or $y = 2, 0$ FT their x provided real If B0B0, award SC1 for incorrectly paired coordinates
b)		G1 G1 G1 [3] **Total [12]**	Circle, centred in 4th quadrant, extending into all 3 other quads Line, positive gradient, intersecting circle twice FT their line Fully correct, with points of intersection clearly labelled

Qu	Solution	Mark	Notes
7.	When $n = 1$, $13^1 + 8 = 21$ which is a multiple of 7. Therefore, proposition is true for $n = 1$.	B1	
	Assume the proposition is true for $n = k$	M1	
	i.e. $13^{2k-1} + 8$ is a multiple of 7 or $13^{2k-1} + 8 = 7N$		
	Consider $n = k + 1$		
	$13^{2(k+1)-1} + 8 = 13^{2k-1+2} + 8$	M1	
	$= 13^2(13^{2k-1}) + 8$	A1	
	$= 169(7N - 8) + 8$	A1	$169(13^{2k-1} + 8) - 1344$
	$= 1183N - 1344$		
	$= 7(169N - 192)$		
	Since this is a multiple of 7, $13^{2(k+1)-1} + 8$ is also a multiple of 7.	A1	
	So, if the proposition is true for $n = k$, it is also true for $n = k + 1$.		
	Since we have shown it is true for $n = 1$, by mathematical induction, it is true for all positive integers n.	A1	cso
		Total [7]	

Qu	Solution	Mark	Notes
8. a)	METHOD 1:		
	When $\cos\theta = 0.8$ and θ acute, $\sin\theta = 0.6$		
	Rotation matrix: $\begin{bmatrix}0.8 & -0.6\\0.6 & 0.8\end{bmatrix}$	B1	si
	$\begin{bmatrix}0.8 & -0.6\\0.6 & 0.8\end{bmatrix}[\text{Reflection}] = \frac{1}{85}\begin{bmatrix}-84 & -13\\-13 & 84\end{bmatrix}$	M1	Attempt to use rotation matrix in equation
	$[\text{Reflection}] = \frac{1}{85}\begin{bmatrix}0.8 & -0.6\\0.6 & 0.8\end{bmatrix}^{-1}\begin{bmatrix}-84 & -13\\-13 & 84\end{bmatrix}$	m1	Use of inverse
	$\begin{bmatrix}0.8 & -0.6\\0.6 & 0.8\end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix}0.8 & 0.6\\-0.6 & 0.8\end{bmatrix}$	A1	
	$[\text{Reflection}] = \frac{1}{85}\begin{bmatrix}0.8 & 0.6\\-0.6 & 0.8\end{bmatrix}\begin{bmatrix}-84 & -13\\-13 & 84\end{bmatrix}$	m1	Multiplication
	$[\text{Reflection}] = \begin{bmatrix}-\frac{15}{17} & \frac{8}{17}\\ \frac{8}{17} & \frac{15}{17}\end{bmatrix}$	A1	cao
	As $[\text{Reflection}] = \begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ \sin 2\alpha & -\cos 2\alpha\end{bmatrix}$ $\cos 2\alpha = -\frac{15}{17}$ AND $\sin 2\alpha = \frac{8}{17}$ (2α is in the second quadrant)	B1	FT their reflection matrix provided of correct format
	$2\alpha = 151.9(275131\ldots)$ $\alpha = 75.9(6375653\ldots)$	B1	
	$y = (\tan 75.96375653\ldots)x = 4x$ Therefore, $k = 4$	B1	

Qu	Solution	Mark	Notes
8. a)	METHOD 2: When $\cos\theta = 0.8$ and θ acute, $\sin\theta = 0.6$		
	Rotation matrix: $\begin{bmatrix}0.8 & -0.6\\0.6 & 0.8\end{bmatrix}$	(B1)	si
	Let reflection matrix be $\begin{bmatrix}a & b\\c & d\end{bmatrix}$ $\begin{bmatrix}0.8 & -0.6\\0.6 & 0.8\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix} = \frac{1}{85}\begin{bmatrix}-84 & -13\\-13 & 84\end{bmatrix}$	(M1)	
	$0.8a - 0.6c = -\frac{84}{85}$ and $0.6a + 0.8c = -\frac{13}{85}$	(m1)	Attempt to form simultaneous eqn
	$0.8b - 0.6d = -\frac{13}{85}$ and $0.6b + 0.8d = \frac{84}{85}$	(A1)	All correct
	Solving simultaneous equations	(m1)	m0A0 no working
	$a = -\frac{15}{17}$ $b = \frac{8}{17}$ $c = \frac{8}{17}$ $d = \frac{15}{17}$ $[\text{Reflection}] = \begin{bmatrix}-\frac{15}{17} & \frac{8}{17}\\\frac{8}{17} & \frac{15}{17}\end{bmatrix}$	(A1)	cao
	As $[\text{Reflection}] = \begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\\sin 2\alpha & -\cos 2\alpha\end{bmatrix}$ $\cos 2\alpha = -\frac{15}{17}$ AND $\sin 2\alpha = \frac{8}{17}$ (2α is in the second quadrant)	(B1)	FT their reflection matrix provided of correct format
	$2\alpha = 151.9(275131\ldots)$ $\alpha = 75.9(6375653\ldots)$	(B1)	
	$y = (\tan 75.96375653\ldots)x = 4x$ Therefore, $k = 4$	(B1)	

Qu	Solution	Mark	Notes
8. a)	METHOD 3: When $\cos\theta = 0.8$ and θ acute, $\sin\theta = 0.6$		
	Rotation matrix: $\begin{bmatrix}0.8 & -0.6\\0.6 & 0.8\end{bmatrix}$	(B1)	si
	Let reflection matrix be $\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ \sin 2\alpha & -\cos 2\alpha\end{bmatrix}$ $\begin{bmatrix}0.8 & -0.6\\0.6 & 0.8\end{bmatrix}\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ \sin 2\alpha & -\cos 2\alpha\end{bmatrix} = \frac{1}{85}\begin{bmatrix}-84 & -13\\-13 & 84\end{bmatrix}$	(M1)	
	$0.8\cos 2\alpha - 0.6\sin 2\alpha = -\frac{84}{85}$ and $0.6\cos 2\alpha + 0.8\sin 2\alpha = -\frac{13}{85}$	(m1) (A1)	Attempt to form simultaneous eqn All correct
	Solving $\cos 2\alpha = -\frac{15}{17}$	(m1) (A1)	m0A0 no working cao Accept $\sin 2\alpha = \frac{8}{17}$
	As $[\text{Reflection}] = \begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ \sin 2\alpha & -\cos 2\alpha\end{bmatrix}$ $\cos 2\alpha = -\frac{15}{17}$ AND $\sin 2\alpha = \frac{8}{17}$ (2α is in the second quadrant)	(B1)	FT their $\cos 2\alpha$ or $\sin 2\alpha$
	$2\alpha = 151.9(275131\ldots)$ $\alpha = 75.9(6375653\ldots)$	(B1)	
	$y = (\tan 75.96375653\ldots)x = 4x$ Therefore, $k = 4$	(B1) [9]	
8. b)	$T = \frac{1}{85}\begin{pmatrix}-84 & -13\\-13 & 84\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$	M1	
	Giving, $\frac{1}{85}(-84x - 13y) = x$ or $\frac{1}{85}(-13x + 84y) = y$	A1	
	Leading to $169x + 13y = 0$ or $13x + y = 0$ Therefore the line of fixed points is $13x + y = 0$.	A1 [3]	
		Total [12]	

Qu	Solution	Mark	Notes
Qu	**Solution**	**Mark**	**Notes**
9. a)	$\Pi_1: \mathbf{r}.(4\mathbf{i}-3\mathbf{j}+2\mathbf{k})=5$, let $\mathbf{n}_1=4\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ $\Pi_2: \mathbf{r}.(6\mathbf{i}+\mathbf{j}+\mathbf{k})=9$, let $\mathbf{n}_2=6\mathbf{i}+\mathbf{j}+\mathbf{k}$ $\mathbf{n}_1.\mathbf{n}_2=\lvert\mathbf{n}_1\rvert\lvert\mathbf{n}_2\rvert\cos\theta$ $\cos\theta=\frac{(4\times6)+(-3\times1)+(2\times1)}{\sqrt{4^2+(-3)^2+2^2}\sqrt{6^2+1^2+1^2}}$ $\cos\theta=\frac{23}{\sqrt{29}\sqrt{38}}$ $\theta=46.1^{\circ}$	B1 M1 A1 A1 [4]	Both $\mathbf{n}_1, \mathbf{n}_2$ si Use of Mark final answer Accept 0.805 rad
b)	$D=\frac{\lvert(4\times5)+(-3\times-2)+(2\times-6)-5\rvert}{\sqrt{4^2+(-3)^2+2^2}}$ $D=\frac{9}{\sqrt{29}}\quad(1.67\ldots)$	M1 A1 [2]	
c) i)	Point *B*: $(4\times5)+(-3\times5)+(2\times0)=5$ Point *C*: $(6\times1)+(1\times3)+(1\times0)=9$	B1	Both shown convincingly
ii)	Any valid plane equation e.g. $z=0$ (because z-coordinate in *B*, *C* is 0) $-x+2y+kz=5$ (where $k\in R$)	B1 [2] **Total [8]**	

GCE AS MARKING SCHEME

SUMMER 2024

AS
FURTHER MATHEMATICS
UNIT 2 FURTHER STATISTICS A
2305U20-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 2 FURTHER STATISTICS A

SUMMER 2024 MARK SCHEME

Qu.	Solution	Mark	Notes
1(a) (i)	Total number of fish caught, F, is $\text{Po}((3.8 + 4.3) \times 0.5)$ $\text{Po}(4.05)$	M1 M1	M1 for Poisson (si) and adding. M1 for multiplying by 0.5, oe. M0M1 for 4.05 with Poisson not mentioned nor used. M0M1 for $\text{Po}(2.15)$ or $\text{Po}(1.9)$
	$\text{P}(F < 2) = \frac{4.05^1 \times \text{e}^{-4.05}}{1!} + \frac{4.05^0 \times \text{e}^{-4.05}}{0!}$	M1	Use of formula or calculator with their $\lambda \neq 3.8$ or 4.3
	$= 0.08798$	A1	cao Condone 0.088
	ALTERNATIVE SOLUTION Total number of fish caught by Dave, D, is $\text{Po}(4.3 \times 0.5)$ $\text{Po}(2.15)$	(M1)	M1 for both Dave and Llinos
	Total number of fish caught by Llinos, L, is $\text{Po}(3.8 \times 0.5)$ $\text{Po}(1.9)$ Possible combinations are $D = 0$ and $L = 0$ $D = 0$ and $L = 1$ OR $D = 1$ and $L = 0$ $\text{P}(D = 0 \text{ and } L = 0) = \frac{2.15^0 \times \text{e}^{-2.15}}{0!} \times \frac{1.9^0 \times \text{e}^{-1.9}}{0!}$ $\text{P}(D = 0 \text{ and } L = 1) = \frac{2.15^0 \times \text{e}^{-2.15}}{0!} \times \frac{1.9^1 \times \text{e}^{-1.9}}{1!}$ $\text{P}(D = 1 \text{ and } L = 0) = \frac{2.15^1 \times \text{e}^{-2.15}}{1!} \times \frac{1.9^0 \times \text{e}^{-1.9}}{0!}$	(M1)	M1 for use of formula once.
	$\text{P}(F < 2) = 0.017422 + 0.033102 + 0.037458$ $\text{P}(F < 2) = 0.08798$	(M1) (A1)	M1 for addition
(ii)	Valid justification in context. e.g. Fish are caught singly. Fish are caught independently. Catches occur at random. Dave and Llinos catch fish independently. Constant average rate of fish being caught.	E1 **[5]**	Must be in context E0 for constant rate

Qu.	Solution	Mark	Notes
1(b) (i)	Expected time until next fish $= \frac{1}{4.3} \times 8$	M1	oe
	1.86 hours or 1 hour 52 minutes		
	i.e. 12.52pm	A1	Accept 12.51pm A0 for 1.86 only
(ii)	P (Dave doesn't catch a fish for the rest of the day) $= \mathrm{e}^{-4.3\times0.5}$	M1	
	$= 0.1165$	A1	
	<u>Alternative solution</u> Using $\lambda = 2.15$ AND $\mathrm{P}(X = 0)$	(M1)	si
	$\mathrm{P}(X = 0) = 0.1165$	(A1)	
		[4]	
(c)	Let J be the number of trout she catches in a year. $J \sim \mathrm{B}(950,0.02)$	B1	si
	$\mathrm{P}(J \geq 30) = 1 - \mathrm{P}(J \leq 29)$ $= 0.01109$	M1 A1	
		[3]	
(d)	$\mathrm{Po}(19)$ Poisson since n is large and p is small.	B1 E1	Condone similar with values e.g. $n > 50, p < 0.1$ e.g. $np > 10$
		[2]	
	Total for Question 1	**14**	

Qu.	Solution	Mark	Notes
2(a)	Realising Q_3 is in the third part of the CDF.	B1	si
	$\frac{x^2 - x + 3}{5} = 0.75$	M1	Setting $F(x) = 0.75$ Allow $\frac{x+2}{5} = 0.75$ for M1 only
	$x^2 - x - 0.75 = 0$	A1	oe
	$x = -0.5$ or $x = 1.5$	A1	Both values.
	Reject $-0.5 \therefore x = 1.5$	B1	FT provided quadratic, with one answer in the range [1,2] and one outside this range.
		[5]	
(b)	$f(x) = F'(x)$	M1	M1 Attempt at differentiating with at least one power of x decreasing.
	$f(x) = \begin{cases} \frac{1}{5} & -2 \le x < 1 \\ \frac{2x-1}{5} & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$	A1 A1 B1	A1 Correct expression for $f(x)$ for $-2 \le x < 1$. A1 Correct expression for $f(x)$ for $1 \le x \le 2$. B1 for "0 otherwise" and correct ranges.
		[4]	
(c)(i)	$\mathrm{E}(X) = \int_{-2}^{1} \frac{x}{5} \mathrm{d}x + \int_{1}^{2} \frac{2x^2 - x}{5} \mathrm{d}x$	M1	M1 Attempt at integrating $xf(x)$ with at least one power of x increasing (ignore limits here)
	$\mathrm{E}(X) = \left[\frac{x^2}{10}\right]_{-2}^{1} + \left[\frac{2x^3}{15} - \frac{x^2}{10}\right]_{1}^{2}$	A1	A1 correct integration with correct limits. FT 'their $f(x)$' of equivalent difficulty
	$\mathrm{E}(X) = \frac{1}{3}$ (minutes)	A1	cao
(ii)	Valid interpretation e.g. 20 seconds longer than the target time **on average**.	E1	FT their $\mathrm{E}(X)$
		[4]	
	Total for Question 2	**13**	

Qu.	Solution	Mark	Notes
3(a)	H_0:The number of bags sold can be modelled by a Poisson distribution with mean 2.2. H_1:The number of bags sold cannot be modelled by a Poisson distribution with mean 2.2.	B1	
	No. bags sold: 0 | 1 | 2 | 3 | 4 | 5+ Exp Freq: 5.54 | 12.19 | 13.41 | 9.83 | 5.41 | 3.62	B1 B1	B1 for 2 values correct. B1 All correct.
	Combining classes No. bags sold: 0 | 1 | 2 | 3 | 4+ Exp Freq: 5.54 | 12.19 | 13.41 | 9.83 | 9.03	M1	M1 combining classes.
	Use of χ^2 test stat $=\sum\frac{(O-E)^2}{E}$.	M1	SC for solution that does not combine classes. (M0M1m1A0B1B1m1A0) Alternative M1m1
	$=\frac{(7-5.54)^2}{5.54}+\frac{(10-12.19)^2}{12.19}+\frac{(11-13.41)^2}{13.41}+\frac{(9-9.83)^2}{9.83}+\frac{(13-9.03)^2}{9.03}$	m1	$\frac{7^2}{5.54}+\frac{10^2}{12.19}+\frac{11^2}{13.41}+\frac{9^2}{9.83}+\frac{13^2}{9.03}-50$
	$=3.02\ldots$	A1	cao Accept 3.03 Accept 3.016…
	DF = 4	B1	
	10% CV = 7.779	B1	
	Since $3.02\ldots < 7.779$ there is insufficient evidence to Reject H_0.	m1	Dependent on 2nd M1 FT their TS and CV
	There is no evidence to suggest that the number of bags sold cannot be modelled by a Poisson (2.2) distribution.	A1 **[11]**	cso
3(b)	$\mathrm{Po}(2.5)$ is better since the χ^2 test statistic is smaller.	E1 **[1]**	FT their TS from (a)
	Total for Question 3	**12**	

Qu.	Solution	Mark	Notes
4 (a)	$r = \frac{-18895.13043}{\sqrt{11503.91304 \times 78669.30435}}$ $r = -0.628$ Valid interpretation. e.g. The negative value of 0.628 implies that on average the more people rely on cash, the less debt they have as a proportion of their household income. e.g. The correlation is fairly strong between household debt and the use of cash for transactions. e.g. As the percentage of cash transactions increases, net household debt as a percentage of disposable income tends to decrease. e.g. Household debt and percentage of cash transactions are negatively correlated.	M1 A1 E1 **[3]**	Condone +ve numerator 3sf
(b)	$b = \frac{S_{xy}}{S_{xx}}$ $b = -1.64$ $a = \frac{2695}{23} + 1.6424959\ldots \times \frac{1467}{23}$ $a = 221.9\ldots$ $y = 222 - 1.64x$	M1 A1 M1 A1 B1 **[5]**	 FT their b for M1A0 Correct to 3sf, ft their a and b
(c)	Selecting correct equation to use in each case. Malta $p = 222 - 1.64 \times 92$ $p = 71.12$ Denmark $q = -0.24 \times 261 + 91.92$ $q = 29.28$	 B1 B1 **[2]**	FT (b) for Malta Accept anything rounding to 71
(d)	Valid explanation e.g. Not reliable because it's extrapolation, rather than interpolation. e.g. May not be reliable because the values are very close to the extremes of the graph. e.g. May be reliable because there is fairly strong correlation and the values are only just outside the range of the graph.	E1	
(e)	Valid explanation. e.g. Net disposable income may be negative. e.g. Household incomes are less than outgoings leading to a negative value for net disposable income. e.g. Maybe an erroneous value.	E1 **[2]**	
	Total for Question 4	**12**	

Qu.	Solution	Mark	Notes
5(a)	$F = \frac{74 \times 64}{253}$ $= 18.72$ $G = \frac{(8 - 14.33)^2}{14.33}$ $= 2.80$ $H = 8.33 + 0.73 + G + 3.88 + 0.02$ $= 15.76$	 B1 M1 A1 A1 **[4]**	or by adding row and subtracting from 74 or adding column and subtracting from 64. FT their G
(b)	H_0: Attitude towards Welsh is independent of how Welsh was learned. H_1: Attitude towards Welsh is not independent of how Welsh was learned.	B1	both OR there is (not) association …
	$\chi^2 = 20.70 + 3.52 + H$ $\chi^2 = 39.98$	B1	FT their H, or using an appropriate contribution in the comparison with CV
(c)	DF = 8 5% CV = 15.507 Since $39.98 > 15.507$ there is sufficient evidence to reject H_0. There is evidence to suggest that how a student learns Welsh affects their attitude towards the language. Two different valid comments e.g. Students who learned Welsh from two parents are more likely to have a positive attitude towards the language. e.g. There is a real mix of attitudes towards the Welsh language amongst students who learned Welsh at school. e.g. There are far fewer than expected number of students that have a very negative attitude towards Welsh who learnt from two parents/carers.	B1 B1 M1 A1 E1×2 **[8]**	 cso Must comment on more than just size of contributions.
	Total for Question 5	**12**	

Qu.	Solution	Mark	Notes
6	Identifying 140, 200, 260, 320 as the values of Y.	B1	B1 allow one error.
			Table may use rv$X$.
	y | 140 | 200 | 260 | 320 $\mathrm{P}(Y=y)$ | 0.3 | $2p$ | p | $0.7-3p$	B1	B1 for attaching $2p$ and p to 200 and 260 (or 6 and 7)
		B1	B1 for $0.7-3p$.
	$\mathrm{E}(Y)=0.3\times140+2p\times200+p\times260+(0.7-3p)\times320$	M1	Using $\sum x\mathrm{P}(X=x)$ to form an expression in p (or another variable)
	$206=42+400p+260p+224-960p$	M1	Set =206 (or 6.1)
	$p=0.2$	A1	
	ALTERNATIVE SOLUTION SIMULTANEOUS EQUATIONS		
	y | 140 | 200 | 260 | 320 $\mathrm{P}(Y=y)$ | 0.3 | $2p$ | p | q	(B1)	B1 for attaching $2p$ and p to 200 and 260 (or 6 and 7) and labelling q.
	$0.3+3p+q=1$ $3p+q=0.7$	(B1)	Either
	$\mathrm{E}(Y)=0.3\times140+2p\times200+p\times260+q\times320$	(M1)	Using $\sum x\mathrm{P}(X=x)$ to form an expression in p and q
	$206=42+400p+260p+320q$	(M1)	Set =206 (or 6.1)
	Solve simultaneous equations $165p+80q=41$ and $3p+q=0.7$		
	$p=0.2$ or $q=0.1$	(A1)	A1 for either
	y | 140 | 200 | 260 | 320 $\mathrm{P}(Y=y)$ | 0.3 | 0.4 | 0.2 | 0.1	A1	Fully correct probability distribution, cao
		[7]	
	Total for Question 6	**7**	

GCE AS MARKING SCHEME

SUMMER 2024

AS
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1

About this marking scheme

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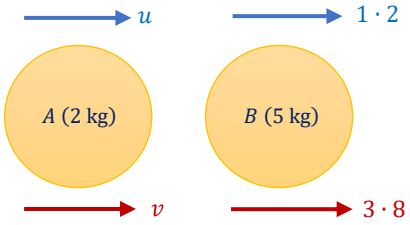
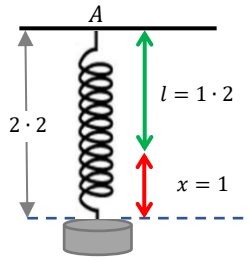
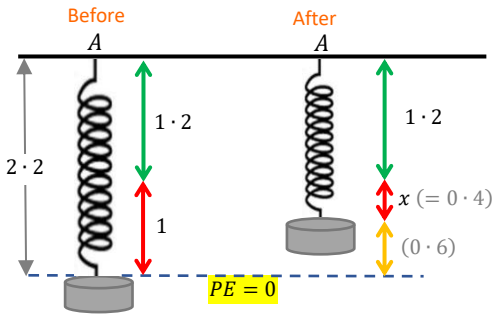
WJEC GCE AS FURTHER MATHEMATICS

UNIT 3 FURTHER MECHANICS A

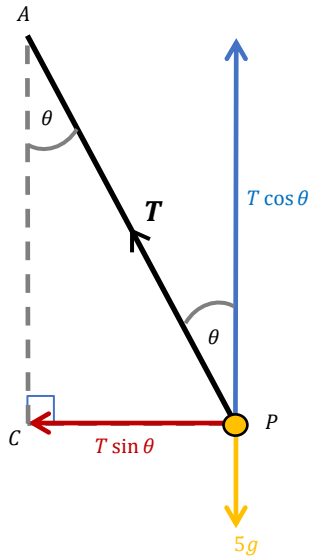
SUMMER 2024 MARK SCHEME

Q1	Solution	Mark	Notes
			$e = 0 \cdot 3$
(a)	(i) Impulse, I = Change in momentum Impulse $= (5)(3 \cdot 8 - 1 \cdot 2)$ Impulse $= 13 \quad (\text{Ns})$	M1 A1	Used cao
	(ii) $I = Ft$ $13 = F \times 0 \cdot 08$ $F = 162 \cdot 5 \quad (\text{N})$	M1 A1 **[4]**	Used to find a force FT Impulse from (i)
(b)	Conservation of momentum $(2)(u) + (5)(1 \cdot 2) = (2)(v) + (5)(3 \cdot 8)$ $2u + 6 = 2v + 19$ $u - v = 6 \cdot 5$ Restitution $3 \cdot 8 - v = -0 \cdot 3(1 \cdot 2 - u)$ $0 \cdot 3u + v = 4 \cdot 16$ $1 \cdot 3u = 10 \cdot 66$ or $1 \cdot 3v = 2 \cdot 21$ $u = 8 \cdot 2 \ (\text{ms}^{-1})$ $v = 1 \cdot 7 \ (\text{ms}^{-1})$	M1 A1 M1 A1 m1 A1 A1 **[7]**	Attempted All correct Attempted All correct, oe Eliminating one variable

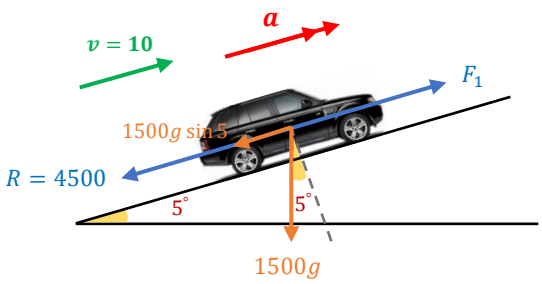
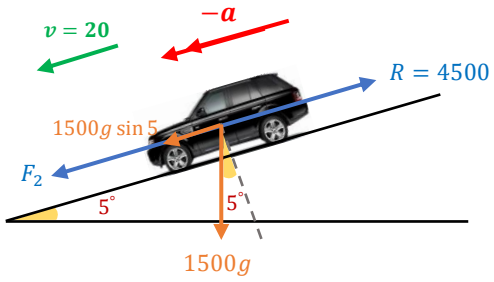
Q1	Solution	Mark	Notes
(c)	(i) $e = 0$	B1	
	(ii) Conservation of momentum	M1	Attempted for **combined** mass
	$(5)(3 \cdot 8) + (4)(0) = (5 + 4)(v)$		Zero term not required
	$19 = 9v$		
	$v = 2 \cdot 11 \ldots \quad (\text{ms}^{-1})$	A1	Any form, $v = \frac{19}{9} = 2 \cdot \dot{1}$
		[3]	
	Total for Question 1	**14**	

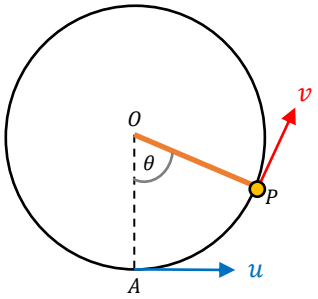
Q2	Solution	Mark	Notes
(a)			
	Using expression for $\text{EE}=\frac{\lambda x^2}{2l}$ $\text{EE}=\frac{84(1)^2}{2(1\cdot2)}$	M1	Used with $\lambda=84, l=1\cdot2$ $x=2\cdot2-1\cdot2=1$
	$\text{EE}=35$ (J)	A1 **[2]**	Convincing
(b)			$m=4$
	Using expression for $\text{EE}=\frac{\lambda x^2}{2l}$	M1	Used with $\lambda=84, l=1\cdot2$ and $5\cdot6$
	$5\cdot6=\frac{84x^2}{2(1\cdot2)}$ $\quad(5\cdot6=35x^2)$ $x=\pm0\cdot4$ $\quad\therefore\quad$ $x=0\cdot4$	A1	Negative case discarded (if considered, not needed)
	Using expression for PE $(\text{PE}=\pm4gh)$	M1	Any h
	$\text{PE}=\pm\begin{cases}4g(2\cdot2-1\cdot2-0\cdot4)\\4g(0\cdot6)=2\cdot4g=23\cdot52\end{cases}$	A1	Any correct form FT derived h
	$\text{KE}=\pm\frac{1}{2}(4)v^2$ $\quad(=2v^2)$	B1	
	Conservation of energy (all energy forms)	M1	KE, EE (35 **and** $5\cdot6$), PE
	$35=5\cdot6+23\cdot52+2v^2$ $v^2=\frac{147}{50}=2\cdot94$	A1	oe
	$v=\frac{7\sqrt{6}}{10}=1\cdot7146\ldots(\text{ms}^{-1})$	A1 **[8]**	
	Total for Question 2	**10**	

Q3	Solution	Mark	Notes
(a)	$\mathbf{R} = (4\mathbf{i} - 7\mathbf{j} + 9\mathbf{k}) + (5\mathbf{i} + 3\mathbf{j} - 8\mathbf{k})$ $+(-2\mathbf{i} + 6\mathbf{j} - 11\mathbf{k})$ $\mathbf{R} = 7\mathbf{i} + 2\mathbf{j} - 10\mathbf{k}$ (N)	B1 **[1]**	
(b)	Displacement vector $\mathbf{r} = (a\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}) - (3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k})$ $= (a - 3)\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ Work done by $\mathbf{R} = \mathbf{R} \,.\, \mathbf{r}$ $21 = (7\mathbf{i} + 2\mathbf{j} - 10\mathbf{k}).\left((a - 3)\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}\right)$ $21 = (7)(a - 3) + (2)(3) + (-10)(2)$ $21 = 7a - 21 + 6 - 20$ $a = 8$	M1 M1 m1 A1 **[4]**	Difference attempted
	Total for Question 3	**5**	

Q4	Solution	Mark	Notes
			$\tan\theta = \frac{20}{21}$ $\sin\theta = \frac{20}{29}$ $\cos\theta = \frac{21}{29}$
	Resolve vertically $T\cos\theta = 5g$	M1 A1	
	$\left(T = \frac{29}{21}\times 5g = \frac{203}{3} = \frac{145}{21}g = 67\cdot 66\ldots\right)$		
	N2L towards centre $T\sin\theta = 5a$	M1	
	$T\sin\theta = 5\frac{v^2}{r}$	m1	Using $a = \frac{v^2}{r}$
	$r = 3\sin\theta \qquad \left(r = 3\times\frac{20}{29} = \frac{60}{29} = 2\cdot 06896\ldots\right)$	B1	
	$\frac{203}{3}\times\frac{20}{29} = 5\frac{v^2}{\frac{60}{29}} \qquad \left(\frac{145}{21}g\times\frac{20}{29} = 5\frac{v^2}{\frac{60}{29}}\right)$	m1	Eliminating T
	$v^2 = \frac{560}{29} = 19\cdot 3103\ldots \qquad \left(v^2 = \frac{400}{203}g\right)$		
	$v = 4\cdot 394\ldots \quad (\text{ms}^{-1})$	A1	
		[7]	
	Total for Question 4	**7**	

Q5	Solution	Mark	Notes
(a)	$\mathbf{a} = \frac{\mathbf{F}}{2} = 2t\mathbf{i} - \frac{1}{2}t^{\frac{1}{2}}\mathbf{j} + 3\mathbf{k}$	B1	
	$\mathbf{v} = \int \mathbf{a}\, dt$	M1	Attempt to integrate $\mathbf{a}$
	$\mathbf{v} = \frac{1}{2} \times 2t^2\mathbf{i} - \frac{1}{\frac{3}{2}} \times \frac{1}{2}t^{\frac{3}{2}}\mathbf{j} + 3t\mathbf{k} \quad (+\mathbf{c})$ $\left(\mathbf{v} = t^2\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + 3t\mathbf{k} \quad (+\mathbf{c})\right)$	A1	Correct integration
	At $t = 1$, $\mathbf{v} = 3\mathbf{i} - \frac{1}{3}\mathbf{j} - \mathbf{k}$ $3\mathbf{i} - \frac{1}{3}\mathbf{j} - \mathbf{k} = \mathbf{i} - \frac{1}{3}\mathbf{j} + 3\mathbf{k} + \mathbf{c}$ $\mathbf{c} = 2\mathbf{i} - 4\mathbf{k}$	m1	Used
	$\mathbf{v} = (t^2 + 2)\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + (3t - 4)\mathbf{k} \quad (\text{ms}^{-1})$	A1 **[5]**	
(b)	$\mathbf{v}.(-1\mathbf{i} + 3\mathbf{k}) = 0$ $\left((t^2 + 2)\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + (3t - 4)\mathbf{k}\right).(-1\mathbf{i} + 3\mathbf{k}) = 0$	M1	
	$(t^2 + 2)(-1) + (3t - 4)(3) = 0$	m1	FT $\mathbf{v}$ from (a)
	$t^2 - 9t + 14 = 0$		
	Solving quadratic $t = 2, 7$	m1 A1	Both values
		[4]	
	Total for Question 5	**9**	

Q6	Solution	Mark	Notes
	Moving **up** the slope  		Moving **down** the slope 
	$F = \frac{P}{v}$	B1	Used, si
	N2L **up** the slope	M1	All forces, dim. correct equation, F_1 and R opposing
	$F_1 - R - mg\sin\theta = ma$	A1	Correct equation
	$\frac{P}{10} - 4500 - 1500g\sin 5^\circ = 1500a$ $\frac{P}{10} - 4500 - 1281 \cdot 189 \ldots = 1500a$ $\frac{P}{10} - 5781 \cdot 189 \ldots = 1500a$	A1	Correct equation in P and a
	N2L **down** the slope	M1	All forces, dim. correct equation, F_2 and R opposing
	$F_2 - R + mg\sin\theta = -ma$	A1	Correct equation
	$\frac{P}{20} - 4500 + 1500g\sin 5^\circ = -1500a$ $\frac{P}{20} - 4500 + 1281 \cdot 189 \ldots = -1500a$ $\frac{P}{20} - 3218 \cdot 81 \ldots = -1500a$	A1	Correct equation in P and a
	Eliminating a $\frac{3P}{20} - 9000 = 0$	m1	Eliminating one variable
	$P = 60\,000$	A1	
	$a = 0 \cdot 145(873 \ldots)$	A1	
		[10]	
	Total for Question 6	**10**	

Q7	Solution	Mark	Notes
(a)			
	Conservation of energy	M1	KE and PE in dim. correct equation
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\left(\frac{5}{7}\right)(1-\cos\theta)$ $v^2 = u^2 - 14(1-\cos\theta)$	A1 A1	KE $\pm\frac{1}{2}mu^2$ OR $\pm\frac{1}{2}mv^2$ PE $\pm mgl(1-\cos\theta)$
	$v^2 = u^2 - 14 + 14\cos\theta$	A1 **[4]**	Convincing
(b)	Expression in (a) used with $v = 0$ when $\theta = 180^0$ (or $\cos\theta = -1$)	M1	
	least value of $u^2 = 28$	A1 **[2]**	
(c)	(i) N2L towards centre	M1	Dimensionally correct equation, T and $mg\cos\theta$ opposing
	$T - mg\cos\theta = \frac{mv^2}{r}$	A1	
	$T = \frac{7m}{5}(32\cdot 2 - 14 + 14\cos\theta) + 9\cdot 8m\cos\theta$	m1	Sub. for v^2 with or without $u^2 = 32\cdot 2$
	$T = \begin{cases} m(25\cdot 48 + 29\cdot 4\cos\theta) \\ mg(2\cdot 6 + 3\cos\theta) \end{cases}$	A1	oe
	(ii) Rod exerts a thrust when $T < 0$ $m(25\cdot 48 + 29\cdot 4\cos\theta) < 0$	M1	Used
	$\cos\theta < -\frac{25\cdot 48}{29\cdot 4}$ $\left(= -\frac{13}{15} = -0\cdot 866\ldots\right)$	A1	
	Thrust exerted in the range $150\cdot 07^0 < \theta < 209\cdot 93^0$	A1 A1 **[8]**	A1 for sight of either $150.073\ldots$ or $209.926\ldots$ Statement (mathematical or otherwise) to the effect that interval is between the given boundaries.
(d)	**No.** Answer to (c) would remain the same as $\cos\theta$ is independent of m.	E1 **[1]**	
	Total for Question 7	**15**	

GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
FURTHER MATHEMATICS
UNIT 4 FURTHER PURE MATHEMATICS B
1305U40-1

About this marking scheme

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WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 4 FURTHER PURE MATHEMATICS B

SUMMER 2024 MARK SCHEME

Qu	Solution	Mark	Notes
1. a)	$r = \sqrt{5^2 + 10^2} = \sqrt{125} = 5\sqrt{5}$	B1	
	$\arg(5 + 10\mathrm{i}) = 1.11$	B1	
	$\sqrt[3]{5 + 10\mathrm{i}} = \sqrt[3]{\sqrt{125}}\mathrm{e}^{\frac{1.11\mathrm{i}}{3}\left(+\frac{2n\pi\mathrm{i}}{3}\right)}$	M1	
	$z_1 = \sqrt{5}\mathrm{e}^{0.37\mathrm{i}}$	A1	One root
	$z_2 = \sqrt{5}\mathrm{e}^{2.46\mathrm{i}}$	m1	$+\frac{2n\pi\mathrm{i}}{3}$
	$z_3 = \sqrt{5}\mathrm{e}^{4.56\mathrm{i}}$	A1	All roots Use of degrees gains B1, B1, M1, A0, M1, A0 at most
		[6]	Penalise -1 for use of $r(\cos\theta + \mathrm{i}\sin\theta)$ or $x + \mathrm{i}y$, unless written as $r\mathrm{e}^{\mathrm{i}\theta}$
b)	METHOD 1: Converting to coordinates: $z_1 = (2.086, 0.807)$ $z_2 = (-1.741, 1.404)$ $z_3 = (-0.344, -2.209)$	B1	FT (a) Any two, si
	Distance between z_1z_2: $\sqrt{(-1.741 - 2.086)^2 + (1.404 - 0.807)^2}$ $= 3.87(3285\ldots)$	M1 A1	Or z_1z_3, z_2z_3 Exact distance is $\sqrt{15}$
	Area = $\frac{1}{2} \times 3.87\ldots \times 3.87\ldots \times \sin\frac{\pi}{3}$	M1	Use of $\frac{1}{2}ab\sin C$, including degrees
	$= 6.5$	A1	cao Must be to 2 sf
	METHOD 2: Splitting triangle into 3 smaller isosceles triangles Angle at centre where 3 isosceles triangles meet = $\frac{2\pi}{3}$	(B1)	si
	Area of 1 isosceles triangle = $\frac{1}{2} \times 125^{\frac{1}{6}} \times 125^{\frac{1}{6}} \times \sin\frac{2\pi}{3}$	(M1) (A1)	Fully correct
	Area of full triangle $= 3 \times \frac{1}{2} \times 125^{\frac{1}{6}} \times 125^{\frac{1}{6}} \times \sin\frac{2\pi}{3}$ $= 6.5$	(B1) (A1)	Area × 3 cao Must be to 2 sf
		[5]	
		Total [11]	

Qu	Solution	Mark	Notes
2. a)	METHOD 1: Use of Maclaurin series, $\cosh x$ with $\left(\frac{x}{2}\right)$		
	$\cosh\left(\frac{x}{2}\right) = 1 + \frac{\left(\frac{x}{2}\right)^2}{2!} + \frac{\left(\frac{x}{2}\right)^4}{4!}$	M1	
	$\cosh\left(\frac{x}{2}\right) = 1 + \frac{x^2}{8} + \frac{x^4}{384}$	A1	
	METHOD 2: Use of Maclaurin series $f(x)$ with $\left(\frac{x}{2}\right)$		
	$f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0) + \frac{x^3}{6}f'''(0) + \frac{x^4}{24}f''''(0)$		
	$f(x) = 1 + (x \times 0) + \left(\frac{x^2}{2} \times \frac{1}{4}\right) + \left(\frac{x^3}{6} \times 0\right) + \left(\frac{x^4}{24} \times \frac{1}{16}\right)$	(M1)	
	$f(x) = 1 + \frac{x^2}{8} + \frac{x^4}{384}$	(A1)	
		[2]	
b)	$1 + \frac{x^2}{8} + \frac{x^4}{384} = x^2 - 2$	M1	FT (a)
	$\frac{x^4}{384} - \frac{7}{8}x^2 + 3 = 0$ $x^4 - 336x^2 + 1152 = 0$	A1	
	Solving e.g. if $y = x^2$, $y^2 - 336y + 1152 = 0$	m1	Solving quartic m0 no working
	$y = 332.5 \ldots$ or $y = 3.46 \ldots$	A1	cao
	Therefore, $x = (\pm)18.2 \ldots$ or $x = (\pm)1.86 \ldots$		
	Since $0 < x < 4$, at the point A, $x = 1.86 \ldots$	B1	cao Unsupported answer 0 marks
		[5]	
c)	Intersection of $g(x)$ and x-axis at $x = \sqrt{2}$	B1	si
	METHOD 1 (Integration of cosh): Area $= \int_0^{1.86} \cosh\left(\frac{x}{2}\right) \mathrm{d}x - \int_{\sqrt{2}}^{1.86} (x^2 - 2) \mathrm{d}x$	M1	FT (b), limits not required
	$= \left[2\sinh\left(\frac{x}{2}\right)\right]_0^{1.86} - \left[\frac{x^3}{3} - 2x\right]_{\sqrt{2}}^{1.86}$	A2	A1 each integration
	$= 2.139 \ldots \ldots - 0.310 \ldots \ldots$	m1	correct limits used
	$= 1.829$	A1	cao
	METHOD 2 (Use of Maclaurin expansion): Area $= \int_0^{1.86} \left(1 + \frac{x^2}{8} + \frac{x^4}{384}\right) \mathrm{d}x - \int_{\sqrt{2}}^{1.86} (x^2 - 2) \mathrm{d}x$	(M1)	FT (a) & (b), limits not required
	$= \left[x + \frac{x^3}{8 \times 3} + \frac{x^5}{384 \times 5}\right]_0^{1.86} - \left[\frac{x^3}{3} - 2x\right]_{\sqrt{2}}^{1.86}$	(A2)	A1 each integration
	$= 2.139 \ldots \ldots - 0.310 \ldots \ldots$	(m1)	correct limits used
	$= 1.829$	(A1)	cao
		[6]	
		Total [13]	

Qu	Solution	Mark	Notes
3.	Dividing both sides by $\cos x$: $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y \sin x}{\cos x} = 4\cos^2 x \sin x + \frac{5}{\cos x}$	M1	
	Integrating factor: $\mathrm{e}^{\int \frac{\sin x}{\cos x}\mathrm{d}x}$ $= \mathrm{e}^{\int \tan x \mathrm{d}x} = \mathrm{e}^{\ln \sec x} = \sec x$	M1 A1	oe
	THEN, METHOD 1: Multiplying both sides: $\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \frac{\sin x}{\cos x} \sec x = 4 \sin x \cos x + \frac{5}{\cos x} \sec x$	M1	
	$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \tan x \sec x = 2 \sin 2x + 5 \sec^2 x$	A1	Form to integrate
	Integrating $y \sec x = -\cos 2x + 5 \tan x \ (+c)$	m1 A1	Both sides of equation
	Substituting $y = 3\sqrt{2}$ when $x = \frac{\pi}{4}$: $3\sqrt{2} \times \sqrt{2} = 0 + 5 \times 1 + c \rightarrow \ c = 1$	m1	
	Solution: $y \sec x = -\cos 2x + 5 \tan x + 1$ $y = -\cos 2x \cos x + 5 \sin x + \cos x$	A1	cao Must be $y = \cdots$
	OR, METHOD 2: Multiplying both sides: $\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \frac{\sin x}{\cos x} \sec x = 4 \sin x \cos x + \frac{5}{\cos x} \sec x$	(M1)	Form to integrate
	$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \tan x \sec x = 4 \sin x \cos x + 5 \sec^2 x$	(A1)	
	Integrating: $y \sec x = 2 \sin^2 x + 5 \tan x \ (+c)$	(m1) (A1)	Both sides of equation
	Substituting $y = 3\sqrt{2}$ when $x = \frac{\pi}{4}$: $3\sqrt{2} \times \sqrt{2} = 2 \times \frac{1}{2} + 5 \times 1 + c \rightarrow \ c = 0$	(m1)	
	Solution: $y \sec x = 2 \sin^2 x + 5 \tan x$ $y = 2 \sin^2 x \cos x + 5 \sin x$	(A1)	cao Must be $y = \cdots$

Qu	Solution	Mark	Notes
3.	OR, METHOD 3: Multiplying both sides: $\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \frac{\sin x}{\cos x} \sec x = 4 \sin x \cos x + \frac{5}{\cos x} \sec x$ $\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \tan x \sec x = 4 \sin x \cos x + 5 \sec^2 x$	(M1) (A1)	Form to integrate
	Integrating: $y \sec x = -2 \cos^2 x + 5 \tan x \, (+c)$	(m1) (A1)	Both sides of equations
	Substituting $y = 3\sqrt{2}$ when $x = \frac{\pi}{4}$: $3\sqrt{2} \times \sqrt{2} = -2 \times \frac{1}{2} + 5 \times 1 + c \rightarrow \ c = 2$	(m1)	
	Solution: $y \sec x = -2 \cos^2 x + 5 \tan x + 2$ $y = -2 \cos^3 x + 5 \sin x + 2 \cos x$	(A1)	cao Must be $y = \cdots$
		[9] **Total [9]**	

Qu	Solution	Mark	Notes
4. a)	$\left(z+\frac{1}{z}\right)^4 = z^4+4z^2+6+4z^{-2}+z^{-4}$	M1 A1	3 correct terms unsimplified
	$= (z^4+z^{-4})+(4z^2+4z^{-2})+6$	m1	si
	$= 2\cos 4\theta + 8\cos 2\theta + 6$	A1	
	$(2\cos\theta)^4 = 2\cos 4\theta + 8\cos 2\theta + 6$ $\therefore 16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$	A1	cao
		[5]	
b)	$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (3-4\cos^2\theta)^2 \,\mathrm{d}\theta$	M1	Condone omission of ½ until final A1
	$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (9-24\cos^2\theta+16\cos^4\theta)\,\mathrm{d}\theta$	A1	
	$24\cos^2\theta = 24\times\frac{1}{2}(1+\cos 2\theta)$	B1	si
	$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (9-12(1+\cos 2\theta)+2\cos 4\theta+8\cos 2\theta+6)\,\mathrm{d}\theta$	A1	FT for A1A1A1 (a) if in form $a\cos 4\theta + b\cos 2\theta + c$
	$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (3-4\cos 2\theta+2\cos 4\theta)\,\mathrm{d}\theta$		
	$= \frac{1}{2}\left[3\theta-2\sin 2\theta+\frac{2}{4}\sin 4\theta\right]_{\pi/6}^{5\pi/6}$	A2	A1 for 2 terms correct
	$= \frac{1}{2}\left(\frac{15\pi}{6}+\frac{2\sqrt{3}}{2}-\frac{\sqrt{3}}{4}\right)-\frac{1}{2}\left(\frac{3\pi}{6}-\frac{2\sqrt{3}}{2}+\frac{\sqrt{3}}{4}\right)$	m1	Use of limits
	$= \pi+\frac{3\sqrt{3}}{4}$ or 4.44…..	A1	cao
		[8]	

Qu	Solution	Mark	Notes
4. c)	$x = r\cos\theta = 3\cos\theta - 4\cos^3\theta$	M1	
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3\sin\theta + 12\cos^2\theta\sin\theta$	A2	-1 each error
	When perpendicular to initial line: $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3\sin\theta + 12\cos^2\theta\sin\theta = 0$	m1	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0$
	$-3\sin\theta(1 - 4\cos^2\theta) = 0$ Therefore, $-3\sin\theta = 0$ or $1 - 4\cos^2\theta = 0$ $\cos^2\theta = \frac{1}{4}$	m1	
	$\sin\theta = 0$ $\cos\theta = \pm\frac{1}{2}$	A1	
	When $\sin\theta = 0$ the curve does not exist.		
	When $\cos\theta = \pm\frac{1}{2}$, $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$	A1	
	$r = 2$ $\left(2, \frac{\pi}{3}\right), \left(2, \frac{2\pi}{3}\right)$	A1	
		[8]	
		Total [21]	

Qu	Solution	Mark	Notes
5. a)	$\frac{3-x}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$	M1	Use of
	$3-x = A(x^2+1) + x(Bx+C)$	A1	
	When $x = 0, 3 = A$	A1	
	Substituting values e.g. When $x = 1, 2 = 2A + B + C \quad \therefore B + C = -4$ When $x = -1, 4 = 2A + B - C \quad \therefore B - C = -2$ Solving, $B = -3$ and $C = -1$	m1 A1	Or comparing coefficients
	$\int \frac{3-x}{x(x^2+1)}\,\mathrm{d}x = \int \left(\frac{3}{x} + \frac{-3x-1}{x^2+1}\right)\mathrm{d}x$		
	$= \int \left(\frac{3}{x} - \frac{3x}{x^2+1} - \frac{1}{x^2+1}\right)\mathrm{d}x$	M1	FT A, B, C if ≠ 0
	$= 3\ln\lvert x\rvert - \frac{3}{2}\ln\lvert x^2+1\rvert - \tan^{-1}x + c$	A2	A1 for two parts
		[8]	
b)	METHOD 1:		
	$\frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}} = \frac{2\sinh x \cosh x}{\cosh x\sqrt{\cosh^2 x - 9}}$	M1	Rewrite $\sinh 2x$ and take out $\sqrt{\cosh^2 x}$
	$= \frac{2\sinh x}{\sqrt{\cosh^2 x - 9}}$	A1	
	Let $u = \cosh x$ $\frac{\mathrm{d}u}{\mathrm{d}x} = \sinh x$ $\int \frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}}\,\mathrm{d}x = \int \frac{2\sinh x}{\sqrt{\cosh^2 x - 9}}\,\mathrm{d}x$	M1	Or equivalent appropriate sub.
	$= \int \frac{2}{\sqrt{u^2-9}}\,\mathrm{d}u$	A1	Form to integrate
	$= 2\cosh^{-1}\left(\frac{u}{3}\right) + c$ OR $2\left(\ln\left\lvert u + \sqrt{u^2-9}\right\rvert\right) + c$	A1	
	$= 2\cosh^{-1}\left(\frac{\cosh x}{3}\right) + c$ OR $2\left(\ln\left\lvert \cosh x + \sqrt{\cosh^2 x - 9}\right\rvert\right) + c$	A1	Mark final answer
		[6]	

Qu	Solution	Mark	Notes
5. b)	METHOD 2:		
	Let $u = \cosh^2 x$	(M1)	
	$\frac{du}{dx} = 2\cosh x \sinh x = \sinh 2x$ $\int \frac{\sinh 2x}{\sqrt{\cosh^4 x - 9\cosh^2 x}}\,dx = \int \frac{1}{\sqrt{u^2 - 9u}}\,du$	(A1)	
	$= \int \frac{1}{\sqrt{\left(u - \frac{9}{2}\right)^2 - \frac{81}{4}}}\,du$	(M1) (A1)	Attempt to complete the square
	$= \cosh^{-1}\left(\frac{u-\frac{9}{2}}{\frac{9}{2}}\right) + c$ OR $\ln\left\lvert\left(u - \frac{9}{2}\right) + \sqrt{\left(u - \frac{9}{2}\right)^2 - \frac{81}{4}}\right\rvert + c$	(A1)	
	$= \cosh^{-1}\left(\frac{\cosh^2 x - \frac{9}{2}}{\frac{9}{2}}\right) + c$ OR $\ln\left\lvert\left(\cosh^2 x - \frac{9}{2}\right) + \sqrt{\left(\cosh^2 x - \frac{9}{2}\right)^2 - \frac{81}{4}}\right\rvert + c$	(A1)	Mark final answer
		[6] **Total [14]**	
	Across parts (a) and (b) – penalise -1 once only for no constant term		

Qu	Solution	Mark	Notes
6. a)	The 3 **planes** will intersect at a single **point**.	B2 [2]	B1 Because det $\mathbf{M} \neq 0$, there is a unique solution.
b)	METHOD 1 (Use of inverse matrix): If $\mathbf{MX} = \mathbf{N}$, then $\mathbf{X} = \mathbf{M}^{-1}\mathbf{N}$.		
	Cofactor matrix: $\begin{pmatrix} -600 & 92 & 425 \\ -80 & 40 & -30 \\ 400 & -96 & -240 \end{pmatrix}$	B1	May be implied by later work At least 5 correct entries
	Inverse matrix $\mathbf{M}^{-1} = \frac{1}{-1040}\begin{pmatrix} -600 & -80 & 400 \\ 92 & 40 & -96 \\ 425 & -30 & -240 \end{pmatrix}$	M1 A1	
	Therefore, $\mathbf{X} = \mathbf{M}^{-1}\mathbf{N}$ $= \frac{1}{-1040}\begin{pmatrix} -600 & -80 & 400 \\ 92 & 40 & -96 \\ 425 & -30 & -240 \end{pmatrix}\begin{pmatrix} 2668 \\ 3402 \\ 4581 \end{pmatrix}$ $\mathbf{X} = \begin{pmatrix} 39 \\ 56 \\ 65 \end{pmatrix}$	m1	
	Single £39, Double £56, Family £65	A2	A1 for two solutions Unsupported answer 0 marks
	METHOD 2 (Simultaneous equations/row operations): Let £x, £y, £z be the price of single, double and family rooms, respectively $12x + 30y + 8z = 2668$ $18x + 25y + 20z = 3402$ $19x + 50y + 16z = 4581$	(B1)	Forming 3 correct equations
	Multiplying each equation to get same coefficient of 1 variable $120x + 300y + 80z = 26680$ e.g. $72x + 100y + 80z = 13608$ $95x + 250y + 80z = 22905$		
	Eliminating 1 variable to arrive at 2 simultaneous equations e.g. $48x + 200y = 13072$ $25x + 50y = 3775$	(M1) (A1)	May be for y or z variables
	Solving pair of simultaneous equations $x = 39, y = 56, z = 65$	(m1)	
	Single £39, Double £56, Family £65	(A2)	A1 for two solutions Unsupported answer 0 marks

Qu	Solution	Mark	Notes
6. b)	METHOD 3 (Row reduction to echelon form): Let £x, £y, £z be the price of single, double and family rooms, respectively $12x + 30y + 8z = 2668$ R1 $18x + 25y + 20z = 3402$ R2 $19x + 50y + 16z = 4581$ R3	(B1)	Forming 3 correct equations
	Row operations on one row to eliminate variable e.g. $2 \times \text{R2} - 3 \times \text{R1}$ to give $12x + 30y + 8z = 2668$ $-40y + 16z = -1200$ $19x + 50y + 16z = 4581$	(M1)	May be on any row
	Row operations on second row to eliminate two variables	(m1)	May be on any row
	Correct echelon form e.g. $12x + 30y + 8z = 2668$ $-40y + 16z = -1200$ $208z = 13520$	(A1)	Must have 0 in 1 row and 0 0 in another row
	Value of 1 variable correct i.e. one of $x = 39, y = 56, z = 65$	(A1)	
	Substituting into remaining rows		
	Remaining two values $x = 39, y = 56, z = 65$		
	Single £39, Double £56, Family £65	(A1)	All correct Unsupported answer 0 marks
		[6] **Total [8]**	

Qu	Solution	Mark	Notes
7. a) i)	$16-6x-x^2=25-(x+3)^2$	B1	
	Mean value $=\frac{1}{1--3}\int_{-3}^{1}\frac{1}{\sqrt{16-6x-x^2}}\,\mathrm{d}x$	M1	
	$=\dfrac{1}{1--3}\displaystyle\int_{-3}^{1}\frac{1}{\sqrt{25-(x+3)^2}}\,\mathrm{d}x$		
	$=\frac{1}{4}\left[\sin^{-1}\left(\frac{x+3}{5}\right)\right]_{-3}^{1}$	B1	For $\sin^{-1}\left(\frac{x+3}{5}\right)$
	$=\frac{1}{4}\left(\sin^{-1}\frac{4}{5}\right)-\frac{1}{4}(\sin^{-1}0)$		
	$=0.232$	A1	If M0, award SC1 for 0.232 unsupported
		[4]	
ii)	Volume $=\pi\int_{-3}^{1}\left(\frac{1}{\sqrt{16-6x-x^2}}\right)^2\mathrm{d}x$	M1	FT completed square from (i) if possible Award B1 from (i) here if not awarded previously
	$=\pi\displaystyle\int_{-3}^{1}\frac{1}{25-(x+3)^2}\,\mathrm{d}x$	A1	
	$=\pi\left[\frac{1}{2\times5}\ln\left\lvert\frac{5+(x+3)}{5-(x+3)}\right\rvert\right]_{-3}^{1}$ OR $=\pi\left[\frac{1}{5}\tanh^{-1}\frac{x+3}{5}\right]_{-3}^{1}$	A1	
	$=\pi\left[\frac{1}{10}\ln\left\lvert\frac{9}{1}\right\rvert-\frac{1}{10}\ln\left\lvert\frac{5}{5}\right\rvert\right]$ OR $=\pi\left[\frac{1}{5}\tanh^{-1}\frac{4}{5}-\frac{1}{5}\tanh^{-1}0\right]$	m1	Use of limits
	$=\frac{\pi}{10}\ln 9$ or $0.69(0278459\ldots)$	A1	If M0, award SC1 for 0.69 unsupported
		[5]	
b)	$\lim_{a\to\infty}\displaystyle\int_{1}^{a}\frac{-8\mathrm{e}^{-2x}}{4\mathrm{e}^{-2x}-5}\,\mathrm{d}x$	M1	
	$=\lim_{a\to\infty}[\ln\lvert4\mathrm{e}^{-2x}-5\rvert]_{1}^{a}$	A1	
	$=\lim_{a\to\infty}[\ln\lvert4\mathrm{e}^{-2a}-5\rvert-\ln\lvert4\mathrm{e}^{-2}-5\rvert]$		
	$=[\ln\lvert-5\rvert-\ln\lvert4\mathrm{e}^{-2}-5\rvert]$		
	$=0.115$	A1	No workings, 0 marks
		[3]	
		Total [12]	

Qu	Solution	Mark	Notes
8. a)	METHOD 1:		
	$4x + 3 = \sinh y$		
	$4\frac{\mathrm{d}x}{\mathrm{d}y} = \cosh y$	M1	oe
	$4\frac{\mathrm{d}x}{\mathrm{d}y} = \pm\sqrt{1 + \sinh^2 y}$	m1	Use of identity
	$4\frac{\mathrm{d}x}{\mathrm{d}y} = \pm\sqrt{1 + (4x+3)^2}$	m1	Sub for $\sinh y$
	$4\frac{\mathrm{d}x}{\mathrm{d}y} = \pm\sqrt{16x^2 + 24x + 10}$	A1	oe
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\pm\sqrt{16x^2 + 24x + 10}}{4}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{16x^2 + 24x + 10}}$ AND Justification e.g. graph of $\sinh^{-1} x$ e.g. derivative of $\sinh x$ is $\cosh x$ which is always positive.	A1	convincing, with explanation over the choice of +
	METHOD 2:		
	$\sinh y = 4x + 3$		
	$\cosh y \frac{\mathrm{d}y}{\mathrm{d}x} = 4$	(M1)	oe
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm\frac{4}{\sqrt{1 + \sinh^2 y}}$	(m1)	Use of identity
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm\frac{4}{\sqrt{1 + (4x+3)^2}}$	(m1)	Sub for $\sinh y$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm\frac{4}{\sqrt{16x^2 + 24x + 10}}$	(A1)	oe
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{16x^2 + 24x + 10}}$ AND Justification e.g. graph of $\sinh^{-1} x$ e.g. derivative of $\sinh x$ is $\cosh x$ which is always positive.	(A1)	convincing, with explanation over the choice of +
		[5]	

Qu	Solution	Mark	Notes
8. b)	METHOD 1:		
	$y = \dfrac{\sinh 2x}{\mathrm{e}^{-3x}} = \dfrac{\mathrm{e}^{2x} - \mathrm{e}^{-2x}}{2\mathrm{e}^{-3x}}$	M1	Rewrite and substitute $\sinh 2x$
	$y = \dfrac{\mathrm{e}^{5x}}{2} - \dfrac{\mathrm{e}^{x}}{2}$	A1	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{5\mathrm{e}^{5x}}{2} - \dfrac{\mathrm{e}^{x}}{2}$	A1	
	When stationary, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	m1	
	$\dfrac{5\mathrm{e}^{5x}}{2} - \dfrac{\mathrm{e}^{x}}{2} = 0$		
	$\dfrac{\mathrm{e}^{x}}{2}(5\mathrm{e}^{4x} - 1) = 0$		
	$\frac{\mathrm{e}^{x}}{2} = 0$ or $5\mathrm{e}^{4x} - 1 = 0$	A1	
	$\frac{\mathrm{e}^{x}}{2} = 0$ does not lead to a solution for x		
	$5\mathrm{e}^{4x} - 1 = 0$		
	$x = \dfrac{1}{4}\ln\dfrac{1}{5}$		
	Therefore, there is only one stationary point.	A1	
	METHOD 2:		
	$\mathrm{e}^{-3x}y = \sinh 2x$		
	$\mathrm{e}^{-3x}\dfrac{\mathrm{d}y}{\mathrm{d}x} - 3\mathrm{e}^{-3x}y = 2\cosh 2x$	(M1)	
	$\mathrm{e}^{-3x}\dfrac{\mathrm{d}y}{\mathrm{d}x} - 3\sinh 2x = 2\cosh 2x$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2\cosh 2x + 3\sinh 2x}{\mathrm{e}^{-3x}}$	(A1)	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{e}^{2x} + \mathrm{e}^{-2x} + \frac{3}{2}\mathrm{e}^{2x} - \frac{3}{2}\mathrm{e}^{-2x}}{\mathrm{e}^{-3x}}$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{5}{2}\mathrm{e}^{5x} - \dfrac{1}{2}\mathrm{e}^{x}$	(A1)	
	When stationary, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	(m1)	
	$\dfrac{5\mathrm{e}^{5x}}{2} - \dfrac{\mathrm{e}^{x}}{2} = 0$		
	$\dfrac{\mathrm{e}^{x}}{2}(5\mathrm{e}^{4x} - 1) = 0$		
	$\frac{\mathrm{e}^{x}}{2} = 0$ or $5\mathrm{e}^{4x} - 1 = 0$	(A1)	
	$\frac{\mathrm{e}^{x}}{2} = 0$ does not lead to a solution for x		
	$5\mathrm{e}^{4x} - 1 = 0$		
	$x = \dfrac{1}{4}\ln\dfrac{1}{5}$		
	Therefore, there is only one stationary point.	(A1)	

Qu	Solution	Mark	Notes
8. b)	METHOD 3: $e^{-3x}y = \sinh 2x$ $y = e^{3x}\sinh 2x$	(M1)	
	$\frac{dy}{dx} = 3e^{3x}\sinh 2x + 2e^{3x}\cosh 2x$	(A2)	A1 each part
	When stationary, $\frac{dy}{dx} = 0$ $e^{3x}(3\sinh 2x + 2\cosh 2x) = 0$	(m1)	
	$e^{3x} = 0$ or $3\sinh 2x + 2\cosh 2x = 0$	(A1)	
	$e^{3x} = 0$ does not lead to a solution for x		
	$3\sinh 2x + 2\cosh 2x = 0$ $\frac{\sinh 2x}{\cosh 2x} = -\frac{2}{3} \rightarrow \tanh 2x = -\frac{2}{3}$ $2x = -0.8047$ ... ... $x = -0.402$ ... ... or $\frac{1}{4}\ln\frac{1}{5}$ Therefore, there is only one stationary point.	(A1)	
		[6] **Total [11]**	

Qu	Solution	Mark	Notes
9.	Rewriting, $\sin 6\theta + \sin 2\theta = 3\cos 2\theta - 2\cos^2\theta + 1$		
	Use of summing formula, $2\sin\frac{6\theta+2\theta}{2}\cos\frac{6\theta-2\theta}{2} = 2\sin 4\theta\cos 2\theta$	M1 A1	
	Use of identity, $3\cos 2\theta - (2\cos^2\theta - 1) = 3\cos 2\theta - \cos 2\theta$ $= 2\cos 2\theta$	M1 A1	
	Therefore, $2\sin 4\theta\cos 2\theta = 2\cos 2\theta$ $2\sin 4\theta\cos 2\theta - 2\cos 2\theta = 0$	m1	FT provided M1M1, for method marks only
	$2\cos 2\theta\,(\sin 4\theta - 1) = 0$	A1	Solvable form
	$2\cos 2\theta = 0$ or $\sin 4\theta - 1 = 0$ $\cos 2\theta = 0$ $2\theta = \frac{\pi}{2} + n\pi$ $\theta = \frac{\pi}{4} + \frac{n\pi}{2}$	m1 A1	Either cao
	$\sin 4\theta = 1$ $4\theta = \frac{\pi}{2} + 2n\pi$ $\theta = \frac{\pi}{8} + \frac{n\pi}{2}$	 A1	 cao
		[9]	
		Total [9]	

Qu	Solution	Mark	Notes
10. a)	METHOD 1: Rearrange first equation: $\frac{\mathrm{d}x}{\mathrm{d}t} - 4x - 6\mathrm{e}^{3t} = 2y$ $\frac{1}{2}\frac{\mathrm{d}x}{\mathrm{d}t} - 2x - 3\mathrm{e}^{3t} = y$	M1	
	Substituting into second equation: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\frac{\mathrm{d}x}{\mathrm{d}t} - 2x - 3\mathrm{e}^{3t}\right) = 6x + 8\left(\frac{1}{2}\frac{\mathrm{d}x}{\mathrm{d}t} - 2x - 3\mathrm{e}^{3t}\right) + 15\mathrm{e}^{3t}$	M1	
	$\frac{1}{2}\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 2\frac{\mathrm{d}x}{\mathrm{d}t} - 9\mathrm{e}^{3t} = 6x + 4\frac{\mathrm{d}x}{\mathrm{d}t} - 16x - 24\mathrm{e}^{3t} + 15\mathrm{e}^{3t}$	A2	-1 each error
	$\frac{1}{2}\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 6\frac{\mathrm{d}x}{\mathrm{d}t} + 10x = 0$	A1	Convincing
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 20x = 0$	[5]	
	METHOD 2: Differentiating: $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 4\frac{\mathrm{d}x}{\mathrm{d}t} + 2\frac{\mathrm{d}y}{\mathrm{d}t} + 18\mathrm{e}^{3t}$	(M1)	
	Substituting for $\frac{\mathrm{d}y}{\mathrm{d}t}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 4\frac{\mathrm{d}x}{\mathrm{d}t} + 2(6x + 8y + 15\mathrm{e}^{3t}) + 18\mathrm{e}^{3t}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 4\frac{\mathrm{d}x}{\mathrm{d}t} + 12x + 16y + 30\mathrm{e}^{3t} + 18\mathrm{e}^{3t}$	(A1)	
	Substituting for y $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 4\frac{\mathrm{d}x}{\mathrm{d}t} + 12x + 8\left(\frac{\mathrm{d}x}{\mathrm{d}t} - 4x - 6\mathrm{e}^{3t}\right) + 48\mathrm{e}^{3t}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 4\frac{\mathrm{d}x}{\mathrm{d}t} + 12x + 8\frac{\mathrm{d}x}{\mathrm{d}t} - 32x - 48\mathrm{e}^{3t} + 48\mathrm{e}^{3t}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 12\frac{\mathrm{d}x}{\mathrm{d}t} - 20x$	(M1) (A1)	
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 20x = 0$	(A1) **(5)**	Convincing

Qu	Solution	Mark	Notes
10. b)	The auxiliary equation is: $m^2 - 12m + 20 = 0$ $(m-2)(m-10) = 0$ $m = 2$ or $m = 10$ Therefore, the general solution is: $x = Ae^{2t} + Be^{10t}$	 M1 A1	Condone use of x and y
	Differentiating $\frac{dx}{dt} = 2Ae^{2t} + 10Be^{10t}$ $\frac{d^2x}{dt^2} = 4Ae^{2t} + 100Be^{10t}$	M1 A1	 Both $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$
	Substituting and solving $2A + 10B = 9$ $4A + 100B = 10$ Solving, $A = 5$ and $B = -\frac{1}{10}$	 m1 A1	
	Therefore, $x = 5e^{2t} - \frac{1}{10}e^{10t}$	 B1	 cao Must be $x = f(t)$
		[7] **Total [12]**	

GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
FURTHER MATHEMATICS
UNIT 5 FURTHER STATISTICS B
1305U50-1

About this marking scheme

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WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 5 FURTHER STATISTICS B

SUMMER 2024 MARK SCHEME

Qu.	Solution	Mark	Notes
1(a)	$\overline{x} = 29.95$	B1	
	Standard error = $\sqrt{\frac{0.6}{8}}$ $(= 0.2738\ldots)$	B1	$SE^2 = \frac{0.6}{8}$
	Use of $\bar{x} \pm z \times \text{SE}$	M1	FT their $\bar{x}$ and SE $\neq \sqrt{0.6}$ for M1A1 Allow z values for M1
	$=29.95 \pm 1.96 \times \sqrt{\frac{0.6}{8}}$	A1	1.96 or better
	$[29.41, 30.49]$	A1	cao 3sf or greater.
		[5]	
(b)	Two valid comments. e.g. We would have to use the t-distribution instead. e.g. Use 2.365 instead of 1.96. e.g. We would have to use t-tables. e.g. We would use the sample variance. e.g. We need to find s. e.g. Calculate an unbiased estimator of the variance.	E1 E1	Do not allow same reason twice.
		[2]	
(c)(i)	Valid explanation. e.g. The mean is outside the confidence interval so it may have been a different player. e.g. The average time is much higher than any of the other times recorded by the first player.	E1	
(ii)	Valid explanation. e.g. In part (a), the times refer to the first line drill of a practice session. Here it includes all the line drills in a practice session so the player may be tired and as a result have a higher mean time. e.g. The player may be returning from injury / may be injured. e.g. The player may be having an 'off' day.	E1	Do not condone there is a small probability that the player might score 35.6 at random. (The probability for this is in the order of magnitude $\times 10^{-95}$) Do not condone "5% chance it could be the player"
		[2]	
	Total for Question 1	**9**	

Qu.	Solution	Mark	Notes
2 (a)	H_0: The median daily caffeine intake per student from Country B who drinks coffee is 120mg H_1: The median daily caffeine intake per student from Country B who drinks coffee is greater than 120mg	B1	Let η be the median daily caffeine intake per student from Country B who drinks coffee. Both. $\eta = 120, \eta > 120$
	<table><tr><td>Diff x-120</td><td>16</td><td>29</td><td>82</td><td>-10</td><td>-20</td><td>60</td><td>67</td><td>18</td><td>77</td><td>-5</td></tr><tr><td>Rank</td><td>3</td><td>6</td><td>10</td><td>2</td><td>5</td><td>7</td><td>8</td><td>4</td><td>9</td><td>1</td></tr></table>	M1 A1	Attempt to find differences All correct
	$w^+ = 3 + 6 + 10 + 7 + 8 + 4 + 9$ $w^+ = 47$ OR $w^- = 2 + 5 + 1$ $w^- = 8$	M1 A1	M1 Attempt at summing ranks. cao
	Upper CV = 44 OR Lower CV $(= \frac{1}{2} \times 10 \times (10+1) - 44) = 11$ Since $47 > 44$ (OR $8 < 11$) there is sufficient evidence to reject H_0. It is reasonable to believe that the coffee-drinking students from country B drink more coffee than the students from country A.	B1 B1 E1 **[8]**	 FT their w^+/w^- and their CV cso
(b)	Valid comment on caffeine consumption. e.g. Caffeine intake may be from other sources so this should be taken into account. OR Valid comment on the exclusion of a sizable proportion of students. e.g. Whilst there may be evidence to suggest that the foreign students who drink coffee may drink more coffee it would be unwise to make this conclusion for the whole population because 1/3 of our sample did not drink any coffee. e.g Any conclusion made would be based on the coffee drinkers, which accounts for only 2/3 of the sample. e.g. Ignores students who have not drunk coffee on that particular day but maybe they usually do. OR Valid comment on experimental design. e.g. Small sample size. e.g. Could take a sample from both countries and compare. e.g. Data is only taken on one day.	E1 **[1]**	E0 for The time the caffeine intake was measured. E0 for The four students with zero caffeine intake may drink coffee late at night.
	Total for Question 2	**9**	

Qu.	Solution	Mark	Notes
3(a)	$\hat{p} = \frac{55}{80} = \frac{11}{16} = 0.6875$	B1	
	$\text{ESE} = \sqrt{\frac{0.6875 \times (1 - 0.6875)}{80}}$	M1	FT their $\hat{p}$ for M1A1
	$= 0.0518\ldots$ or $\frac{\sqrt{11}}{64}$	A1	si
	$\hat{p} \pm z \times \text{ESE}$	M1	FT their $\hat{p}$ and ESE for M1A1
	$0.6875 \pm 1.6449 \times 0.0518\ldots$	A1	1.645 or better
	$[0.602, 0.773]$	A1	cao
		[6]	
(b)	(90% of 50 =) 45	B1	
		[1]	
	Total for question 3	**7**	

Qu.	Solution	Mark	Notes
4(a)	$H_0: \mu_x - \mu_y = 0$ $H_1: \mu_x - \mu_y \neq 0$	B1 **[1]**	Both hypotheses. oe
(b)	Standard error of difference of means = $\sqrt{\frac{8^2}{40} + \frac{10^2}{40}}$	M1	
	$= 2.024(845673 \ldots)$	A1	si
	p-value $= 0.0692 \times 2$	A1	For 0.0692 Allow 0.06944 from tables
	$= 0.1384$	A1	Allow 0.1389
	Since $0.1384 > 0.05$ insufficient evidence to reject H_0.	m1	Accept other significance levels.
	There is not enough evidence to suggest that there is a difference in specific gravity of blood of cyclists and runners.	A1 **[6]**	cso
(c)	SE of difference in means = $\sqrt{\frac{8^2}{n} + \frac{10^2}{n}}\left(= \sqrt{\frac{164}{n}}\right)$ and difference of means $= 3$	M1	
	$\mathrm{P}\left(Z > \frac{3-0}{\sqrt{\frac{164}{n}}}\right) < 0.005$	M1	M1 Probability statement with "their mean and variance" and 0.005. May be implied by A1 oe Allow = or ≤ (or ≥ for M1A1)
	$\frac{3-0}{\sqrt{\frac{164}{n}}} > 2.5758$	A1	Inequality ft "their mean and variance" and 2.576 or better.
	$\frac{3-0}{2.5758} > \sqrt{\frac{164}{n}}$		
	$n > 120.8998 \ldots$		$n < 120.8998 \ldots$ leading to $n = 121$ scores M1A0A1
	$n = 121$	A1 **[4]**	cao
	Total for question 4	**11**	

Qu.	Solution	Mark	Notes
5 (a) (i)	$\mathrm{E}(\overline{X}) = \mathrm{E}(X)$		
	$\mathrm{E}(X) = \int_0^{\alpha} x.\frac{3x^2}{\alpha^3}\,\mathrm{d}x$	M1	M1 for integrating $xf(x)\mathrm{d}x$ Limits not required
	$\mathrm{E}(X) = \left[\frac{3x^4}{4\alpha^3}\right]_0^{\alpha}$	A1	Correct integration, limits required
	$\mathrm{E}(X) = \frac{3\alpha}{4}$	A1	
	$\mathrm{E}(U) = \frac{4}{3}\mathrm{E}(\overline{X}) = \frac{4}{3} \times \frac{3\alpha}{4}$	M1	
	$\mathrm{E}(U) = \alpha$ Therefore U is an unbiased estimator for α.	A1 **[5]**	convincing
(ii)	$\mathrm{E}(X^2) = \int_0^{\alpha} x^2.\frac{3x^2}{\alpha^3}\,\mathrm{d}x = \left[\frac{3x^5}{5\alpha^3}\right]_0^{\alpha}$	M1	Attempt to integrate Limits not required
	$\mathrm{E}(X^2) = \frac{3\alpha^2}{5}$	A1	
	$\mathrm{Var}(X) = \frac{3\alpha^2}{5} - \left(\frac{3\alpha}{4}\right)^2$	M1	FT their $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$
	$\mathrm{Var}(X) = \frac{3\alpha^2}{80}$	A1	cao
	$\mathrm{Var}(U) = \left(\frac{4}{3}\right)^2 \mathrm{Var}(\overline{X})$	M1	Use of $a^2\mathrm{Var}(\bar{X})$.
	$\mathrm{Var}(U) = \frac{16}{9} \times \frac{3\alpha^2}{80n}$	M1	FT their $\mathrm{Var}(X)$
	$\mathrm{Var}(U) = \frac{\alpha^2}{15n}$	A1	cao
	$\mathrm{SE} = \sqrt{\frac{\alpha^2}{15n}}$	M1	FT their $\mathrm{Var}(U)$
	therefore $n = 15$	A1 **[9]**	cao
(b)(i)	$\mathrm{Var}(V) = 4^2\mathrm{Var}(\overline{X}_1) + \left(\frac{8}{3}\right)^2 \mathrm{Var}(\overline{X}_2)$	M1	Use of $a^2\mathrm{Var}(\bar{X})$.
	$\mathrm{Var}(V) = 16 \times \frac{3\alpha^2}{80n} + \frac{64}{9} \times \frac{3\alpha^2}{80n}$	m1	FT their $\mathrm{Var}(\bar{X})$
	$\mathrm{Var}(V) = \frac{13\alpha^2}{15n}$	A1	cao
	$\frac{\mathrm{Var}(U)}{\mathrm{Var}(V)} = \frac{\alpha^2}{15n} \times \frac{15n}{13\alpha^2} = \frac{1}{13}$ **ag***	B1 **[4]**	Convincing
(b)(ii)	Since $\frac{\mathrm{Var}(U)}{\mathrm{Var}(V)} < 1$, U is the better estimator.	E1 **[1]**	
	Total for Question 5	**19**	

Qu.	Solution	Mark	Notes
6	H_0: The median numbers of words memorised by Group A and Group B are the same. H_1: The median number of words memorised by Group B is more than the median number of words memorised by Group A.	B1	Accept $H_0: \eta_A = \eta_B$ $H_1: \eta_B > \eta_A$ Also accept $H_1: \eta_A < \eta_B$
	Use of the formula $U = \sum\sum z_{ij}$	M1	Attempt to use
	$U = 2+9+6+7+9+6+6+6+6+6+5+7$ OR $U = 1+1+10+2+0+0+1+10+8$		Accept ranks with opposite signs ALTERNATIVE METHOD t_A = sum of ranks for $A = 111$
	$U = 75$ OR $U = 33$	A1	$U_A = 111 - \frac{13\times12}{2} = 33$
	Upper critical value is 78 OR Lower CV is 30	B1	B = sum of ranks for $B = 120$ $U_A = 120 - \frac{9\times10}{2} = 75$
	75 < 78 OR 33 >30, there is insufficient evidence to reject H_0.	m1	
	There is insufficient evidence to suggest that memorising words that are different in meaning is easier than memorising words that are similar in meaning. There is insufficient evidence to suggest that group B are better at memorising words than group A.	A1 **[6]**	cso
	Total for Question 6	**6**	

Qu.	Solution	Mark	Notes
7(a)	$\mathrm{P}(W < 19) = 0.1$ $\mathrm{P}(W < 19) = P\left(Z < \frac{19-\mu}{0.6}\right)$ $\frac{19-\mu}{0.6} = -1.282$ $\mu = 19.8$	 M1 A1 A1 **[3]**	 M1 for attempt to standardise with 19 and 0.6 oe
(b)	$\overline{X} \sim \mathrm{N}\left(20.1, \frac{1.2^2}{8}\right)$ $\mathrm{P}(\overline{X} > 20) = 0.593\ldots$ ***ALTERNATIVE SOLUTION*** Let $L = X + X + X + X + X + X + X + X$ $L \sim \mathrm{N}(160.8, 8 \times 1.2^2)$ $\mathrm{P}(L > 160) = 0.593\ldots$	B1 M1A1 (B1) (M1) (A1) **[3]**	si M1 for $z = \frac{20-20.1}{\sqrt{0.18}}$ or $z = -0.24$ A1 for 0.59483 FT for M1 only provided $\sigma \neq 1.2$ M1 for $z = \frac{160-160.8}{\sqrt{11.52}}$ or $z = -0.24$ A1 for 0.59483
(c) (i)	Let $T = Y + Y + Y - X - X - X$ $\mathrm{E}(T) = 6.3$ $\mathrm{Var}(T) = 3 \times 1.5^2 + 3 \times 1.2^2$ $\mathrm{Var}(T) = 11.07$ $\mathrm{P}(\text{move the einkorn}) = \mathrm{P}(T > 0) = 0.97085$	 B1 M1 A1 A1	 cao
(ii)	$\mathrm{P}(\text{move the corn}) = \mathrm{P}(T < 0) = 1 - \mathrm{P}(T > 0)$ $\mathrm{P}(\text{move the corn}) = \mathrm{P}(T < 0) = 0.02915$	 B1 **[5]**	 FT $1 - (i)$
(d) (i)	Let $U = X - 3E$ $\mathrm{P}(U > 0) = 0.35208$ $\mathrm{E}(U) = -11.4$ $\mathrm{P}(Z > \frac{0--11.4}{\sigma_U}) = 0.35208$ $\frac{11.4}{\sigma_U} = 0.37971$ $\sigma_U = 30.022912\ldots$ $\mathrm{Var}(U) = \mathrm{Var}(X) + 9\mathrm{Var}(E)$ $901.37525 = 1.2^2 + 9\mathrm{Var}(E)$ $\sigma = 10$	 B1 B1 B1 M1 A1 M1 A1	 B1 for use of 0.35208, si B1 for E(U) B1 for ± 0.37971 M1 for equation si M1 for use of variance formula.

Qu.	Solution	Mark	Notes
7(d) (i)	***ALTERNATIVE SOLUTION*** $\mathrm{P}(X > 3E) = 0.35208$ $X - 3E \sim N(20.1 - 3 \times 10.5, 1.2^2 + 9\sigma^2)$ $X - 3E \sim N(-11.4, 1.2^2 + 9\sigma^2)$ $\mathrm{P}(X > 3E) > 0$ $\mathrm{P}\left(Z > \frac{0 - -11.4}{\sqrt{1.2^2 + 9\sigma^2}}\right) = 0.35208$ OR $\mathrm{P}\left(Z < \frac{0 - -11.4}{\sqrt{1.2^2 + 9\sigma^2}}\right) = 0.64792$ $\frac{11.4}{\sqrt{1.2^2 + 9\sigma^2}} = 0.3797$ $11.4^2 = 0.3797^2 \times (1.2^2 + 9\sigma^2)$ $\sigma^2 = \frac{129.96 - 0.20761}{0.3797^2 \times 9}$ $\sigma^2 = 99.99808023$ $\sigma = 10$	(B1) (B1) (M1) (B1) (M1) (A1) (A1)	B1 for use of 0.35208, si B1 for -11.4 M1 for Variance. B1 for ± 0.3797 M1 for equation A1 rearranging to get $\sigma^2 = k$
7(d) (ii)	Valid comment e.g. (The value of σ is far too big.) It gives a high probability that the mass would be negative, which is obviously impossible. e.g. (The value for σ is far too big.) It's almost as big as the mean.	E1 **[8]**	Must give reason more than just "σ is too big".
	Total for Question 7	**19**	

GCE A LEVEL MARKING SCHEME

SUMMER 2024

A LEVEL
FURTHER MATHEMATICS
UNIT 6 FURTHER MECHANICS B
1305U60-1

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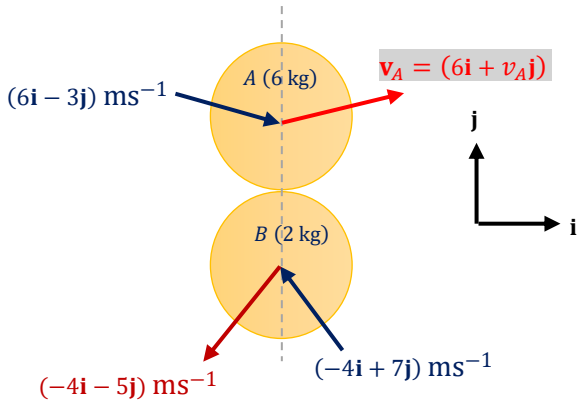
WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 6 FURTHER MECHANICS B

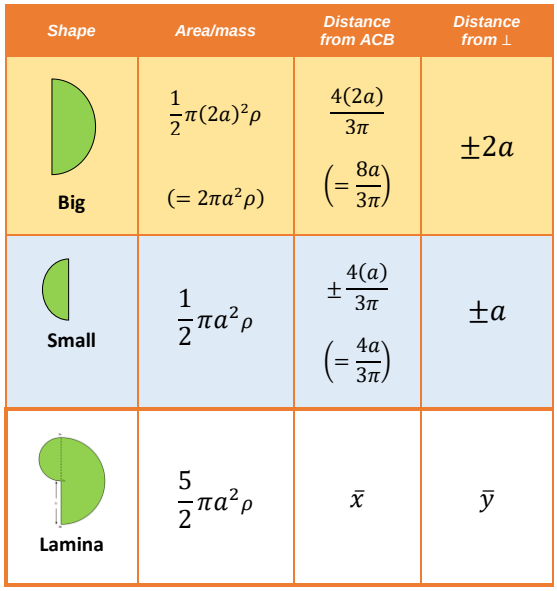
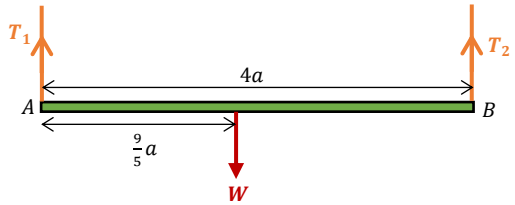
SUMMER 2024 MARK SCHEME

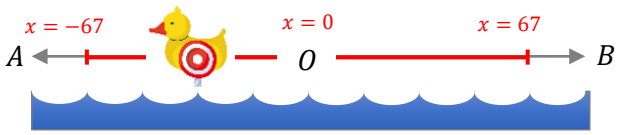
Q1	Solution	Mark	Notes
			Before collision After collision
(a)	Con. of momentum (along line of centres)	M1	Attempted
	$6(-3)+2(7)=6v_A+2(-5)$	A1	All correct, oe $-4=6v_A-10$
	$(v_A=1)$		
	$\mathbf{v}_A=(6\mathbf{i}+\mathbf{j})$ (ms^{-1})	A1 **[3]**	
	<u>Alternative Solution</u> (Vector Method)		
	Conservation of momentum $6\mathbf{u}_A+2\mathbf{u}_B=6\mathbf{v}_A+2\mathbf{v}_B$	(M1)	Attempted
	$6(6\mathbf{i}-3\mathbf{j})+2(-4\mathbf{i}+7\mathbf{j})=6\mathbf{v}_A+2(-4\mathbf{i}-5\mathbf{j})$	(A1)	All correct, oe
	$\mathbf{v}_A=(6\mathbf{i}+\mathbf{j})$ (ms^{-1})	(A1) **([3])**	
(b)	Restitution (along line of centres)	M1	Attempted
	$(-5)-(1)=-e(7--3)$ OR $e=\frac{1--5}{7--3}=\frac{-5-1}{-3-7}$ OR $-e=\frac{-5-1}{7--3}=\frac{1--5}{-3-7}$	A1	All correct, oe FT component $\mathbf{j}$ of $\mathbf{v}_A$ from (a)
	$e=\frac{3}{5}=0\cdot6$	A1 **[3]**	

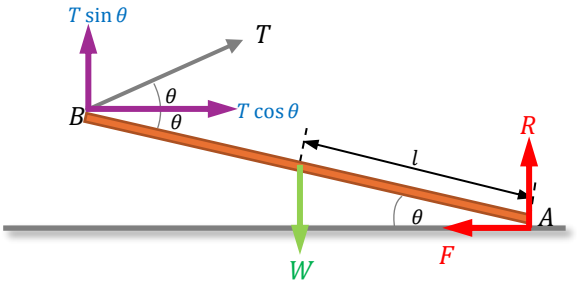
Q1	Solution	Mark	Notes
(c)	$\cos\theta = \dfrac{(-4\mathbf{i}+7\mathbf{j})\cdot(-4\mathbf{i}-5\mathbf{j})}{\lvert-4\mathbf{i}+7\mathbf{j}\rvert\lvert-4\mathbf{i}-5\mathbf{j}\rvert}$ $\cos\theta = \dfrac{16-35}{\sqrt{65}\sqrt{41}}$ $(=-0\cdot368048451\ldots)$ $\theta = 112°$ (nearest degree)	M1 A1 A1 **[3]**	Use of (any form) $\cos\theta = \dfrac{\mathbf{u}_B\cdot\mathbf{v}_B}{\lvert\mathbf{u}_B\rvert\lvert\mathbf{v}_B\rvert}$ oe or $360-112=248°$
	Alternative Solution Sight of either one from **BOTH** rows $\theta = \tan^{-1}\left(\pm\frac{7}{4}\right)$ OR $\theta = \tan^{-1}\left(\pm\frac{4}{7}\right)$ $(\theta = \pm60\cdot2\ldots)$ $(\theta = \pm29\cdot7\ldots)$ $\theta = \tan^{-1}\left(\pm\frac{5}{4}\right)$ OR $\theta = \tan^{-1}\left(\pm\frac{4}{5}\right)$ $(\theta = \pm51\cdot3\ldots)$ $(\theta = \pm38\cdot6\ldots)$ $\theta = 60\cdot255\ldots + 51\cdot340\ldots$ $\theta = 112°$ (nearest degree)	(M1) (A1) (A1) **([3])**	oe Correct calculation or $360-112=248$
(d)	Impulse, $\mathbf{I}$ = change in momentum $(-20\mathbf{i}+18\mathbf{j}) = 2\mathbf{v}_B - 2(-4\mathbf{i}-5\mathbf{j})$ $\mathbf{v}_B = (-14\mathbf{i}+4\mathbf{j})$ (ms^{-1})	M1 A1 **[2]**	Difference in Momentum used, $(-20\mathbf{i}+18\mathbf{j}) = -2\mathbf{v}_B + 2(-4\mathbf{i}-5\mathbf{j})$
	Total for Question 1	**11**	

Q2	Solution	Mark	Notes
(a)	Applying N2L (mg, $0\cdot 2v^2$ opposing),	M1	Dimensionally correct equation
	$1\cdot 8g - 0\cdot 2v^2 = 1\cdot 8a$	A1	Correct equation
	Dividing (by $0\cdot 2, 1\cdot 8$) and using $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ $9\frac{\mathrm{d}v}{\mathrm{d}t} = 9g - v^2$ $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{9g-v^2}{9}$	A1 **[3]**	Convincing
(b)	$9\int \frac{1}{9g-v^2}\,\mathrm{d}v = \int \mathrm{d}t$	M1	Separating variables
	$t\ \ (+C) = \begin{cases} \frac{9}{2\times 3\sqrt{g}}\ln\left\lvert\frac{3\sqrt{g}+v}{3\sqrt{g}-v}\right\rvert \\ \frac{9}{3\sqrt{g}}\tanh^{-1}\left(\frac{v}{3\sqrt{g}}\right) \end{cases}$	A1 A1	$\ln\left\lvert\frac{3\sqrt{g}+v}{3\sqrt{g}-v}\right\rvert$ or $\tanh^{-1}\left(\frac{v}{3\sqrt{g}}\right)$ Everything correct, oe
	When $t=0, v=\sqrt{g}$	m1	Use of initial conditions
	$C = \begin{cases} \frac{9}{2\times 3\sqrt{g}}\ln(2) \\ \frac{9}{3\sqrt{g}}\tanh^{-1}\left(\frac{1}{3}\right) \end{cases}$	A1	Finding C Notes • These will be negative if $+C$ features on the RHS above • The numerator of 9 will not be present when the LHS is $\frac{t}{9}$
	Using $v=8$, $t = \begin{cases} \frac{9}{2\times 3\sqrt{g}}\ln\left\lvert\frac{3\sqrt{g}+8}{3\sqrt{g}-8}\right\rvert - \frac{9}{2\times 3\sqrt{g}}\ln(2) \\ \frac{9}{3\sqrt{g}}\tanh^{-1}\left(\frac{8}{3\sqrt{g}}\right) - \frac{9}{3\sqrt{g}}\tanh^{-1}\left(\frac{1}{3}\right) \\ 0\cdot 878(03761\ldots) \end{cases}$ Time taken = $0\cdot 878(03761\ldots)$ (s)	A1 **[6]**	$t = \begin{cases} \frac{3}{2\sqrt{g}}\ln\left\lvert\frac{3\sqrt{g}+8}{2(3\sqrt{g}-8)}\right\rvert \\ \frac{3}{\sqrt{g}}\tanh^{-1}\left(\frac{8}{3\sqrt{g}}\right) - \frac{3}{\sqrt{g}}\tanh^{-1}\left(\frac{1}{3}\right) \end{cases}$
(c)	Using $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ to get $v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{9g-v^2}{9}$	M1	Used with DE
	$9\int \frac{v}{9g-v^2}\,\mathrm{d}v = \int \mathrm{d}x$ $\quad \frac{9}{-2}\int \frac{-2v}{9g-v^2}\mathrm{d}v = \int \mathrm{d}x$	m1	Separating variables
	$-\frac{9}{2}\ln\lvert 9g - v^2\rvert = x\ \ (+C)$	A1 A1	$\ln\lvert 9g-v^2\rvert$ Everything correct
	When $x=0, v=\sqrt{g}$ $-\frac{9}{2}\ln\lvert 8g\rvert = C$	m1	Use of initial conditions

Q2	Solution	Mark	Notes
	$x = \begin{cases} -\frac{9}{2}\ln\lvert 9g - v^2\rvert + \frac{9}{2}\ln\lvert 8g\rvert \\ \frac{9}{2}\ln\left\lvert\frac{8g}{9g - v^2}\right\rvert \end{cases}$ When $v = 8$, $x = \begin{cases} -\frac{9}{2}\ln\left\lvert 9g - 8^2\right\rvert + \frac{9}{2}\ln\lvert 8g\rvert \\ \frac{9}{2}\ln\left\lvert\frac{8g}{9g - 8^2}\right\rvert \end{cases}$ $x = 5 \cdot 289(620824) \quad (\text{m})$	A1 A1 **[7]**	oe, for correct expression Notes • Signs will be reversed if $+C$ features on the LHS • The numerator of 9 will not be present when the RHS is $\frac{x}{9}$
Total for Question 2		**16**	

Q3	Solution	Mark	Notes
(a)	 (i) Moments about AB $\frac{5}{2}\pi a^2\bar{x} = (2\pi a^2)\left(\frac{8a}{3\pi}\right) - \left(\frac{1}{2}\pi a^2\right)\left(\frac{4a}{3\pi}\right)$ $\bar{x} = \frac{28}{15\pi}a$	B1 B1 B1 B1 M1 A1 A1	Condone omission of ρ (mass per unit area) B3 6 B2 any 4 or 5, B1 any 2 or 3 correct Addition of areas/masses Masses and moments consistent Convincing
	(ii) Moments about ⊥ through A $\frac{5}{2}\pi a^2\bar{y} = (2\pi a^2)(2a) + \left(\frac{1}{2}\pi a^2\right)(a)$ OR $\frac{5}{2}\pi a^2\bar{y} = (2\pi a^2)(-2a) + \left(\frac{1}{2}\pi a^2\right)(-a)$ $\bar{y} = \pm\frac{9}{5}a = \pm 1\cdot 8a$ Distance is $\frac{9}{5}a$ or $1\cdot 8a$	M1 A1 A1 **[10]**	Masses and moments consistent
(b)	 Moments about A $\frac{9}{5}a \times W = 4a \times T_2$ $T_2 = \frac{9}{20}W$ (Required fraction is $\frac{9}{20}$)	M1 A1 A1 **[3]**	W = weight
	Total for Question 3	**13**	

Q4	Solution	Mark	Notes
(a)	Acceleration, $\frac{d^2x}{dt^2} = \pm\omega^2 x$ $1344 = \pm\omega^2(\pm 84)$ $\omega^2 = 16$ $\omega = 4$ Period, $T = \frac{2\pi}{\omega}$ $= \frac{\pi}{2}$ (s)	M1 A1 A1 **[3]**	Used with $a = 1344$, $x = \pm 84$ Convincing
(b)	$v^2 = \omega^2(a^2 - x^2)$, $\omega = 4$, $x = \pm 84$, $v = \pm 52$ $(52)^2 = 4^2(a^2 - (84)^2)$ $a = 85$ Maximum speed $= a\omega = (85)(4)$ $= 340$ (cms^{-1})	M1 A1 A1 M1 A1 **[5]**	Used FT ω from (a) Used FT ω from (a) and corresponding a
(c)			
	Using $x = \pm a\cos(\omega t)$ or $x = \pm a\sin(\omega t + \frac{\pi}{2})$ $\pm 67 = \pm 85\cos(4t)$ (Same/opposing signs) $4t = \cos^{-1}\left(\frac{67}{85}\right)$ or $4t = \cos^{-1}\left(\frac{-67}{85}\right)$ $4t = 0 \cdot 66286\ldots$ $4t = 2 \cdot 47873\ldots$ $t = 0 \cdot 16571\ldots$ $t = 0 \cdot 61968\ldots$ Times $= T + 0 \cdot 16571\ldots$, $T + 0 \cdot 61968\ldots$ $= 1 \cdot 73651\ldots$, $2 \cdot 190479\ldots$ (s)	M1 m1 m1 A1 A1 **[5]**	Allow $x = \pm a\sin(\omega t)$ FT $a > 67$ 1st correct method Remaining method Corresponding time for 1st correct method. A1
	Total for Question 4	**13**	

Q5	Solution	Mark	Notes
			Length of rod $= 2l$
(a)	(i) Moments about A	M1	Dim. correct, no missing/extra terms
	$T \times 2l \sin 2\theta = W \times l \cos\theta$	A1	All correct
	$T \times 2l \times 2\sin\theta\cos\theta = W \times l\cos\theta$	m1	Use of $\sin 2\theta = 2\sin\theta\cos\theta$
	$T = \frac{W}{4}\operatorname{cosec}\theta$	A1	Convincing
	(ii) Resolve vertically	M1	Dim. correct equation with 3 terms
	$R + T\sin\theta = W$	A1	
	$R + \frac{W}{4}\operatorname{cosec}\theta\sin\theta = W$	m1	Elimination of T
	$R = \frac{3}{4}W$	A1	
		[8]	
(b)	Resolve horizontally	M1	Dimensionally correct equation, no missing/extra terms
	$F = T\cos\theta$	A1	
	$F = \frac{W}{4}\operatorname{cosec}\theta\cos\theta = \frac{W}{4}\cot\theta$		
	Use of $F \le \mu R$ with $\mu = \frac{\sqrt{3}}{3}$	M1	si (or equality $F_{lim} = \mu R$)
	$\frac{W}{4}\cot\theta \le \frac{\sqrt{3}}{3} \times \frac{3}{4}W$		
	$\cot\theta \le \sqrt{3}$ or $\tan\theta \ge \frac{\sqrt{3}}{3}$	A1	
	$\theta \ge 30^\circ$	A1	
		[5]	
	Total for Question 5	**13**	

Q6	Solution	Mark	Notes
(a)	$(V\bar{x} =)\ \pi\int_0^b xy^2\,\mathrm{d}x$ $(V\bar{x} =)\ \pi\dfrac{a^2}{b^2}\displaystyle\int_0^b x(b^2-x^2)\,\mathrm{d}x$ $(V\bar{x} =)\ \pi\dfrac{a^2}{b^2}\displaystyle\int_0^b (b^2x-x^3)\,\mathrm{d}x$ $(V\bar{x} =)\ \pi\dfrac{a^2}{b^2}\left[\frac{b^2}{2}x^2-\frac{1}{4}x^4\right]_0^b$ $(V\bar{x} =)\ \dfrac{1}{4}\pi a^2b^2$ Using $V=\frac{2}{3}\pi a^2b$ and dividing to get $\bar{x}=\frac{3}{8}b$	M1 A1 m1 A1 A1 **[5]**	Used All correct, oe At least one term correctly integrated AND sight of correct limits Convincing
(b)	 $\frac{2}{3}\pi\left(\frac{h}{4}\right)^2 h\rho \quad \left(=\frac{1}{24}h^3\pi\rho\right)$ — $50+\frac{3h}{8}$  $\pi(25)^2\times 50\times 20\rho \quad (=625000\pi\rho)$ — 25  $\pi\rho\left[\frac{1}{24}h^3+(25)^2\times 50\times 20\right]$ — $\bar{h}$, 50 Moments about horizontal ground $\pi\rho\left(\frac{1}{24}h^3\left(50+\frac{3h}{8}\right)+(25)^2\times 50\times 20\times 25\right)$ $=\pi\rho\left(\frac{1}{24}h^3+(25)^2\times 50\times 20\right)\times\bar{h}$ Using $\bar{h}=50$ and rearranging to get $\frac{h^4}{64}=25^3\times 50\times 20$ $h=177\cdot 82(7941\ldots)$	B1 B1 B1 B1 M1 A1 m1 A1 **[8]**	Condone omission of ρ (mass per unit area) For Distance of COM For Mass Both correct Must be addition of volumes Notes $h^4=10^9$ $h=(10^9)^{\frac{1}{4}}=10^{\frac{9}{4}}$
(c)	One possible limitation identified.	E1 **[1]**	For example, • Modelling the tree as a (uniform) solid • Considering tree/pot to have constant density • Shape of tree perfectly matches the region R
	Total for Question 6	**14**	