GCE AS MARKING SCHEME

SUMMER 2022

AS (NEW)
MATHEMATICS
UNIT 1 PURE MATHEMATICS A
2300U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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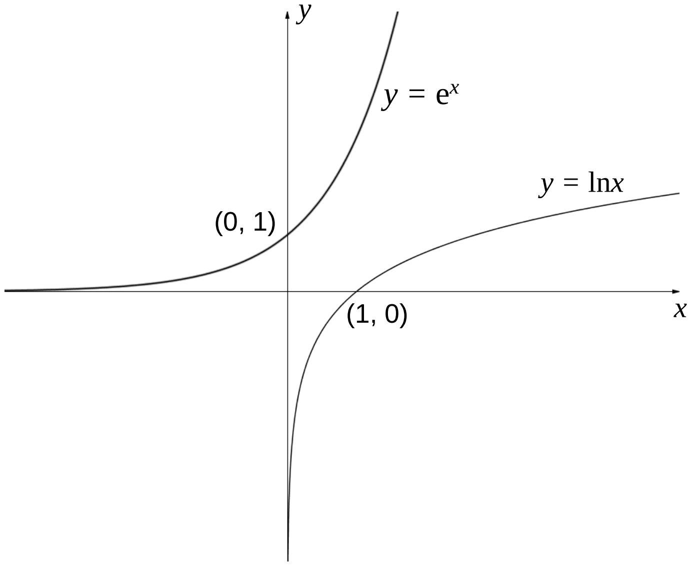
WJEC GCE AS MATHEMATICS

UNIT 1 PURE MATHEMATICS A

SUMMER 2022 MARK SCHEME

Q	Solution	Mark	Notes
1	$y = \ln x$	B1	Allow $y = \log_e x$ May be seen on graph



		B1	graph of $y = e^x$ and (0,1)
		B1	graph of $y = \ln x$ and (1,0)

<u>If B0 B0</u>

SC1 both graphs correctly drawn,
but intercepts missing or incorrect

OR

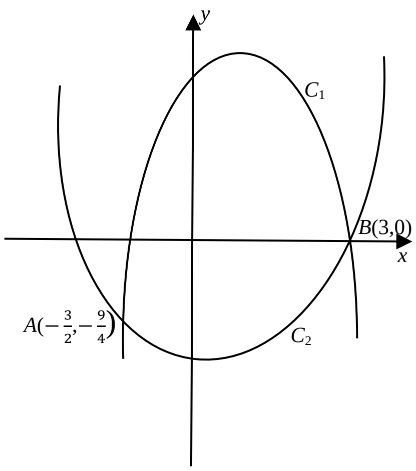
SC1 correct intercepts but incorrect graphs

Q	Solution	Mark	Notes
2	$5\sqrt{48} = 20\sqrt{3}$	B1	
	$(2\sqrt{3})^3 = 24\sqrt{3}$	B1	
	$\frac{2+5\sqrt{3}}{5+3\sqrt{3}} = \frac{(2+5\sqrt{3})(5-3\sqrt{3})}{(5+3\sqrt{3})(5-3\sqrt{3})}$	M1	multiplying by conjugate M0 if multiplying by conjugate not shown
	$= -\frac{1}{2}(10 - 6\sqrt{3} + 25\sqrt{3} - 45)$	A1	for numerator
		A1	for denominator (25 – 27)
	$= -\frac{1}{2}(19\sqrt{3} - 35)$		
	Expression = $\frac{1}{2}(35 - 27\sqrt{3})$	A1	cao, any correct simplified form

Q	Solution	Mark	Notes
3(a)	Grad. of $L_1 = \frac{increase\ in\ y}{increase\ in\ x}$	M1	
	Grad. of $L_1 = \frac{-1-5}{3-0} = -2$	A1	
	Equ of L_1 is $y - 5 = -2x$	A1	any correct form Mark final answer
	$y + 2x = 5$		
3(b)	$y = \frac{1}{2}x$	B1	ft grad L_1 any correct form Mark final answer
3(c)	At C, $\frac{1}{2}x + 2x = 5$	M1	oe
	$x = 2$, $y = 1$	A1	ft their (a) and (b)
	C is the point (2, 1)		
	Area $OAC = \frac{1}{2} \times OA \times$ (x-coord of C)	M1	
	Area $OAC = (\frac{1}{2} \times 5 \times 2) = 5$	A1	ft their ‘x-coord of C’
	OR		
	Area $OAC = \frac{1}{2} \times OC \times AC$	(M1)	
	$OC = \sqrt{2^2 + 1^2} = \sqrt{5}$		
	$AC = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$		
	Area $OAC = (\frac{1}{2} \times \sqrt{5} \times 2\sqrt{5}) = 5$	(A1)	ft their coordinates of C

Q	Solution	Mark	Notes
3(d)	Gradient of L_3 = -2	M1	
	Either		
	Equ of L_3 is y -2 = -2(x -4)	A1	ft their gradient of L_1 any correct form ISW
	OR		
	Equ of L_3 is $y = -2x + c$		
	2 = -2 × 4 + c		
	c = 10		
	Equ of L_3 is $y = -2x + 10$	(A1)	ft their gradient of L_1
3(e)	Using similar triangles,		
	Area $ODE = 2^2 \times 5 = 20$	B1	ft their (c)
	OR		
	Area = $\frac{1}{2} \times OE \times$ (x-coord of D)		
	Area = $\frac{1}{2} \times 10 \times 4 = 20$	(B1)	

Q	Solution	Mark	Notes
4	$x^2 + 3x - 6 > 4x - 4$		
	$x^2 - x - 2$ (> 0)	M1	oe Allow 1 slip terms all collected on one side
	$(x + 1)(x - 2)$ (> 0)	A1	si condone '=' ft their quadratic
	Critical values, -1 and 2	A1	si cao
	$x < -1$ or $x > 2$	A1	ft their critical values condone ',', or nothing A0 for 'and' Mark final answer

	Solution	Mark	Notes
5(a)	$-x^2 + 2x + 3 = x^2 - x - 6$	M1	
	$2x^2 - 3x - 9 = 0$	A1	
	$(2x + 3)(x - 3) = 0$		
	$x = -\frac{3}{2}, 3$	A1	or one correct pair A0 A0 if no workings seen
	$y = -\frac{9}{4}, 0$	A1	all correct
	$A(-\frac{3}{2}, -\frac{9}{4})$ $\quad B(3,0)$		or other way round
			If 0 marks, award SC1 for sight of (3,0)
5(b)			
		M1	at least one quadratic curve
		A1	one cup, one hill
		A1	graphs all correct with correct points of intersection FT points of intersection where possible

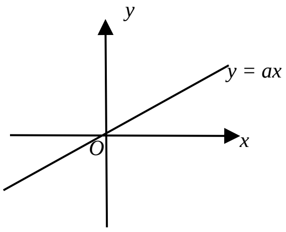
5(c)	Area enclosed by curves to the right of the *y*-axis		ft for equivalent diagram
		B1	for 1 correct region
		B1	for 2nd correct region -1 for each additional incorrect region

Q	Solution	Mark	Notes
6(a)	Statement B is false		
	<u>Two negative numbers</u>:		
	Correct choice of numbers, eg		
	$x = -25$, $y = -4$,	M1	
	Correct verification, eg		
	$x + y = -29$		
	$2\sqrt{xy} = 2 \times \sqrt{(-25) \times (-4)}$	A1	both substitutions
	$2\sqrt{xy} = 20$		
	Since $-29 < 20$ statement *B* is false.	A1	oe
	<u>One positive number, one negative number</u>:		
	Correct choice of numbers, eg		
	$x = 1$, $y = -4$,	(M1)	
	Correct verification, eg		
	$x + y = -3$		
	$2\sqrt{xy} = 2 \times \sqrt{(1) \times (-4)}$	(A1)	both substitutions
	$2\sqrt{xy} = 2\sqrt{-4}$		
	$2\sqrt{-4}$ is not real, statement *B* is false.	(A1)	oe

Q	Solution	Mark	Notes
6(b)	Statement A is true		
	Either		
	$x^2+y^2 \geq 2xy$		
	$x^2 - 2xy + y^2 \geq 0$	M1	
	$(x - y)^2 \geq 0$, which is always true	A1	
	Therefore, Statement A is true		
OR			
	Consider $(x - y)^2 \geq 0$	(M1)	
	$x^2 - 2xy + y^2 \geq 0$		
	$x^2+y^2 \geq 2xy$	(A1)	

Q	Solution	Mark	Notes
7(a)	$A(2, 3)$	B1	
	A correct method for finding the radius,		
	e.g., $(x - 2)^2 + (y - 3)^2 = 4^2$	M1	
	Radius = 4	A1	
7(b)	At points of intersection		
	$x^2 + (x + 5)^2 - 4x - 6(x + 5) - 3 = 0$	M1	
	$2x^2 - 8 = 0$	A1	oe or $2y^2 - 20y + 42 = 0$ All terms collected
	$x = -2, 2$	A1	or $y = 3, 7$ or 1 correct pair
	$y = 3, 7$	A1	or $x = -2, 2$ all correct
	$P(-2, 3)$ $Q(2,7)$		or $P(2,7)$, $Q(-2, 3)$
7(c)	Attempt to find, B, the midpoint of PQ	M1	ft their P and Q
	$B(0, 5)$		
	$PB=\sqrt{(-2-0)^2+(3-5)^2} = \sqrt{8} = 2\sqrt{2}$	A1	ft their P and Q
	OR		
	$PB = \frac{1}{2} PQ = \frac{1}{2}\sqrt{(-2-2)^2+(3-7)^2}$	(M1)	
	$PB = \frac{1}{2} 4\sqrt{2}$		
	$PB = 2\sqrt{2}$	(A1)	ft their P and Q

7(d)	Area = quarter circle – triangle *APQ*	M1	
	Area = $\frac{1}{4} \times \pi \times 4^2 - \frac{1}{2} \times 4 \times 4$	A1	
	Area = $4\pi - 8$		answer given

Q	Solution	Mark	Notes
8(a)		B1	Straight line through the origin, positive or negative gradient
8(b)	Mary's pay = $120 \times \frac{2}{3}$	M1	Divide by 3 oe e.g. $3m = 120$
		M1	oe × by 2
	Mary's pay = £80	A1	Unsupported answer of £80 award M1A1A1
8(c)	$P = 1013 \times 0.88^{\frac{H}{1000}}$	B1	
	When $H = 8848$, $P = 1013 \times 0.88^{\frac{8848}{1000}}$	M1	e.g. $P = 1013 \times 0.88^{H}$ Allow $P = 1013 \times 0.988^{H}$
	$P = 326.8828$ or 327 (units)	A1	Allow answers in the range 324 to 330

Q	Solution	Mark	Notes
9	Discriminant = $(2k)^2 – 4 \times 1 \times 8k$	B1	An expression for $b^2 – 4ac$
	Discriminant = $4k^2 – 32k$		
	If no real roots, discriminant < 0	M1	May be implied by later work M0 if discriminant given in terms of *k* **and** *x*
	$k(k – 8) < 0$		
	Critical values, $k = 0, 8$	B1	si ft their quadratic discriminant if B0 awarded previously
	$0< k < 8$	A1	ft their 2 critical values provided M1 awarded

Q	Solution	Mark	Notes
10	$\ln 2^x = \ln 53$	M1	taking ln or log to any base of both sides.
	$x\ln 2 = \ln 53$	A1	use of power law
	$x = \frac{\ln 53}{\ln 2}$		
	$x = 5.727920455$		
	$x = 5.73$	A1	cao Must be to 2dp

Note:

- No workings M0
- $x = \log_2 53$, award M1A1

Q	Solution	Mark	Notes
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 10 + 6x - 3x^2$	M1	At least one correct term
	Attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$	m1	
	Grad of tangent at $C = 10$	A1	cao
	Equation of tangent at C is		
	$y - 24 = 10(x - 2)$	m1	oe
	$y = 10x + 4$		
	D is the point $(0, 4)$	A1	cao
11(b)	Area of trapezium = $\frac{1}{2}(4 + 24) \times 2$ (= 28)	B1	ft their $D(0,k)$, $0 < k < 24$
	A under curve = $\int_0^2 (10x + 3x^2 - x^3)\mathrm{d}x$	M1	attempt to integrate, at least one term correct, limits not required
	$= \left[5x^2 + x^3 - \frac{x^4}{4}\right]_0^2$	A1	correct integration, limits not required
	$= (20 + 8 - 4) - (0)$	m1	use of limits
	(= 24)		
	Shaded area = area (trap – under curve)	m1	
	Shaded area = 4	A1	cao

Note: Must be supported by workings

Q	Solution	Mark	Notes
11(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 10 + 6x - 3x^2$		FT their $\frac{\mathrm{d}y}{\mathrm{d}x}$ where possible
	At stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1	
	$10 + 6x - 3x^2 = 0$		
	$3x^2 - 6x - 10 = 0$		
	$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times (-10)}}{6}$	m1	attempt to solve quadratic
	$x = -1.08, 3.08$ or $\frac{3 \pm \sqrt{39}}{3}$	A1	any correct form
	Required range is $-1.08 < x < 3.08$	A1	
Alternative Solution			
11(c)	$f'(x) = 10 + 6x - 3x^2$		FT their $f'(x)$ where possible
	For increasing function, $f'(x) > 0$	(M1)	
	$10 + 6x - 3x^2 > 0$		
	$3x^2 - 6x - 10 < 0$		
	$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times (-10)}}{6}$	(m1)	attempt to solve quadratic
	$x = -1.08, 3.08$ or $\frac{3 \pm \sqrt{39}}{3}$	(A1)	any correct form
	Required range is $-1.08 < x < 3.08$	(A1)	

Q	Solution	Mark	Notes
12(a)	$f(x) = 2x^3 - x^2 - 5x - 2$		
	$f(-1) = -2 - 1 + 5 - 2 = 0$	M1	one use of factor theorem
	$(x + 1)$ is a factor	A1	oe
	$f(x) = (x + 1)(2x^2 + px + q)$	M1	at least one of p, q correct
	$f(x) = (x + 1)(2x^2 - 3x - 2)$	A1	oe (see note below*) cao
	$f(x) = (x + 1)(2x + 1)(x - 2)$	m1	coeffs of x^2 multiply to give 2 constant terms multiply to their q or formula with correct a,b,c
	$x = -1, -\frac{1}{2}, 2$	A1	cao

<u>Note</u>:

- Answers only with no workings 0 marks
- * $f(x) = (x - 2)(2x^2 + 3x + 1)$
- * $f(x) = (2x + 1)(x^2 - x - 2)$

Q	Solution	Mark	Notes
12(b)	$\cos(2\theta - 51^\circ) = 0.891$		
	$2\theta - 51^\circ = 27^\circ, (-27^\circ)$	B1	
	$\theta = 39^\circ$	B1	
	$\theta = 12^\circ$	B1	
			-1 each extra root up to 2
			Ignore roots outside $0° < \theta < 180°$

Q	Solution	Mark	Notes
13	Required term = $\binom{5}{3}(2)^{5-3}(-3)^3$	B1	$\binom{5}{3}$ oe
		B1	$(2)^{5-3}$ oe
		B1	$(-3)^3$ oe
	Required term = $10 \times 4 \times (-27)$		
	Required term = -1080	B1	ISW

Q	Solution	Mark	Notes
14(a)	Attempt to differentiate	M1	
	$f'(x) = 9x^2 - 10x + 1$	A1	
	$9x^2 - 10x + 1 = 0$	m1	
	$(9x - 1)(x - 1) = 0$		
	$x = \frac{1}{9}, y = -\frac{1445}{243} = -5.9465$	A1	or $x = \frac{1}{9}, 1$
	$x = 1, y = -7$	A1	all correct
	$f''(x) = 18x - 10$	M1	oe ft quadratic $f'(x)$
	$x = \frac{1}{9}$,($f(x) = -5.9465$) is a maximum	A1	ft their x value
	$x = 1$, ($f(x) = -7$) is a minimum	A1	ft their x value provided different conclusion
	Note: if $f''(x)$ is incorrectly found from their $f'(x)$, maximum marks M1A1A0		
14(b)(i)	Rewriting the equation		
	To give $f(x) = 3x^3 - 5x^2 + x - 6$ on one side.	M1	oe
	$3x^3 - 5x^2 + x - 6 = -7$,		
	2 (distinct roots)	A1	
14(b)(ii)	To give $f(x) = 3x^3 - 5x^2 + x - 6$ on one side	M1	oe
	$3x^3 - 5x^2 + x - 6 = -6.5$		
	3 (distinct roots)	A1	

Note: 14b – 0 marks for unsupported answers

Q	Solution	Mark	Notes
15	$\frac{(x^2y)^3}{x^2y^2} \times \frac{9}{x^2y^2} = 36$	B1	one use of subtraction law
		B1	one use of addition law
		B1	one use of power law
	$4y = x^2$	B1	oe for a correct equation after the removal of logs
	$\log_a\left(\frac{y}{x+3}\right) = \log_a 1$	(B1)	for use of the subtraction law if not previously awarded.
	$y = x + 3$	B1	or $x = y - 3$
	$4y = 4x + 12 = x^2$	M1	or $4y = (y - 3)^2$
	$x^2 - 4x - 12 = 0$		or $y^2 - 10y + 9 = 0$
	$(x + 2)(x - 6) = 0$		or $(y - 1)(y - 9) = 0$
	$x = -2, 6$	A1	cao or $y = 1, 9$ or 1 correct pair
	$y = 1, 9$	A1	cao or $x = -2, 6$ all correct
	$x = -2$ and $y = 1$, $x = 6$ and $y = 9$		
OR			
	$3(2\log_a x + \log_a y) - 2(\log_a x + \log_a y)$	(B1B1B1)	one for each use of laws
	$+ \log_a 9 - 2(\log_a x + \log_a y) = \log_a 36$	(B1)	correct equation
	$2\log_a x - \log_a y = \log_a 4$		
	$\log_a y - \log_a (x + 3) = 0$		
	$2\log_a x - \log_a (x + 3) = \log_a 4$	(M1)	solve simultaneously
	$x^2 - 4x - 12 = 0$	(A1)	
	$(x + 2)(x - 6) = 0$		
	$x = -2, 6$	(A1)	
	$y = 1, 9$	(A1)	
	$x = -2$ and $y = 1$, $x = 6$ and $y = 9$		

Q	Solution	Mark	Notes
16(a)	$\lvert\mathbf{a}\rvert = \sqrt{2^2 + 1^2}$	M1	correct method
	$\lvert\mathbf{a}\rvert = \sqrt{5}$		
	Required unit vector = $\frac{1}{\sqrt{5}}(2\mathbf{i} - \mathbf{j})$	A1	
16(b)	$\theta = \tan^{-1}(\pm 3)$	M1	
	$\theta = (\pm)71.6°\ (288.4°)$	A1	Accept 72° or 288°
16(c)(i)	$\mu\mathbf{a} + \mathbf{b} = \mu\,(2\mathbf{i} - \mathbf{j}) + (\mathbf{i} - 3\mathbf{j})$	B1	Mark final answer
	$\mu\mathbf{a} + \mathbf{b} = (2\mu + 1)\mathbf{i} - (\mu + 3)\mathbf{j}$		
16(c)(ii)	If parallel to $4\mathbf{i} - 5\mathbf{j}$,		
	$(2\mu + 1)\mathbf{i} - (\mu + 3)\mathbf{j} = k(4\mathbf{i} - 5\mathbf{j})$	M1	or $k\,((2\mu + 1)\mathbf{i} - (\mu + 3)\mathbf{j}) = (4\mathbf{i} - 5\mathbf{j})$ Both sides in terms of **i** and **j**
	$2\mu + 1 = 4k$ and $\mu + 3 = 5k$	A1	ft (c)(i)
	Solving simultaneously	m1	any correct method
	$(k = \frac{5}{6})$		
	$\mu = \frac{7}{6}$	A1	cao
	<u>Alternative solution</u>		
	If parallel to $4\mathbf{i} - 5\mathbf{j}$,		
	$\frac{2\mu+1}{\mu+3} = \frac{4}{5}$	(M1A1) ft (c)(i)	
	$10\mu + 5 = 4\mu + 12$	(m1)	
	$6\mu = 7$		
	$\mu = \frac{7}{6}$	(A1)	cao

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UNIT 2 APPLIED MATHEMATICS A

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SECTION A – Statistics

Qu. No.	Solution	Mark	Notes
1(a)	$\mathrm{P}(A \cap B) = 0.3 + 0.6 - 0.82$	M1	Use of addition formula
	$\mathrm{P}(A \cap B) = 0.08$	A1	Sight of 0.08 in a Venn diagram earns M1A1.
	$\mathrm{P}(A) \times \mathrm{P}(B) = 0.3 \times 0.6 = 0.18$	B1	
	Since $0.18 \neq 0.08$, A and B are not independent.	E1	FT their $P(A)P(B)$ and $P(A \cap B)$ provided one is correct. No FT for negative probabilities.
	Alternative: If A and B are independent $\mathrm{P}(A) \times \mathrm{P}(B) = 0.3 \times 0.6 = 0.18$	(B1)	
	$\mathrm{P}(A \cup B) = 0.3 + 0.6 - 0.18$	(M1)	
	$\mathrm{P}(A \cup B) = 0.72$	(A1)	
	Since $0.72 \neq 0.82$, A and B are not independent.	(E1)	FT their $P(A)P(B)$ and $P(A \cup B)$ provided one is correct. No FT for negative probabilities.
(b)	P(Exactly one of A and B) $= P(A \cup B) - \mathrm{P}(A \cap B)$	M1	Writing or using formula.
	$= 0.82 - 0.08$	M1	FT 'their 0.08' providing it is a valid probability ≠ 0.18
	$= 0.74$	A1	CAO
	Alternative 1: P(Exactly one of A and B) $= P(A \cap B') + P(B \cap A')$. OR Sight of 0.22 or 0.52	(M1)	FT 'their 0.08' provided it is a valid probability ≠ 0.18
	P(Exactly one of A and B) $= 0.22 + 0.52$	(M1)	Both values correct
	P(Exactly one of A and B) $= 0.74$	(A1)	CAO
	Alternative 2: P(Exactly one of A and B) $= \mathrm{P}(A) + P(B) - 2\mathrm{P}(A \cap B)$	(M1)	
	$= 0.3 + 0.6 - 2 \times 0.08$	(M1)	FT 'their 0.08' providing it is a valid probability ≠ 0.18.
	$= 0.74$	(A1) Total: **[7]**	CAO

2 (a)	Height $= 40 \times 0.45 \div 3$	M1	M1 for $40 \times 0.45 \div (\text{their width})$
	$= 6$ units	A1	
(b)	Valid explanation. e.g., We don't know how the probability is distributed within the two groups.	E1	E0 for an explanation that refers to the probability of individual integer values of X e.g., we don't know the probability that $X = 268$. E0 for "We don't know the probability" Condone "We don't know how it is distributed within the two groups."
(c)	Valid explanation. Must imply different samples are being considered. e.g., Different samples will lead to different results. e.g., The lifetimes of the light bulbs that Celyn collects will be different from those considered in (a). e.g., If the differences are big enough this would suggest that something might have gone wrong. This explanation is the exception to the requirement to refer to samples or collections of light bulbs.	E1 **Total [4]**	Allow explanations that compare a sample with the expected values, e.g. The histogram drawn from the table of probabilities only shows the expected values, whereas the histogram that Celyn draws represents a single sample (of 40 light bulbs). Condone histogram drawn using different intervals. E0 for every light bulb is different. E0 for anything that implies it's from a distribution other than the one in the question. e.g., It might be a different type of lightbulb.

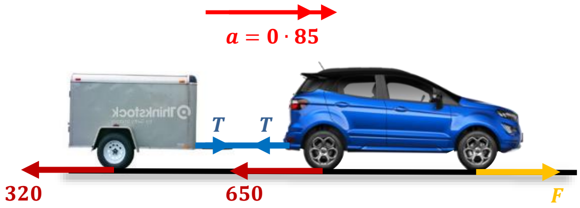
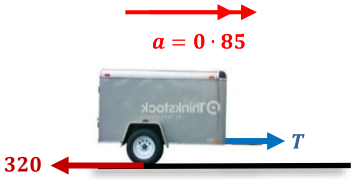
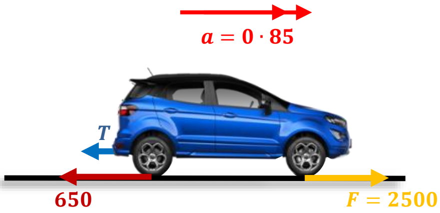
3(a)	**Earthworms** need to occur • at a uniform/constant average **rate**. **AND one of** • **independently** / **singly** / **randomly.**	E1	Accept equivalent statement.
(b)	The number of earthworms X is $Po(2.75)$.	B1	si
	$\mathrm{P}(X=5)=\frac{2.75^5 \times \mathrm{e}^{-2.75}}{5!}$	M1	FT their derived mean or 11 FYI $P(X=5)=0.0224$ for $\lambda=11$ $P(X=5)$ correct for any λ other than 2.75 earns M1A0
	$\mathrm{P}(X=5)=0.0837861\ldots$	A1	CAO M1A1 for use of calculator 3sf (awrt 0.0838)
(c)	$\mathrm{P}(X \leq 12)=0.3585$	M1	M1A0 for 13.98 or 13.99
	$\lambda=14$	A1 **Total [6]**	If no marks awarded, SC1 for $\lambda=15$ Sight of $\lambda=14$ earns M1A1

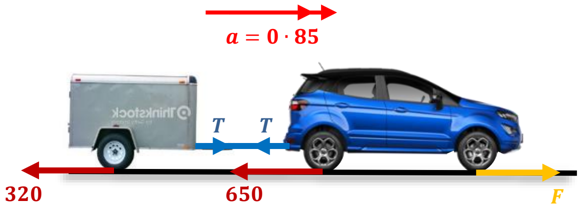
4(a)(i)	$64 + (64 - 49.5) \times 1.5$ $(= 64 + 14.5 \times 1.5 = 85.75)$	M1	Use of rule $Q_3 + 1.5\text{IQR}$
	$87.2 > 85.75$ therefore, it's an outlier.	A1	Correct calculation and conclusion.
(ii)	Valid reason. e.g. May distort calculations and therefore inferences about population.	E1	Condone "so that the mean will not be affected by outliers." Allow "there may have been an error in measurement". Allow "may skew the data". E0 for "median does not change". E0 for "make calculations more reliable".
(iii)	Valid reason. e.g., It's only just an outlier. Still a valid measurement so should include it.	E1	Condone "to use all of the data".
(b)	The larger the hip girth, the larger the thigh girth, **on average**.	E1	oe Condone 'tends' or 'in general' in place of on average. Penalise first omission of on average only.
(c)	Each increase of 1 cm of hip girth corresponds to a 0.69 cm increase in thigh girth **on average**.	E1	Penalise first omission of on average only. Watch out for hip girth and thigh girth in the wrong order.
(d)	Using all of the data instead of a sample would lead to more accurate results.	E1 **Total [7]**	Condone increase sample size provided no nonsensical statements follow. Do not allow reference to other sampling methods.

5(a)	$H_0: p = 0.7$ $H_1: p < 0.7$	B1	Allow other letters if defined. Allow worded hypotheses or use of 70%. B0 for 0.7% B0 for omission of p or for a non-strict inequality in H_1
(b)(i)	The critical region is the **range of values** of the **number of people** that know the name of the company that would lead us to reject H_0.	E1	Condone "The critical region is the **range of values** of the **test statistic** that would lead us to reject H_0."
(ii)	Under H_0, $X \sim \mathrm{B}(60, 0.7)$ si $P(X \leq 35) = 0.0362$	B1 M1	Award if seen in part (iii) M0 for $P(X = k)$ FT their hypotheses
	CR $X \leq 35$	A1	Do not accept as probability statement, i.e. $P(X \leq 35)$. CAO
(iii)	40 is not in the critical region so there is insufficient evidence to reject H_0.	M1	Allow use of p-value method in part (iii), $P(X \leq 40) = 0.3308$ and correct comparison with 0.05. M0 for conclusion based on $P(X = k)$ p-value not in critical region earns M0A0 FT their hypotheses
	There is insufficient evidence to say that fewer than 70% of participants know the name of the sponsoring company.	A1	Do not allow categorical statements *without reference to insufficient evidence or suggests.* FT their hypotheses
(c)	Valid comment with a reason e.g., It's worth sponsoring the event because the result of the hypothesis test suggests it is an effective way of getting brand recognition. e.g., It's unclear whether the brand recognition provides the necessary monetary compensation for the sponsorship money. e.g., The test implies that it's likely that a reasonable proportion of participants know the name of the sponsor so it may be worth doing. e.g., *Run4Lyfe* may be concerned about the proportion who know the name of the sponsor and so may wish to discontinue sponsorship. e.g., They may feel that the evidence is inconclusive and so my wish to continue for another year. e.g., Continue with their existing approaches as there is insufficient evidence to substantiate their concern.	E1 **Total [8]**	FT based on their (possible incorrect) test conclusion. Must mention sponsorship (or imply it). Do not allow e.g. "Need to advertise more" without valid justification that refers to the conclusion reached in (b).

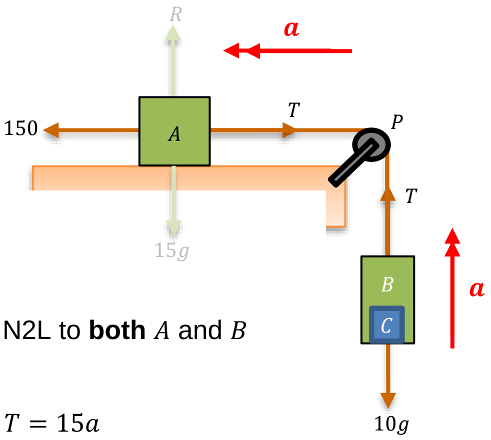
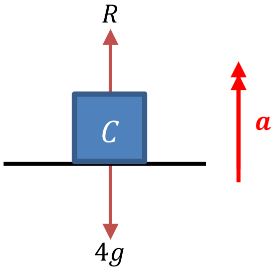
6(a)	Two valid comments. e.g. 1914 is negatively skewed. 2014 is positively skewed. Fertility rates were larger in 1914 than 2014 on average.	E2	Condone 1914 is skewed to the left. 2014 is skewed to the right. Allow “people tended to have more children in 1914 than in 2014”. E1 for each valid comment.
(b)	That fertility rates are increasing in (at least) one country i.e., women had more babies on average in 2014 than 1914 in that country.	E1	E0 for decreasing by -0.61.
(c)(i)	Attempt to find decrease for either.	M1	Do not allow values other than 2.5 and 6.5 without a valid justification.
	Approximately $2.5 - 1.98 = 0.52$	A1	Allow in percentage terms i.e., France fell by 20.8%
(ii)	Approximately $6.5 - 4.4 = 2.1$	A1	Allow in percentage terms, i.e., Ethiopia fell by 32.3%
(iii)	Valid reason must address the decrease. e.g., Countries with a higher fertility rate in 1914 have more of an opportunity for it to decrease. e.g., Ethiopia is a developing country and its fertility rate is likely to have decreased more rapidly in the last 100 years than France which is a developed country.	E1	Do not allow comparison of birth rates in both countries.
(iv)	Valid explanation. e.g., We have used the midpoint of the group to estimate. e.g., We have no way of knowing what the exact fertility rates of France and Ethiopia are in 1914. e.g., Exact fertility rates in 1914 are unknown.	E1 **Total [8]**	Allow responses that imply that the fertility rate may not have been measured accurately, e.g., data collection methods may differ between the countries or across the 100 years.

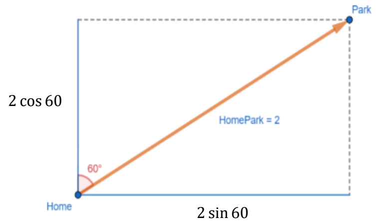
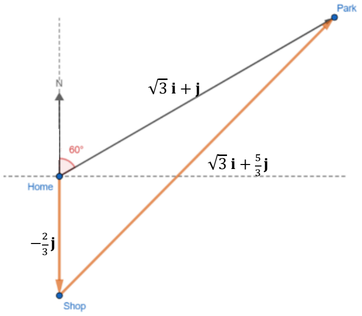
SECTION B – Mechanics

Q7	Solution	Mark	Notes
	Method 1 (Combining as one particle)		
			
	Apply N2L to vehicle **and** trailer **combined**	M1	Dimensionally correct equation F and 970 opposing
	$F-(650+320)=(1300+500)a$	A1	
	$F-970=1800(0\cdot85)$ $(F-970=1530)$		
	$F=2500$	A1	Convincing
			
	Apply N2L to trailer	M1	Dimensionally correct equation T and 320 opposing
	$T-320=500a$	A1	
	$T-320=500(0\cdot85)$ $(T-320=425)$		
	$T=745$ (N)	A1	cao
		[6]	
	<u>Alternative solution for finding T</u>		
			
	Apply N2L to vehicle	(M1)	Dimensionally correct equation with all forces, T and 2500 opposing, ±650
	$2500-650-T=1300a$	(A1)	
	$2500-650-T=1300(0\cdot85)$ $(1850-T=1105)$		
	$T=745$ (N)	(A1)	cao
	Total for Question 7	6	

Q7	Solution	Mark	Notes
	Method 2 (Car and Trailer separate particles)		M1 available once for N2L with this method
			
	Apply N2L to trailer	M1	Dimensionally correct equation T and 320 opposing
	$T - 320 = 500a$	A1	
	$T - 320 = 500(0 \cdot 85)$ $(T - 320 = 425)$		
	$T = 745$ (N)	A1	
	Apply N2L to vehicle	(M1)	Dim. correct. All terms. F and T opposing, ± 650
	$F - 650 - T = 1300a$	A1	
	$F - 650 - 745 = 1300(0 \cdot 85)$ $(F - 1395 = 1105)$	m1	For substituting their T
	$F = 2500$	A1	Convincing
		[6]	
	Alternative Solution using elimination of T		
	Apply N2L to trailer	M1	Dimensionally correct equation T and 320 opposing
	$T - 320 = 500a$	A1	
	Apply N2L to vehicle	(M1)	Dim. correct. All terms. F and T opposing, ± 650
	$F - 650 - T = 1300a$	A1	
	Adding	m1	Eliminating T
	$F - 970 = 1800a$		
	$F - 970 = 1800(0 \cdot 85)$ $(F - 970 = 1530)$		
	$F = 2500$	A1	Convincing
	$T = 745$ (N)	A1	cao
		[6]	
	Total for Question 7	6	

Q8	Solution	Mark	Notes
(a)	$v = u + at,\ u = 4, a = 1\cdot 5, t = 8$ $v = 4 + (1\cdot 5)(8)$ $v = 16 \quad (\text{ms}^{-1})$	M1 A1 **[2]**	Used cao
(b)	$v^2 = u^2 + 2as,\ v = 78, u = 4, a = 1\cdot 5$ $(78)^2 = (4)^2 + 2(1\cdot 5)s \quad (6084 = 16 + 3s)$ minimum $AB, s = \frac{6068}{3} = 2022\cdot 66 \ldots \quad (\text{m})$	M1 A1 A1 **[3]**	Used, FT their velocity from (a) cso, allow answer rounding to 2020 (3sf)
	Total for Question 8	5	

Q9	Solution	Mark	Notes
(a)			
	Apply N2L to **both** A and B	M1	Dimensionally correct equation for at least 1 object T and $10g/150$ opposing
	$150 - T = 15a$	B1	1st correct equation
	$T - 10g = 10a$	A1	2nd correct equation
	Eliminating T $150 - 10g = 25a$	m1	
	$a = 2\cdot08 \quad (\mathrm{ms}^{-2})$	A1	cao
	$T = 118\cdot8 \quad (\mathrm{N})$	A1	FT their a if substituted into a correct equation
		[6]	
(b)			
	Apply N2L to C	M1	Dimensionally correct equation R and $4g$ opposing
	$R - 4g = 4a$ $R = 4(2\cdot08) + 4(9\cdot8)$	A1	
	$R = 47\cdot52$ (N)	A1	FT their a
		[3]	
	Total for Question 9	9	

Q10	Solution	Mark	Notes
(a)	 		
	$\mathbf{r}_{park} = (2\sin 60)\mathbf{i} + (2\cos 60)\mathbf{j}$	M1	Allow sin/cos error $\mathbf{i}, \mathbf{j}$ not necessary with supporting diagram
	$\left(= 2\left(\frac{\sqrt{3}}{2}\right)\mathbf{i} + 2\left(\frac{1}{2}\right)\mathbf{j}\right)$		
	$= \sqrt{3}\,\mathbf{i} + \mathbf{j}$	A1	Convincing
		[2]	
(b)	 		
	$SP = \sqrt{3}\,\mathbf{i} + \mathbf{j} - \left(-\frac{2}{3}\mathbf{j}\right)$		
	$= \sqrt{3}\,\mathbf{i} + \frac{5}{3}\mathbf{j}$	B1	si, or $SP = -PS$ $\mathbf{i}, \mathbf{j}$ not necessary with supporting diagram
	$\lvert SP\rvert = \sqrt{\left(\sqrt{3}\right)^2 + \left(\frac{5}{3}\right)^2} \quad \left(= \frac{2\sqrt{13}}{3} = 2\cdot 4037\ldots\right)$	M1	
	distance travelled $= \frac{2\sqrt{13}}{3} + \frac{2}{3} = 3\cdot 0703\ldots \quad (\mathrm{km})$	A1	cao
		[3]	
	<u>Alternative Solution (Cosine Rule)</u> $SP^2 = \left(\frac{2}{3}\right)^2 + (2)^2 - 2\left(\frac{2}{3}\right)(2)\cos 120^\circ$	M1	$SP^2 = \frac{52}{9}$
	$\lvert SP\rvert = \frac{2\sqrt{13}}{3} = 2\cdot 4037\ldots$	A1	
	distance travelled $= \frac{2\sqrt{13}}{3} + \frac{2}{3} = 3\cdot 0703\ldots \quad (\mathrm{km})$	A1	cso, allow answer rounding to $3\cdot 1$ (1 dp)
		[3]	
(c)	Any sensible assumption, for example • Unlikely to walk in straight lines • Actual route may not be a straight line • Route may not be flat (be hilly)	E1	
		[1]	
	Total for Question 10	**6**	

Q11	Solution	Mark	Notes
(a)	At rest, $v = 0$ $3t^2 - 24t + 36 = 0$ $3(t-2)(t-6) = 0$ $t = 2, 6 \quad (\mathrm{s})$	M1 A1 **[2]**	Used cao
(b)	Velocity decreasing $\frac{\mathrm{d}v}{\mathrm{d}t} = a = 6t - 24 \quad (< 0),$ $t < 4$ For $0 < t < 2$, Distance $= \int_0^2 (3t^2 - 24t + 36)\mathrm{d}t$ $= [t^3 - 12t^2 + 36t]_0^2$ $= 32$ For $2 < t < 4$, Distance $= -\int_2^4 (3t^2 - 24t + 36)\mathrm{d}t$ $= -[t^3 - 12t^2 + 36t]_2^4$ $= -[-16]$ $= 16$ Required distance $= 32 + 16$ $= 48$	M1 A1 M1 A1 A1 A1 A1 **[7]**	Attempt to differentiate, at least one term correct, oe si Not dependent on above M1 Use of integration, limits not needed, at least one term correct Correct integration Condone –16 cao
Total for Question 11		9	

GCE A LEVEL MARKING SCHEME

SUMMER 2022

A LEVEL (NEW)
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL MATHEMATICS

UNIT 3 PURE MATHEMATICS B

SUMMER 2022 MARK SCHEME

Q	Solution	Mark	Notes
1	$6(1 + \tan^2 x) - 8 = \tan x$	M1	use of $\sec^2 x = 1 + \tan^2 x$ Must be seen for M1
	$a\tan^2 x + b\tan x + c = 0$		
	$6\tan^2 x - \tan x - 2 = 0$		
	$(A\tan x + B)(C\tan x + D) = 0$	m1	$AC = a$ and $BD = c$, $c \neq 0$ oe
	$(3\tan x - 2)(2\tan x + 1) = 0$		
	$\tan x = -\frac{1}{2}, \frac{2}{3}$	A1	cao
	$\tan x = \frac{2}{3}$, $x = 33.69°, 213.69°$	B1	first 2 correct solutions Condone 0.588^c, 3.730^c
	$\tan x = -\frac{1}{2}$, $x = 153.43°$,	B1	3rd correct solution Condone 2.678^c
	$x = 333.43°$	B1	4th correct solution Condone 5.820^c

Notes: If one or two roots obtained for tan x, even if incorrectly obtained, full follow through from these values for B1 B1 B1, provided one +ve and one -ve root. If only one sign obtained, only B1 available for one pair of correct angles.

Do not follow through for sin, cos or anything else.

Ignore all roots outside range $0° \leq x \leq 360°$.

For 5th, 6th, 7th extra root within range, -1 mark each extra root.

If all answers in radians, but radians **not** specified, penalise -1.

Accept all answers correctly rounded to the nearest whole number or better.

Q	**Solution**	**Mark**	**Notes**
2(a)	$y = x^3\ln(5x)$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2\ln(5x) + x^3\frac{5}{5x}$	M1	$f(x)\ln(5x) + x^3g(x)$
			M0 if $f(x) = 0$ or1 or $g(x) = 0$ or 1
		A1	$3x^2\ln(5x)$
		A1	$x^3\frac{5}{5x}$
			ISW
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2\ln(5x) + x^2 = x^2(3\ln(5x) + 1)$		
2(b)	$y = (x + \cos 3x)^4$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4(x + \cos 3x)^3(1 - 3\sin 3x)$	M1	$4(x + \cos 3x)^3\ f(x)$
			M0 if $f(x) = 1$
		A1	$f(x) = (1 - 3\sin 3x)$
			Condone absence of brackets
			for M1 A0, unless corrected for A1.
			ISW

Q	Solution	Mark	Notes
3	$OB\left(=\frac{4}{\cos\frac{\pi}{3}}\right) = 8$ or $OA\ \left(=\frac{4}{\tan 30°}\right) = 4\sqrt{3}$	B1	si ($OA = 6.928….$)
	Area $OAB = \frac{1}{2}\times 4\times 8\sin\frac{\pi}{3}$	M1	Use of $A = \frac{1}{2}\times AB\times OA$
	$= 8\sqrt{3} = 13.856…$		
	Area $OBC = \frac{1}{2}\times 8\times 8\times\frac{\pi}{3}$	M1	Use of $A = \frac{1}{2}r^2\theta$
			Or $A = \frac{1}{6}\pi r^2$
	$= \frac{32\pi}{3} = 33.510…$		ft OB, OA
	Required area $OABC$ = 47.37 (m^2)	A1	cao Must be to 2dp

Q	Solution	Mark	Notes
4	$\frac{a}{1-r} = 120$	B1	si
	$\frac{a}{1-4r^2} = 112\frac{1}{2}$	B1	si
	$120(1 - r) = \frac{225}{2}(1 - 4r^2)$	M1	or elimination of *r*
	$900r^2 - 240r + 15 = 0$ or $a^2 - 208a + 10800 = 0$	m1	attempt to solve their quadratic equation Implied by correct answers
	$60r^2 - 16r + 1 = 0$		
	$(6r - 1)(10r - 1) = 0$		
	$r = \frac{1}{6}, r = \frac{1}{10}$	A1	One correct pair, cao
	$a = 100, a = 108$	A1	all correct, cao

Q	**Solution**	**Mark**	**Notes**
5(a)	$\left(\frac{6x+4}{(x-1)(x+1)(2x+3)}=\right)\ \frac{A}{(x-1)}+\frac{B}{(x+1)}+\frac{C}{(2x+3)}$	M1	correct form
			Implied by equation below
	$6x+4=A(x+1)(2x+3)+B(x-1)(2x+3)$		
	$+\,C(x+1)(x-1)$	M1	si correct equation
	Put $x=-1$, $-2=B(-2)(1)$		
	$B=1$		
	Put $x=-\frac{3}{2}$, $-9+4=C(-\frac{1}{2})(-\frac{5}{2})$		
	$C=-4$	A1	two correct constants
	Put $x=1$, $10=A(2)(5)$		
	$A=1$	A1	third constant correct
	$f(x)=\frac{1}{(x-1)}+\frac{1}{(x+1)}-\frac{4}{(2x+3)}$		
5(b)	$\int\frac{3x+2}{(x-1)(x+1)(2x+3)}\,\mathrm{d}x$		
	$=\int\frac{1}{2}\left[\frac{1}{(x-1)}+\frac{1}{(x+1)}-\frac{4}{(2x+3)}\right]\mathrm{d}x$		
	$=\frac{1}{2}[\ln\|x-1\|+\ln\|x+1\|-2\ln\|2x+3\|\ (+\ln C)]$	B3	B1 correct int of $\frac{1}{(x-1)}$
			B1 correct int of $\frac{1}{(x+1)}$
			B1 correct int of $\frac{K}{(2x+3)}$
			Condone no modulus signs for B3
		M1	attempt to tidy up into one ln term M0 if extra terms seen
	$=\frac{1}{2}\left[\ln\left\|\frac{C(x+1)(x-1)}{(2x+3)^2}\right\|\right]$ or $\left[\ln\left\|\frac{\sqrt{C(x+1)(x-1)}}{(2x+3)}\right\|\right]$	A1	cao accept $+C$
			A0 if no C. ISW

Q	**Solution**	**Mark**	**Notes**
6(a)	$T_{12} = 10 + (12 - 1) \times 0.2$	M1	use of $a + (n - 1)d$ Allow d = 20 for M1. Implied by correct answer.
	T_{12} = £12.20	A1	
6(b)	(954 =) $\frac{n}{2}[2 \times 10 + (n - 1) \times 0.2]$	M1	use of $\frac{n}{2}[2a + (n - 1)d]$ Allow d = 20 for M1.
	$9540 = n[100 + n - 1]$		
	$n^2 + 99n - 9540 = 0$	m1	equating to 954 and writing as quadratic Implied by n = 60
	$(n - 60)(n + 159) = 0$		
	$n = 60$	A1	cao Dependent on M1 A0 if n = –159 present in final answer
	60 (months)		

Q	Solution	Mark	Notes
7	$x^2 = 8\sqrt{x}$ or $y = \left(\frac{y^2}{64}\right)^2$	M1	equating y's
	$x^4 = 64x$ or $y^4 = 4096y$		
	$x(x^3 - 64) = 0$ or $y(y^3 - 4096) = 0$	A1	oe e.g. $x^{\frac{3}{2}} = 8$
	$x = (0,)\ 4$ or $y = (0,)\ 16$	A1	si, 0 not required.
	Area = $\int_0^4 \left(8x^{\frac{1}{2}} - x^2\right) \mathrm{d}x$	M1	oe allow $x^2 - 8x^{\frac{1}{2}}$ limits not required
	Area = $\left[\frac{16}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right]_0^4$	A1	one correct term Must be seen
		A1	other correct term
	Area = $\frac{16}{3} \times 8 - \frac{1}{3} \times 64$		
	Area = $\frac{64}{3}$	A1	cso A0 if integral gives negative answer, unless corrected without any incorrect statements.

<u>Alternative Solution for last 4 marks</u>

	$A = \int_0^4 8x^{\frac{1}{2}}\,\mathrm{d}x$	(M1)	
	$= \left[\frac{16}{3}x^{\frac{3}{2}}\right]_0^4$	(A1)	Must be seen
	$= \frac{16}{3} \times 8 \ \left(= \frac{128}{3}\right)$		
	$B = \int_0^4 x^2\,\mathrm{d}x$		Award M1 if not awarded previously
	$= \left[\frac{1}{3}x^3\right]_0^4$	(A1)	Must be seen
	$\left(= \frac{64}{3}\right)$		
	Required area = $A - B = \frac{64}{3}$	(A1)	

<u>Note</u>: Answer only, M1 A0 A0 A0

Q	**Solution**	**Mark**	**Notes**
8	$\frac{2-x}{\sqrt{1+3x}} = (2 - x)(1 + 3x)^{-1/2}$		
	$(1 + 3x)^{-1/2} = (1+ \left(-\frac{1}{2}\right)(3x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(3x)^2+\ldots)$	B1	$1+ \left(-\frac{1}{2}\right)(3x)$
		B1	$\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(3x)^2$
	$\frac{2-x}{\sqrt{1+3x}} = (2 - x)(1 - \frac{3}{2}x + \frac{27}{8}x^2 + \ldots.)$		
	$= 2 - 3x + \frac{27}{4}x^2 - x + \frac{3}{2}x^2 +\ldots$		
	$= 2 - 4x + \frac{33}{4}x^2 + \ldots$	B3	B1 each term
			Ignore further terms, ISW
	Expansion valid for $\lvert 3x\rvert < 1$		
	$\lvert x\rvert < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$	B1	B1 for $x < \frac{1}{3}$ and $x > -\frac{1}{3}$
			B0 anything else
	When $x = \frac{1}{22}$,		
	$\frac{2-\frac{1}{22}}{\sqrt{1+\frac{3}{22}}} \approx 2 - \frac{4}{22} + \frac{33}{4}\left(\frac{1}{22}\right)^2$	M1	sub into LHS and RHS
	$\frac{\frac{43}{22}}{\frac{5\sqrt{22}}{22}} = \frac{43}{5\sqrt{22}} \approx \frac{323}{176}$ or $\frac{43\sqrt{22}}{110} \approx \frac{323}{176}$		
	$\sqrt{22} \approx \frac{7568}{1615}$ or $\frac{1615}{344}$	A1	cao
	(= 4.686068111…, or 4.694767442…, actual value is 4.69041576…)		

Special case for $(1 + 3x)^{1/2}$ used

$(1 + 3x)^{1/2} = (1 + \left(\frac{1}{2}\right)(3x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(3x)^2 + \ldots)$ (B0)

(B0)

$\frac{2-x}{\sqrt{1+3x}} = (2 - x)(1 + \frac{3}{2}x - \frac{9}{8}x^2 + \ldots.)$

$= 2 + 3x - \frac{9}{4}x^2 - x - \frac{3}{2}x^2 + \ldots$

$= 2 + 2x - \frac{15}{4}x^2 + \ldots$ (B3) B1 each term

Ignore further terms, ISW

Expansion valid for $|3x| < 1$

$|x| < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$ (B1) B1 for $x < \frac{1}{3}$ and $x > -\frac{1}{3}$

B0 anything else

Correct substitution (M1)

(A0)

Q	Solution	Mark	Notes
9(a)	$u_1 = \sin\left(\frac{\pi}{2}\right) = 1$		
	$u_2 = \sin\left(\frac{2\pi}{2}\right) = 0$		
	$u_3 = \sin\left(\frac{3\pi}{2}\right) = -1$		
	$u_4 = \sin\left(\frac{4\pi}{2}\right) = 0$		
	$u_5 = \sin\left(\frac{5\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$	B1	All 5 terms
	Sequence is periodic (with period 4)	B1	Condone ‘Repeats every 4 terms’ or ‘Oscillates’
9(b)	$u_5 = 17$	B1	
	$(u_5 = 17)$, $u_4 = 9$, $u_3 = 5$, $u_2 = 3$, $u_1 = 2$	B1	
	Sequence is increasing.	B1	Accept ‘Divergent’

Q	Solution	Mark	Notes
10	$\frac{6x^5-17x^4-5x^3+6x^2}{(3x+2)} = \frac{(x^2)(6x^3-17x^2-5x+6)}{(3x+2)}$	M1	or removing x^2 from pentic
	$= \frac{(x^2)(3x+2)(2x^2-7x+3)}{(3x+2)}$	M1	divide by $(3x + 2)$, or realising $(3x + 2)$ is a factor of the cubic **and** cancelling
		A1	Sight of $(2x^2 - 7x + 3)$
	$= x^2(2x - 1)(x - 3) = 0$	A1	Must be seen
	$x = 0$(twice), $\frac{1}{2}$, 3.	A1	cao A0 if $-\frac{2}{3}$ present

Note: $(6x^3 - 17x^2 - 5x + 6) = (x - 3)(6x^2 + x - 2)$

$(6x^3 - 17x^2 - 5x + 6) = (2x - 1)(3x^2 - 7x - 6)$

Alternative Solution

10	$\frac{6x^5-17x^4-5x^3+6x^2}{(3x+2)} = \frac{(x^2)(6x^3-17x^2-5x+6)}{(3x+2)}$	(M1)	or removing x^2 from pentic
	$= \frac{(x^2)(3x+2)(2x^2-7x+3)}{(3x+2)}$	(M1)	any linear factor or divide by $(3x + 2)$
		(A1)	Sight of $(2x^2 - 7x + 3)$ oe or second factor from factor theorem
	$= x^2(2x - 1)(x - 3) = 0$	(A1)	$(3x + 2)$ must be cancelled or solution discarded
	$x = 0$ (twice), $\frac{1}{2}$, 3.	(A1)	cao A0 if $-\frac{2}{3}$ present

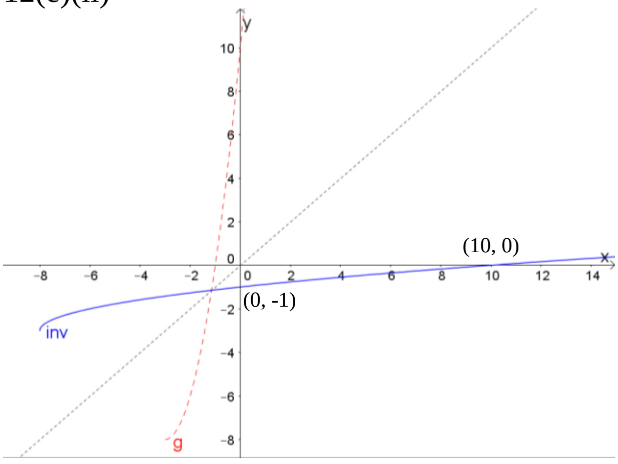
Note: $(6x^3 - 17x^2 - 5x + 6) = (x - 3)(6x^2 + x - 2)$

$(6x^3 - 17x^2 - 5x + 6) = (2x - 1)(3x^2 - 7x - 6)$

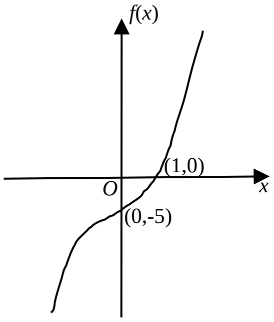
Q	Solution	Mark	Notes
11(a)	$9\cos x + 40\sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$		
	$R\cos\alpha = 9$ and $R\sin\alpha = 40$	M1	implied by correct α if nothing seen. M0 for incorrect equations
	$R = \sqrt{9^2 + 40^2} = 41$	B1	
	$\alpha = \tan^{-1}\left(\frac{40}{9}\right) = 77.32°$	A1	accept 1.349 rad, not 1.349 ft R if $\alpha = \sin^{-1}\left(\frac{40}{R}\right) = \cos^{-1}\left(\frac{9}{R}\right)$
	$9\cos x + 40\sin x \equiv 41\cos(x - 77.32°)$		
11(b)	$y = \frac{12}{9\cos x + 40\sin x + 47}$		
	Maximum y when denominator is minimum,		
	i.e. when $\cos(x - 77.32°) = -1$	M1	implied by correct max
	Max $y \left(= \frac{12}{-41+47}\right) = 2$	A1	ft R

Q	Solution	Mark	Notes
12(a)	$ff(p) = f(0) = 10$	B1	
12(b)	$2x^2 + 12x + 10 = 0$ $2(x^2 + 6x + 5) = 0$ $2(x + 5)(x + 1) = 0$	M1	may be implied by solution
	$p = -5,\ q = -1$	A1	both
12(c)	$f(x) = 2[x^2 + 6x + 5]$		
	$= 2[(x + 3)^2 - 4]$	M1	condone absence of ‘2’
	$= 2(x + 3)^2 - 8$	A1	cao
	Min point at (–3, –8)	B1	
12(d)	$f(x)$ is not a one-to-one function (on its domain).	B1	
12(e)(i)	Let $y = 2(x + 3)^2 - 8$		
	$(x + 3)^2 = \frac{y+8}{2}$	M1	ft similar form from (c)
	$x = -3 \pm \sqrt{\frac{y+8}{2}}$	A1	Condone $x = -3 + \sqrt{\frac{y+8}{2}}$
	since $x \geq -3$, $x = -3 + \sqrt{\frac{y+8}{2}}$	A1	Must discard negative root
	$g^{-1}(x) \ = -3 + \sqrt{\frac{x+8}{2}}$	A1	interchange x and y, could be done earlier

12(e)(ii)



Mark	Notes
B1	Correct shape
B1	(10, 0) (0, -1), cao

Q	Solution	Mark	Notes
13(a)	$f'(x) = 6x^2 + 3$	B1	
	Hence $f'(x) > 0$ for all x,		
	i.e. $f(x)$ does not have a stationary point.	E1	oe
			e.g. $f'(x) = 0$ has no real roots
			discriminant = $0^2 - 4(6)(3) < 0$, no real roots
13(b)	$f''(x) = 12x$	M1	
	At point of inflection $f''(x) = 0$, $x = 0$	m1	
	$f'(x) > 0$ when $x < 0$ and when $x > 0$.		
	Therefore, when $x = 0$,		
	there is a point of inflection.	A1	oe cubic curve no max/min
			must have a point of inflection.
			OR $x > 0$, $f''(x) > 0$; $x < 0$, $f''(x) < 0$
	The point of inflection is (0, −5)	B1	
13(c)			
		G1	cubic curve no max/min
			ft point in (b) coords not required.
			(1,0) not required.
Q	**Solution**	**Mark**	**Notes**

14	$I = [\pm \cos x \, . \, x^2]_0^\pi - \int_0^\pi \pm \cos x \, . \, 2x \, \mathrm{d}x$	M1	attempt at parts, 2 terms, at least one term correct. Limits not required
	$I = [- \cos x \, . \, x^2]_0^\pi - \int_0^\pi - \cos x \, . \, 2x \, \mathrm{d}x$	A1	
	$I = [- \cos x \, . \, x^2]_0^\pi + [\sin x . \, 2x]_0^\pi$ $- \int_0^\pi 2\sin x \, \mathrm{d}x$	A1	correct integration of $\int_0^\pi \pm \cos x \, . \, 2x \, \mathrm{d}x$
	$I = [- \cos x \, . \, x^2]_0^\pi + [2\sin x . \, x]_0^\pi + [2\cos x]_0^\pi$	A1	correct integration of $\int_0^\pi \pm \sin x \, \mathrm{d}x$
	$I = [2x\sin x + (2 - x^2)\cos x]_0^\pi$		
	$I = \pi^2 + 0 + 2(-1 - 1)$	m1	correct use of correct limits Implied by correct answer
	$I = \pi^2 - 4$ (= 5.87)	A1	cao

Note

No marks for answer unsupported by workings.

If integration is incorrect and answer of 5.87 seen with **no working**, m0 A0. If substitution seen m1 is available.

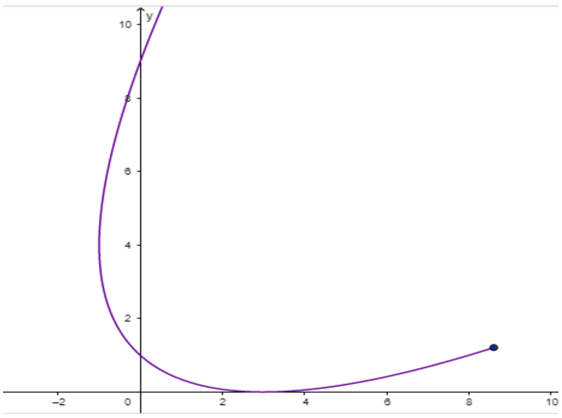
Be careful of use of calculators to obtain correct answer after incorrect integration.

Condone missing d*x*.

M1A0 only for $I = \left[\sin x \, . \, \frac{x^3}{3}\right]_0^\pi - \int_0^\pi \frac{x^3}{3} \cos x \, \mathrm{d}x$

Q	**Solution**	**Mark**	**Notes**
15(a)	$y = \sqrt{16 - x^2}$ OR $A = 2xy$	B1	
	$A = 2x\sqrt{16 - x^2}$	B1	
15(b)	$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}[2x(16 - x^2)^{1/2}]$	M1	$f(x)\ (16 - x^2)^{1/2} + 2xg(x)$
			M0 if $f(x) = 0$ or 1 or $g(x) = 0$ or 1
			Only ft if product with $Bx\sqrt{K - x^2}$
	$\frac{\mathrm{d}A}{\mathrm{d}x} = 2(16 - x^2)^{1/2} + 2x \times \frac{1}{2}(16 - x^2)^{-1/2}(-2x)$	A1A1	one each term, ft (a)
	$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{4}{(16-x^2)^{1/2}}[8 - x^2]$		
	At max, $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$	m1	
	$x^2 = 8$	A1	cao
	$x = 2\sqrt{2}$ (-ve value inadmissible)		
	$y = \sqrt{16 - x^2} = \sqrt{16 - 8} = 2\sqrt{2}$	A1	cao accept $y^2 = 8$
	therefore $y = x$.		
	Justification of maximum	B1	$\frac{\mathrm{d}^2A}{\mathrm{d}x^2} = -22$ when $x = 2\sqrt{2}$
	OR		
	$A^2 = 4x^2(16 - x^2) = 64x^2 - 4x^4$		
	$\frac{\mathrm{d}A^2}{\mathrm{d}x} = 128x - 16x^3$	(M1A1A1)	
	At max, $\frac{\mathrm{d}A^2}{\mathrm{d}x} = 0$	(m1)	
	$x^2 = 8$, $x = 2\sqrt{2}$ (-ve value inadmissible)	(A1)	cao
	$y = \sqrt{16 - x^2} = \sqrt{16 - 8} = 2\sqrt{2}$	(A1)	cao accept $y^2 = 8$
	therefore $y = x$.		
	Justification of maximum	(B1)	$\frac{\mathrm{d}^2A^2}{\mathrm{d}x^2} = -256$ when $x = 2\sqrt{2}$

Q	**Solution**	**Mark**	**Notes**
16(a)	Where C meets the y-axis,		
	$3 - 4t + t^2 = 0$	M1	
	$(t - 1)(t - 3) = 0$		
	$t = 1$, point is (0, 9)	A1	or $t = 1, 3$
	$t = 3$, point is (0, 1)	A1	all correct
16(b)	$\frac{\mathrm{d}y}{\mathrm{d}t} = -2(4 - t)$	B1	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = -4 + 2t$	B1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2(4-t)}{-4+2t}$	B1	ft their dy/dt and dx/dt
	Note: May be seen in (a)		
	At stationary point, $\frac{-2(4-t)}{-4+2t} = 0$	M1	
	$t = 4$		
	At stationary point, $y = (4 - 4)^2 = 0$.		
	Hence the x-axis is a tangent to the curve C.	A1	



Q	**Solution**	**Mark**	**Notes**
17(a)	$\cos(\alpha - \beta) + \sin(\alpha + \beta)$		
	$= \cos\alpha\cos\beta + \sin\alpha\sin\beta + \sin\alpha\cos\beta + \cos\alpha\sin\beta$	B1	expand $\cos(\alpha - \beta)$, $\sin(\alpha + \beta)$
	$=\cos\alpha(\cos\beta + \sin\beta) + \sin\alpha(\cos\beta + \sin\beta)$		
	$=(\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$	B1	convincing
	OR		
	$(\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$		
	$= \cos\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta + \sin\alpha\sin\beta$	(B1)	remove brackets
	$= \cos\alpha\cos\beta + \sin\alpha\sin\beta + \sin\alpha\cos\beta + \cos\alpha\sin\beta$		
	$= \cos(\alpha - \beta) + \sin(\alpha + \beta)$	(B1)	convincing
	OR		
	$\cos(\alpha - \beta) + \sin(\alpha + \beta)$		
	$= \cos\alpha\cos\beta + \sin\alpha\sin\beta + \sin\alpha\cos\beta + \cos\alpha\sin\beta$	(B1)	expand $\cos(\alpha - \beta)$, $\sin(\alpha + \beta)$
	$(\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$		
	$= \cos\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta + \sin\alpha\sin\beta$	(B1)	remove brackets
	Hence $\cos(\alpha - \beta) + \sin(\alpha + \beta)$		
	$= (\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta)$		

Q	**Solution**	**Mark**	**Notes**
17(b)(i)	Put $\alpha = 4\theta$, $\beta = \theta$	M1	
	$\cos(4\theta - \theta) + \sin(4\theta + \theta)$		
	$= (\cos 4\theta + \sin 4\theta)(\cos\theta + \sin\theta)$		
	$\frac{\cos 3\theta + \sin 5\theta}{\cos 4\theta + \sin 4\theta} = \cos\theta + \sin\theta$	A1	convincing
17(b)(ii)	When $\theta = \frac{3\pi}{16}$,		
	$\cos 4\theta + \sin 4\theta = \cos\frac{3\pi}{4} + \sin\frac{3\pi}{4} = 0$		
	So $\frac{\cos 3\theta + \sin 5\theta}{\cos 4\theta + \sin 4\theta}$ is undefined.	B1	oe
	OR		
	$\cos 4\theta + \sin 4\theta \neq 0$		
	$\tan 4\theta \neq -1$		
	$4\theta \neq \frac{3\pi}{4}$		
	$\theta \neq \frac{3\pi}{16}$	B1	

Q	Solution	Mark	Notes
18(a)	Put $u = x + 3$	B1	
	$\int \frac{x^2}{(x+3)^4}\mathrm{d}x = \int \frac{(u-3)^2}{u^4}\mathrm{d}u$	M1	Allow one slip
	$= \int \frac{u^2-6u+9}{u^4}\mathrm{d}u$		
	$= \int (u^{-2} - 6u^{-3} + 9u^{-4})\mathrm{d}u$	A1	integrable form ft $(u + 3)$ only
	$= \frac{u^{-1}}{-1} - \frac{6u^{-2}}{-2} + \frac{9u^{-3}}{-3}(+C)$	A1	correct integration ft $(u + 3)$ only
	$= -\frac{1}{u} + \frac{3}{u^2} - \frac{3}{u^3}(+C)$		
	$= -\frac{1}{x+3} + \frac{3}{(x+3)^2} - \frac{3}{(x+3)^3} + C$	A1	cao Correct expression in terms of x Must include + C
18(b)	$\int_0^1 \frac{x^2}{(x+3)^4}\mathrm{d}x = \left[-\frac{1}{x+3} + \frac{3}{(x+3)^2} - \frac{3}{(x+3)^3}\right]_0^1$		
	$= \left(-\frac{1}{4} + \frac{3}{16} - \frac{3}{64}\right) - \left(-\frac{1}{3} + \frac{1}{3} - \frac{1}{9}\right)$	M1	correct use of correct limits ft for equivalent difficulty for M1 only
	$= \frac{1}{576}$ (= 0.001736)	A1	cao No workings, 0 marks
OR			
	$\int_0^1 \frac{x^2}{(x+3)^4}\mathrm{d}x = \left[-\frac{1}{u} + \frac{3}{u^2} - \frac{3}{u^3}\right]_3^4$		
	$= \left(-\frac{1}{4} + \frac{3}{16} - \frac{3}{64}\right) - \left(-\frac{1}{3} + \frac{1}{3} - \frac{1}{9}\right)$	(M1)	correct use of correct limits ft for equivalent difficulty for M1 only
	$= \frac{1}{576}$ (= 0.001736)	(A1)	cao No workings, 0 marks

GCE A LEVEL MARKING SCHEME

SUMMER 2022

A LEVEL (NEW)
MATHEMATICS
UNIT 4 APPLIED MATHEMATICS B
1300U40-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL MATHEMATICS

UNIT 4 APPLIED MATHEMATICS B

SUMMER 2022 MARK SCHEME

<u>SECTION A – Statistics</u>

Qu. No.	Solution	Mark	Notes
1	Mark for selection to stage three = $66 + k \times 14$	M1	k = 1.645 or better M1 implied by correct answer from calculator. Allow M1 for $\frac{x-66}{14} = 1.645$.
	= 89.03	A1	A1 for sight of either value Condone sight of 89.
	Mark for non-selection = $66 - k \times 14$	(M1)	M1 may be awarded here if not previously awarded. Allow M1 for $\frac{x-66}{14} = -1.645$.
	= 42.97	(A1)	A1 for sight of either value Condone sight of 43.
	Candidates can obtain scores between 43 and 89 in order to be selected for stage two of the interview process.	A1	Must be a range. Accept 42.97 to 89.03 Allow calculation of range between highest and lowest scores. Correct answer only scores M1A1A1 SC1 for 44 to 88 from use of 1.64.
	Total for Question 1	**3**	

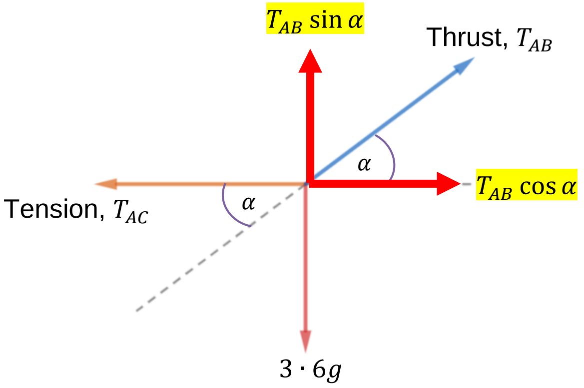
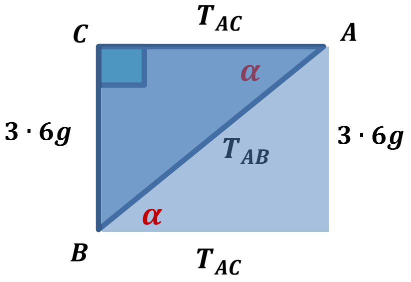
Qu. No.	Solution	Mark	Notes
2(a)	P($F1$) = 0.4 × 0.70 + 0.35 × 0.30	M1	Allow one slip.
	= 0.385	A1	
	Part (a) Total	**[2]**	
(b)	$P(C \mid F1) = \frac{P(C \cap F1)}{P(F1)}$ $= \frac{0.4 \times 0.7}{0.385}$	M1	FT their (a) provided it gives a valid probability as the final answer. If $P(F1) = 1$, must see division by 1.
	= $\frac{8}{11}$ or $0.\dot{7}\dot{2}$	A1	CAO (3sf required) Condone 0.73 from correct working
	Part (b) Total	**[2]**	
(c)	$P(F2') = 0.4 \times 0.8 + 0.35 \times 0.95 + 0.25 \times 0.15$	M1	si Allow one slip OR for $P(F2') = 1 - P(F2)$ with at most one slip in $P(F2)$ calculation
	$P(G \mid F2') = \frac{P(G \cap F2')}{P(F2')}$ $= \frac{0.35 \times 0.95}{0.4 \times 0.8 + 0.35 \times 0.95 + 0.25 \times 0.15}$	M1	Correct numerator (calculation or sight of 0.3325). May be seen as $0.35 \times 0.3 + 0.35 \times 0.65$. (Must be part of fraction)
		m1	Dependent on first M1. Correct denominator (calculation or sight of 0.69) (Must be part of a fraction)
	= $\frac{133}{276}$ or 0.481884….	A1	CAO (3sf required) Condone 0.48 from correct working
	Part (c) Total	**[4]**	
	Total for Question 2	**8**	

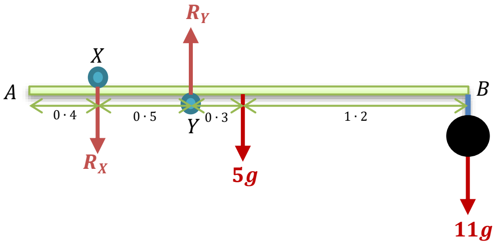
Qu. No.	Solution	Mark	Notes
3(a)	$X \sim \mathrm{U}(0,10)$	M1	Seen, or implied by correct values or calculation of $E(X)$ and $\mathrm{Var}(X)$
	$E(X) = 5$	A1	Must be from correct distribution
	$\mathrm{Var}(X) = \frac{25}{3}$	A1	Must be from correct distribution Condone 8.33 (condone 8.3)
			If no marks awarded, SC1 for $E(X)$ and $\mathrm{Var}(X)$ correct for their uniform distribution (stated or implied, e.g., implied by a diagram or for consistent use of a and b in the mean and variance formulae) e.g., SC1 for $E(X) = 10$ and $\mathrm{Var}(X) = \frac{100}{3}$.
	Part (a) Total	**[3]**	
(b)	$A = X(20 - X)$ $= 20X - X^2$	M1	Stating the area of the rectangle or consideration of relevant products.
	$P(20X - X^2 > 96) = P(X^2 - 20X + 96 < 0)$	M1	Forming a quadratic inequality or equation. Condone omission of $P()$ Stating $X > 8$ with no incorrect working scores M1M1.
	$= P(8 < X < 12)$	A1	Solving quadratic inequality or equation, may be implied by next A1 Condone $P(X > 8)$ if using correct distribution
	$= P(8 < X < 10)$	A1	si (may be implied by a diagram) FT if equivalent difficulty for restricting their range of values for X
	$= \frac{2}{10}$	A1	CSO (correct solution only)
			SC3 (M1M1A1A0A0) for 0.2 from $X \sim U(0,20)$ or with no working.
	Part (b) Total	**[5]**	
Total for Question 3		**8**	

Qu. No.	Solution	Mark	Notes
4(a)	(Let the random variable W be the stopping distance in metres of a car travelling at 30mph.) $W \sim \mathrm{N}(23, 3.8^2)$ $\mathrm{P}(W < 30) = 0.96727$	M1A1	M1 implied by correct answer from calculator or for correctly standardising $Z = \frac{30-23}{3.8} = 1.84$. Gives 0.96712 from tables. 3sf required (0.97 earns M1A0).
	Part (a) Total	**[2]**	
(b)	(Let the random variable X be the stopping distance in metres of a car travelling at 20mph.) $X \sim \mathrm{N}(12, 3.5^2)$ $\mathrm{P}(X > 20) = 0.011135$	M1	Either method correct (see note above). M1 for 2.29 or -0.79 if standardised.
	$\mathrm{P}(W > 20) = 0.78508$	A1	A1 for both probabilities, with at least one probability to 3sf.
	$\frac{0.78508}{0.011135}$	M1	Alternatively, $\frac{0.78508}{50}$ or 0.011135×50. FT their probabilities. Condone division of $P(X < 20)$ by $P(W < 20)$ that leads to 4.6.
	Appropriate conclusion with a valid justification, e.g., You're about 70 times more likely to collide travelling at 30mph than 20mph, so Dafydd is incorrect.	A1	Allow e.g., $0.0157016 \neq 0.011135$ or $0.55675 \neq 0.78508$ so Dafydd is incorrect. FT their calculation for possible M1A1.
	Part (b) Total	**[4]**	
(c)	(Let μ be the population mean stopping distances for cars travelling at 30mph) $H_0: \mu = 23$ $H_1: \mu < 23$	B1	Allow other letters if defined. Allow worded hypotheses. B0 for $H_0: \mathrm{mean} = 23$, must imply or refer to population. B0 for omission of μ or use of $\bar{x}$ B0 for a non-strict inequality in H_1.
	$\bar{X} \sim \mathrm{N}\left(23, \frac{3.8^2}{40}\right)$ under H_0	B1	Distribution of $\bar{X}$ si (condone if used correctly). FT their hypotheses for 2nd B1 only
	$\mathrm{P}(\bar{X} < 21.5 \mid H_0)$	M1	M1 for $\mathrm{P}\left(Z < \frac{21.5-23}{\frac{3.8}{\sqrt{40}}}\right) = \mathrm{P}(Z < -2.50)$, -2.50 scores M1 only if using the p-value method
	$= 0.0062706 \ldots$	A1	0.00621 from tables, M0A0 for use of 21.5 and 23 the wrong way around.
	Since $0.00627 < 0.01$, there is sufficient evidence to reject H_0.	m1	Dependent on previous M1. FT their p-value. m0 for incorrect comparison such as p-value is in the critical region.
	Alternative 1: CV = 21.602	(M1A1)	M1 implied by correct answer from calculator or for correctly standardising $\frac{\mathrm{CV}-23}{\frac{3.8}{\sqrt{40}}} = -2.3263$
	Since $21.5 < 21.602$, there is sufficient evidence to reject H_0.	(m1)	Dependent on previous M1. FT their CV. m0 for incorrect comparison such as CV is less than significance level.
	Alternative 2: TS = $\frac{21.5-23}{\frac{3.8}{\sqrt{40}}}$	(M1)	
	$= -2.50$	(A1)	-2.50 scores M1A1 if used as a TS
	Since $-2.50 < -2.326$, there is sufficient evidence to reject H_0.	(m1)	Dependent on previous M1. FT their TS. m0 for incorrect comparison such as the TS is less than significance level. Condone accept H_1.
	There is sufficient evidence to suggest that stopping distances are less than previously thought.	A1	CSO (correct solution only). Do not allow categorical statements (condone categorical if "sufficient evidence" seen in m1 statement). Allow equivalent statements, e.g., there is sufficient evidence to support the claim.
	Part (c) Total	**[6]**	
(d)	Valid limitation, e.g. These are likely to be mostly young people which may mean they have a faster reaction time than average.	E1	Must address (young people having) faster reaction times. Condone reference to bias in the sample. Minimum response condoned "Only first year students used" or "Inexperienced drivers". Do not allow reference to sample size. Do not allow reference to driving slower.
	Part (d) Total	**[1]**	
	Total for Question 4	**13**	

Qu. No.	Solution	Mark	Notes
5(a)	(Let ρ denote the population correlation coefficient between average house price and average score in the national reading test). $H_0: \rho = 0$ $H_1: \rho > 0$	B1	Allow other letters if defined. Allow worded hypotheses. B0 for H_0: correlation = 0. Population must be stated or implied. B0 for omission of ρ or use of r B0 for a non-strict inequality in H_1
	TS = 0.86371…	B1	Labelled as TS or used in comparison B0 for $TS = \pm 0.86371$ unless the positive value correctly used later.
	CV = 0.3687	B1	FT their hypotheses (e.g., 0.4329 for two-tailed)
	Since TS > 0.3687, there is sufficient evidence to reject H_0.	B1	FT for using 0.746 FT their CV
	Sufficient evidence to suggest there is **positive** correlation between the average house price and average national reading test score.	E1	CSO (correct solution only). E0 for categorical statements or omission of the word positive (unless positive implied by contextualised comment). E0 for conclusion not in context
	Part (a) Total	**[5]**	
(b)	Valid comment saying the two variables are linked, i.e., giving a reason for the headline. e.g., The data support the idea that the more expensive houses are correlated with better reading scores.	E1	
	Valid comment saying why the headline is unreasonable. e.g., It's unreasonable to suggest that a more expensive house will improve a child's reading ability.	E1	E0 for correlation does not imply causation unless explained in context. Condone responses that give a valid alternative explanation for the correlation.
	Part (b) Total	**[2]**	
(c)	Possible explanation. e.g., parents who can afford better houses may have a better education so are more likely to help their children to read.	E1	E0 for comments such as "those with higher reading scores can afford better houses". Do not accept "small sample size" or "it is a coincidence". Condone a repetition of a valid alternative explanation that was given in (b). Do not condone a valid alternative explanation given in (b) only.
	Part (c) Total	**[1]**	
Total for Question 5		**8**	

SECTION B – Mechanics

Q6	Solution	Mark	Notes
			$\tan\alpha = \frac{3}{4}$ $\sin\alpha = \frac{3}{5} = 0\cdot 6$ $\cos\alpha = \frac{4}{5} = 0\cdot 8$ $3\cdot 6g = \frac{882}{25} = 35\cdot 28$
	Resolving horizontally **OR** vertically	M1	Attempt at resolution to get at least one dim. correct equation with no missing or extra forces
	$T_{AB}\sin\alpha = 3\cdot 6g$ $(T_{AB}\times 0\cdot 6 = 3\cdot 6g)$ $T_{AB}\cos\alpha = T_{AC}$ $(T_{AB}\times 0\cdot 8 = T_{AC})$	A1 A1	First correct equation Second correct equation
	$T_{AB} = 58\cdot 8$ (N) $\left(T_{AB} = \frac{294}{5} = 6g\right)$	A1	cso, allow answer rounding to $58\cdot 8$ (1dp)
	$T_{AC} = 47\cdot 04$ (N) $\left(T_{AC} = \frac{1176}{25} = 4\cdot 8g\right)$	A1 [5]	FT their T_{AB} if substituted into a correct equation (if M awarded)
	Alternative Solution (Triangle of forces) 		$\tan\alpha = \frac{3}{4}$ $\sin\alpha = \frac{3}{5} = 0\cdot 6$ $\cos\alpha = \frac{4}{5} = 0\cdot 8$ $3\cdot 6g = \frac{882}{25} = 35\cdot 28$
	Evidence of one of the trig. ratios below (Resolving horizontally **OR** vertically)	M1	Attempt at resolution to get at least one dim. correct equation with no missing or extra forces
	$\sin\alpha = \frac{3\cdot 6g}{T_{AB}}$ $\cos\alpha = \frac{T_{AC}}{T_{AB}}$ $\tan\alpha = \frac{3\cdot 6g}{T_{AC}}$	A1 A1	First correct equation Second correct equation
	$T_{AB} = 58\cdot 8$ (N) **or** $T_{AC} = 47\cdot 04$ (N) $\left(T_{AB} = \frac{294}{5} = 6g\right)$ $\left(T_{AC} = \frac{1176}{25} = 4\cdot 8g\right)$	A1	cso, allow answer rounding to $58\cdot 8$, allow answer rounding to $47\cdot 0$
	$T_{AC} = 47\cdot 04$ (N) **or** $T_{AB} = 58\cdot 8$ (N)	A1 [5]	FT their T_{AB}/T_{AC} if substituted into a correct equation even if $\alpha = 37°$
	Total for Question 6	**5**	

Q7	Solution	Mark	Notes
(a)		B1	Any correct moment with pivot clearly indicated
	Moments about X	M1	Dim. correct equation, oe, no extra/missing forces
	$0\cdot5R_Y = 0\cdot8\times5g+2\times11g$	A1	Correct equation
	$0\cdot5R_Y = 4g+22g$		$0\cdot5R_Y = 26g$
	$R_Y = 52g$	A1	cao
	Moments about Y	(M1)	Dim. correct equation, oe, no extra/no missing forces
	$0\cdot5R_X = 0\cdot3\times5g+1\cdot5\times11g$	(A1)	Correct equation
	$0\cdot5R_X = 1\cdot5g+16\cdot5g$		$0\cdot5R_X = 18g$
	$R_X = 36g$	(A1)	cao
	Resolve vertically	M1	Equation attempted, no extra/missing forces (or 2nd moment equation)
	$R_Y = R_X+5g+11g$ $(R_Y = R_X+16g)$	A1	oe
	$R_X = 36g$ OR $R_Y = 52g$	A1	FT R_X or R_Y
		[7]	
(b)	On the point of turning about Y, $R_X = 0$.	M1	si
	Moments about Y $Mg\times0\cdot9 = 5g\times0\cdot3+11g\times1\cdot5$	m1	Equation, no additional forces
	$0\cdot9Mg = 1\cdot5g+16\cdot5g$ $(0\cdot9Mg = 18g)$		
	$M = 20$	A1	cao
		[3]	
	Total for Question 7	**10**	

Q8	Solution	Mark	Notes
(a)	$R = 90g\cos\alpha \qquad (= 90g\cos 10° = 868\cdot 600\ldots)$	B1	si
	$F = \frac{2}{9} \times R \qquad \left(F = \frac{2}{9} \times 90g\cos\alpha = 20g\cos\alpha\right)$	B1	si
	Apply N2L up slope	M1	Dim. correct, no missing/extra forces,
	$380 - 90g\sin 10° - F = 90a$	A1	
	$380 - 153\cdot 15769\ldots \quad - 193\cdot 02231\ldots \quad = 90a$		$(F = 20g\cos 10° = 193\cdot 022\ldots)$
	$a = 0\cdot 375(7776\ldots) \qquad (\mathrm{ms}^{-2})$	A1	cso, allow answers rounding to $0\cdot 38$
		[5]	
(b)	If object remains stationary, component of weight down slope ≤ Limiting Friction $90g\sin\alpha \leq F$	M1	si
	$90g\sin\alpha \leq 20g\cos\alpha$	A1	$882\sin\alpha \leq 196\cos\alpha$
	$\alpha_{max} = \tan^{-1}\left(\frac{2}{9}\right)$		$\frac{2}{9} = \frac{196}{882}$
	$= 12\cdot 5(288\ldots)^0$	A1	cao
		[3]	
	Total for Question 8	8	

Q9	Solution	Mark	Notes
(a)	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta + 18)$	B1	oe
		[1]	
(b)	$\int \frac{1}{\theta+18}\mathrm{d}\theta = -k\int \mathrm{d}t$	M1	Separating variables
	$\ln(\theta + 18) = -kt\ \ (+C)$	A1	Correct integration $\ln\lvert\theta + 18\rvert$ not needed as $\theta > -18$. FT
	When $t = 0,\ \theta = 10$ $C = \ln(28)$ $(k = 3\cdot3322\ldots)$	m1	Used
	$kt = \ln(28) - \ln(\theta + 18)$		
	$kt = \ln\left(\frac{28}{\theta + 18}\right)$	A1	Convincing
		[4]	
(c)	Using $t = 1,\ \theta = 6$, in given result		
	$k = \ln\left(\frac{28}{6+18}\right)$ $\left(k = \ln\left(\frac{28}{24}\right) = \ln\left(\frac{7}{6}\right) = 0\cdot15415\ldots\right)$	M1	Conditions used
	At $\theta = -5$,		
	$kt = \ln\left(\frac{28}{-5 + 18}\right)$	m1	Their k sustituted
	$t = 4\cdot9773\ldots$		$t = \frac{1}{\ln\left(\frac{7}{6}\right)}\ln\left(\frac{28}{13}\right)$
	$t = 5$ hours	A1	cao
		[3]	
	Total for Question 9	**8**	

Q10	Solution	Mark	Notes
(a)	Horizontally		
	$t = \frac{x}{35\cos\theta}$	B1	oe, $x = (35\cos\theta)t$
	Vertically		
	$y = (35\sin\theta)t \pm \frac{1}{2}gt^2$	M1	$s = ut + \frac{1}{2}at^2$, $a = \pm g$, $u = 35\sin\theta\ /35\cos\theta$
	$y = (35\sin\theta)\left(\frac{x}{35\cos\theta}\right) + \frac{1}{2}(-9\cdot8)\left(\frac{x}{35\cos\theta}\right)^2$	A1	Correct equation
	$y = x\tan\theta - \frac{x^2}{250}\sec^2\theta$		
	$y = x\tan\theta - \frac{x^2}{250}(1+\tan^2\theta)$	A1 **[4]**	Convincing with evidence, e.g $\frac{1}{\cos^2\theta} = 1 + \tan^2\theta$
(b)	(i) $20 = 100\tan\theta - \frac{100^2}{250}(1+\tan^2\theta)$	M1	Correct use of $(100\mathbf{i} + 20\mathbf{j})$ i.e. $x = 100$, $y = 20$
	$2\tan^2\theta - 5\tan\theta + 3 = 0$	m1	An attempt to collect terms, form and solve a quadratic equation in $\tan\theta$.
	$(2\tan\theta - 3)(\tan\theta - 1) = 0$		
	$\tan\theta = \frac{3}{2}, 1$	A1	Both values, isw
	(ii) $0 = x(a) - \frac{x^2}{250}(1 + a^2)$	M1	Using $a = \tan\theta = 1$ **or** $\frac{3}{2}$ **and** $y = 0$ FT their $\tan\theta$ from (i)
	$x = 125$ (or $x = \frac{1500}{13} = 115\cdot38\ldots$)		
	Shortest distance from F is $130 - 125 = 5$ (m)	A1 **[5]**	cao
	Total for Question 10	9	

GCE AS MARKING SCHEME

SUMMER 2022

AS (NEW)
FURTHER MATHEMATICS
UNIT 1 FURTHER PURE MATHEMATICS A
2305U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 1 FURTHER PURE MATHEMATICS A

SUMMER 2022 MARK SCHEME

1. a) i)	METHOD 1: $zw = (3-4\mathrm{i})(2-\mathrm{i}) = 6-3\mathrm{i}-8\mathrm{i}+4\mathrm{i}^2$		
	$zw = 2-11\mathrm{i}$	B2	B1 for unsimplified expansion with 3 correct terms
	$\lvert zw\rvert = \sqrt{2^2+(-11)^2} = 5\sqrt{5}$	B1	FT their zw (zw must be seen)
	$\arg zw = \tan^{-1}\left(-\frac{11}{2}\right) = -1.39$ or -79.7°	B1	oe FT their zw if not in 1st quadrant
	METHOD 2: $\lvert z\rvert = \sqrt{3^2+(-4)^2} = 5$ $\lvert w\rvert = \sqrt{2^2+(-1)^2} = \sqrt{5}$	(B1)	Both mods
	$\arg z = \tan^{-1}\left(-\frac{4}{3}\right) = -0.927$ or -53.13°		oe
	$\arg w = \tan^{-1}\left(-\frac{1}{2}\right) = -0.464$ or -26.57°	(B1)	Both args oe
	Therefore, $\lvert zw\rvert = 5\times\sqrt{5} = 5\sqrt{5}$	(B1)	FT args and mods
	$\arg zw = -\,0.927 + \ -0.464 = -1.39$ or -79.7°	(B1)	oe FT args and mods (mods and args must be seen)
		[4]	
ii)	$\therefore\ 5\sqrt{5}(\cos(-1.39) + \mathrm{i}\sin(-1.39))$ OR $5\sqrt{5}(\cos(-79.7^{o}) + \mathrm{i}\sin(-79.7^{o}))$	B1 [1]	oe FT their mod and arg
b)	METHOD 1: $\frac{1}{v} = \frac{1}{2-\mathrm{i}} - \frac{1}{3-4\mathrm{i}}$		
	$\frac{1}{v} = \frac{3-4\mathrm{i}-2+\mathrm{i}}{(3-4\mathrm{i})(2-\mathrm{i})}$	M1	Attempt to combine
	$\frac{1}{v} = \frac{1-3\mathrm{i}}{2-11\mathrm{i}}$	A1	
	$v = \frac{2-11\mathrm{i}}{1-3\mathrm{i}}$	A1	
	$v = \frac{2-11\mathrm{i}}{1-3\mathrm{i}} \times \frac{1+3\mathrm{i}}{1+3\mathrm{i}}$	M1	FT their v M0 for no working
	$v = \frac{35-5\mathrm{i}}{10}\left(=\frac{7-\mathrm{i}}{2}\right)$		
	$v = 3.5 - 0.5\mathrm{i}$	A1	oe cao
	METHOD 2: $\frac{1}{v} = \frac{1}{2-\mathrm{i}} - \frac{1}{3-4\mathrm{i}}$		

	$\frac{1}{v}=\frac{z-w}{zw}$	(M1)	Attempt to combine
	$v=\frac{zw}{z-w}$ or $\frac{1}{v}=\frac{1-3\mathrm{i}}{2-11\mathrm{i}}$	(A1)	
	$v=\frac{2-11\mathrm{i}}{1-3\mathrm{i}}$	(A1)	
	$v=\frac{2-11\mathrm{i}}{1-3\mathrm{i}}\times\frac{1+3\mathrm{i}}{1+3\mathrm{i}}$	(M1)	FT their v M0 no working
	$v=\frac{35-5\mathrm{i}}{10}\left(=\frac{7-\mathrm{i}}{2}\right)$ $v=3.5-0.5\mathrm{i}$	(A1)	oe cao
	METHOD 3: Attempt to realise at least one fraction e.g. $\frac{1}{2-\mathrm{i}}\times\frac{2+\mathrm{i}}{2+\mathrm{i}}$ OR $\frac{1}{3-4\mathrm{i}}\times\frac{3+4\mathrm{i}}{3+4\mathrm{i}}$	(M1)	M0 no working
	$\frac{1}{v}=\frac{2+\mathrm{i}}{5}-\frac{3+4\mathrm{i}}{25}$		
	$\frac{1}{v}=\frac{7+\mathrm{i}}{25}$	(A1)	
	$v=\frac{25}{7+\mathrm{i}}$	(A1)	
	$v=\frac{25}{7+\mathrm{i}}\times\frac{7-\mathrm{i}}{7-\mathrm{i}}$	(M1)	FT their v M0 no working
	$v=\frac{35-5\mathrm{i}}{10}\left(=\frac{7-\mathrm{i}}{2}\right)$ $v=3.5-0.5\mathrm{i}$	(A1) [5]	oe cao
c)	$\bar{v}=\frac{7+\mathrm{i}}{2}$	B1	FT their v provided complex
	$v\bar{v}=\frac{7-\mathrm{i}}{2}\times\frac{7+\mathrm{i}}{2}=\frac{25}{2}$	B1 [2]	oe
		[12]	

2.a)	METHOD 1: Let $X = \begin{pmatrix}a\\b\end{pmatrix}$ $\begin{pmatrix}3&4\\-1&-2\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}-11\\7\end{pmatrix}$ Therefore,		
	$3a+4b=-11$	M1	Attempt to form 2 sim eqns
	$-a-2b=7$	A1	
	Solving, $a=3$ and $b=-5$	M1	Attempt to solve
	$X=\begin{pmatrix}3\\-5\end{pmatrix}$	A1	Must be in matrix form
	METHOD 2: $det\ A=(3\times -2)-(4\times -1)=-2$	(B1)	si
	$A^{-1}=\frac{1}{-2}\begin{pmatrix}-2&-4\\1&3\end{pmatrix}$	(B1)	
	Therefore, $X=A^{-1}B=\frac{1}{-2}\begin{pmatrix}-2&-4\\1&3\end{pmatrix}\begin{pmatrix}-11\\7\end{pmatrix}$	(M1)	
	$X=\begin{pmatrix}3\\-5\end{pmatrix}$	(A1)	Must be in matrix form
		[4]	
b) (i)	If reflection in $y=-2x$,		
	then $\tan\theta=-2$	B1	si
	$\therefore\ \theta=\tan^{-1}(-2)$		
	Reflection matrix: $\begin{pmatrix}-\frac{3}{5}&-\frac{4}{5}\\-\frac{4}{5}&\frac{3}{5}\end{pmatrix}$	B2	B1 for 1 error (possibly repeated)
		[3]	If B2 then -1 for PA
b) (ii)	METHOD 1: Therefore,		FT their *T*
	$EF=\begin{pmatrix}-\frac{3}{5}&-\frac{4}{5}\\-\frac{4}{5}&\frac{3}{5}\end{pmatrix}\begin{pmatrix}2&3\\7&1\end{pmatrix}$	M1	For attempt to multiply at least 1 point matrix
	$EF=\begin{pmatrix}-\frac{34}{5}&-\frac{13}{5}\\\frac{13}{5}&-\frac{9}{5}\end{pmatrix}$	A1 A1	Left column Right column May be seen as separate matrices
	Midpoint: $\left(-\frac{47}{10},\frac{2}{5}\right)$	B1	oe, FT their *E* and *F*
	METHOD 2:		FT their *T*
	Midpoint of *CD* = $\left(\frac{2+3}{2},\frac{7+1}{2}\right)=\left(\frac{5}{2},4\right)$	(B1)	
	Therefore,		

	$\begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}\begin{pmatrix} \frac{5}{2} \\ 4 \end{pmatrix}$ $= \begin{pmatrix} -\frac{47}{10} \\ \frac{2}{5} \end{pmatrix}$ Midpoint of *EF*: $\left(-\frac{47}{10}, \frac{2}{5}\right)$	(M1) (A1) (A1) [4]	FT their midpoint oe
		[11]	
3.	$x = -1 + 4\lambda \quad y = 2 - 2\lambda \quad z = -6 + 7\lambda$ Substituting, $\therefore\ -3 + 12\lambda + 16 - 16\lambda + 54 - 63\lambda = 0$ $67 - 67\lambda = 0$ $\lambda = 1$ $\therefore x = 3 \quad y = 0 \quad z = 1$ $\Rightarrow\ (3, 0, 1)$	B1 M1 A1 A1 B1 **[5]**	si FT their λ and their x, y, z provided at least 2 correct
4.	$1^2 + 2^2 + 3^2 + \cdots + N^2$ can be written as $\sum_{r=1}^{N} r^2$ $\sum_{r=1}^{N} r^2 = (3N-2)^2$ $\frac{1}{6}N(N+1)(2N+1) = 9N^2 - 12N + 4$ $2N^3 + 3N^2 + N = 54N^2 - 72N + 24$ $2N^3 - 51N^2 + 73N - 24 = 0$ Finding one factor, eg. $(N-1)$ $\therefore\ (N-1)(2N^2 - 49N + 24) = 0$ $\therefore (N-1)(2N-1)(N-24) = 0$ $\therefore\ N = 1 \text{ or } N = \frac{1}{2} \text{ or } N = 24$ Therefore, $N = 1, 24$	M1 A1 A1 B1 m1 A1 A1 **[7]**	cao $(N-k)$ form Linear × Quadratic (2 terms correct) Must reject $N = \frac{1}{2}$

5. a)	$\|z-3+2\mathrm{i}\| = \|z-3\|$ $\|x+\mathrm{i}y-3+2\mathrm{i}\| = \|x+\mathrm{i}y-3\|$ $\|(x-3)+\mathrm{i}(y+2)\| = \|(x-3)+\mathrm{i}y\|$ $\sqrt{(x-3)^2+(y+2)^2} = \sqrt{(x-3)^2+y^2}$ $x^2-6x+9+y^2+4y+4 = x^2-6x+9+y^2$ $4y+4=0$ $y=-1$	M1 m1 A1 [3]	oe Mark final answer Sight of answer only M1m1A1
b)	It is the perpendicular bisector of the line joining the points **(3, -2)** and **(3, 0)** OR The locus of P is all the points which are equidistant from **(3, -2)** and **(3,0)**.	B1 (B1) [1]	
		[4]	
6.	$\alpha+\beta+\gamma = -\frac{p}{2}$ $\alpha\beta+\beta\gamma+\gamma\alpha = -63$ $\alpha\beta\gamma = -\frac{q}{2}$ Let initial root be α AND use of g.p. property Then other roots are -3α and 9α Therefore $\left(7\alpha = -\frac{p}{2}\right)$ $-21\alpha^2 = -63$ $\left(-27\alpha^3 = -\frac{q}{2}\right)$ $\therefore\ \alpha^2 = 3 \quad \Rightarrow \quad \alpha = \pm\sqrt{3}$ If $\alpha = +\sqrt{3},\ p = -14\sqrt{3}$ and $q = 162\sqrt{3}$ AND If $\alpha = -\sqrt{3},\ p = 14\sqrt{3}$ and $q = -162\sqrt{3}$	B1 B1 B1 M1 A1 A1 A1 A1 **[8]**	May be seen later in working Accept solutions where α, β, γ interchanged oe (e.g. $-3\alpha, 9\alpha, -27\alpha$) provided M1 awarded cao

7.	From lines L_1, L_2:		
a)	$(2 \times 3) + (1 \times n) + (1 \times -3) = 0$	M1	
	$6 + n - 3 = 0$		
	$n = -3$	A1	convincing
	From lines L_1, L_3:		
	$(2 \times p) + (-3 \times 3) + (1 \times 4) = 0$	(M1)	If not awarded for L_1, L_2
	$p = \frac{5}{2}$	A1 [3]	
b)	$\left(3 \times \frac{5}{2}\right) + (1 \times 3) + (-3 \times 4) = -\frac{3}{2}$	B1	si FT their p for B1B1M1
	$\lvert 3\mathbf{i} + \mathbf{j} - 3\mathbf{k} \rvert = \sqrt{19}$		
	$\left\lvert \frac{5}{2}\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} \right\rvert = \sqrt{\frac{125}{4}}$	B1	si Both mods
	Therefore,		
	$\cos\theta = \dfrac{-\frac{3}{2}}{\sqrt{19}\sqrt{\frac{125}{4}}}$	M1	oe
	$\theta = 93.5^o$		
	Therefore, acute angle is $\theta = 86.5^o$	A1 [4]	cao
		[7]	

8.	Rotation matrix: $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	B1	
	$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1	Attempt to multiply Allow 1 error (possibly repeated)
	$= \begin{pmatrix} \frac{1}{2}x - \frac{\sqrt{3}}{2}y \\ \frac{\sqrt{3}}{2}x + \frac{1}{2}y \\ z \end{pmatrix}$	A1	
	Therefore, $x' = \frac{1}{2}x - \frac{\sqrt{3}}{2}y$ $y' = \frac{\sqrt{3}}{2}x + \frac{1}{2}y$		
	$\therefore\ \frac{1}{2}x - \frac{\sqrt{3}}{2}y = \frac{\sqrt{3}}{2}x + \frac{1}{2}y$ $x - \sqrt{3}y = \sqrt{3}x + y$ $x - \sqrt{3}x = y + \sqrt{3}y$ $x(1-\sqrt{3}) = y(1+\sqrt{3})$	M1	FT their images matrix
	$y = \frac{x(1-\sqrt{3})}{1+\sqrt{3}}$	A1	cao
	$\frac{1-\sqrt{3}}{1+\sqrt{3}} = \frac{(1-\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$ $= \frac{1+3-\sqrt{3}-\sqrt{3}}{1-3+\sqrt{3}-\sqrt{3}} = \frac{4-2\sqrt{3}}{-2}$	M1	M0 no working FT their y of equivalent difficulty e.g. $y = \frac{x(a+\sqrt{b})}{c+\sqrt{d}}$
	$y = (-2+\sqrt{3})x$	A1 **[7]**	

9. a)	$\frac{1}{r+1}-\frac{2}{r+2}+\frac{1}{r+3}$ $\frac{(r+2)(r+3)-2(r+1)(r+3)+(r+1)(r+2)}{(r+1)(r+2)(r+3)}$ $\frac{r^2+5r+6-2r^2-8r-6+r^2+3r+2}{(r+1)(r+2)(r+3)}$ $=\frac{2}{(r+1)(r+2)(r+3)}.$	M1 A1 [2]	Convincing
b)	$\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{2}{5}+\frac{1}{6}\right)+\cdots$ $+\left(\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right)+\left(\frac{1}{n+1}-\frac{2}{n+2}+\frac{1}{n+3}\right)$ $=\frac{1}{2}-\frac{2}{3}+\frac{1}{3}$ $+\frac{1}{n+2}-\frac{2}{n+2}+\frac{1}{n+3}$ $=\frac{1}{6}-\frac{n+3-n-2}{(n+2)(n+3)}$ $=\frac{1}{6}-\frac{1}{(n+2)(n+3)}$	M1 A1 A1 A1 A1 [5]	Substituting values – At least three correct sets of brackets Must have at least one correct algebraic set of brackets Convincing
c)	$\sum_{r=1}^{5}A_r=\frac{1}{6}-\frac{1}{7\times 8}=\frac{25}{168}$ AND $\sum_{r=1}^{10}A_r=\frac{1}{6}-\frac{1}{12\times 13}=\frac{25}{156}$ $\frac{25}{168}:\frac{25}{156}$ $13:14$	B1 B1 [2]	Both
		[9]	

GCE AS MARKING SCHEME

SUMMER 2022

AS (NEW)
FURTHER MATHEMATICS
UNIT 2 FURTHER STATISTICS A
2305U20-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

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WJEC GCE AS FURTHER MATHEMATICS

UNIT 2 FURTHER STATISTICS A

SUMMER 2022 MARK SCHEME

Qu. No.	Solution	Mark	Notes
1 (a)	$p = 0.0099$	B1	
(b)	$E(X) = (0 \times 0.9+)2 \times 0.09 + 100 \times 0.0099 + 1000 \times 0.0001$	M1	FT "their p" Allow one slip
	$E(X) = 1.27$	A1	
	$Var(X) = (0^2 \times 0.9+)2^2 \times 0.09 + 100^2 \times 0.0099 + 1000^2 \times 0.0001 - 1.27^2$	M1	FT "their p" and "their$E(X)$" Allow one slip
	$Var(X) = 197.7(471)$	A1	Accept 198 from correct working
(c)(i)	£1.28	B1	FT their E(X)
(ii)	Valid explanation. e.g. People may be willing to pay for the excitement of the lottery. The lottery may be raising money for charity. People don't often make decisions based on mathematics. People could win a lot of money.	E1	
		Total [7]	

2 (a)	$S_{xy} = 113.16 - \frac{62.8 \times 19.4}{10}$ $S_{xy} = -8.672$	B1	B1 for each of S_{xy},S_{xx} and S_{yy}.
	$S_{xx} = 413.44 - \frac{62.8^2}{10}$ $S_{xx} = 19.056$	B1	
	$S_{yy} = 46.16 - \frac{19.4^2}{10}$ $S_{yy} = 8.524$	B1	
	$r = \frac{-8.672}{\sqrt{19.056 \times 8.524}}$ $r = -0.68(0427 \ldots)$	B1	B1 for r.
(b)	H_0: $\rho = 0$ $\quad\quad$ H_1: $\rho \neq 0$	B1	FT their r Accept in context
	5% two tail critical value = –0.6319	B1	Or CV = 0.6319
	Since –0.6804 < –0.6319 reject H_0.	B1	Or 0.6804 > 0.6319
	It suggests that the rate of unemployment and the rate of wage inflation are not independent.	E1	Only award E1 if previous three B1 awarded E0 for categorical statements
(c)	Valid comment. e.g. This should cast doubt on Amy’s opinion based on her answer in (b)	E1	FT their conclusion from (b)
	Valid suggestion. e.g. She could look at more countries. She could come to different conclusions for different countries. She could consider more regions within each country	E1	
(d)	The underlying distribution is bivariate normal. The data come from a bivariate normal distribution.	E1	
		Total [11]	

3 (a)	Total number of baskets, T, is $Po\big((2.1+1.9)\times 4\big)$ or $Po(16)$ or $Po(2.1\times 4+1.9\times 4)$ $P(T=20)=\dfrac{16^{20}\times e^{-16}}{20!}$ $=0.0559$	M2 m1 A1	M1 for Poisson and adding. M1 for multiplying by 4. Dependent on M2 Use of formula or calculator cao
(b) (i)	Exponential distribution Mean time between baskets= standard deviation $=\frac{1}{2.1}\times 12$ 5.7 minutes.	B1 M1 A1	Must be clear that 5.7… is mean AND standard deviation
(b) (ii)	P (Klay doesn't score for the rest of the quarter) = $e^{(-1.9\times 0.75)}$ = 0.2405 Alternative solution $\lambda=1.425$ P(X = 0) = 0.2405	M1 A1 (M1) (A1)	M1 for Po(1.9×0.75) SC1 for ($\mathrm{e}^{(-2.1\times 0.75)}=$)$0.207$
(c)	Let F be the number of free throws he misses. $F \sim B(530, 0.04)$ $P(F>25)=1-P(F\le 25)$ $=0.169(1214\ldots.)$	M1 A1 **Total [11]**	

4 (a)	The pdf must be positive (or zero) $f(r) \geq 0$ Therefore $(b-4) \geq 0$ $b \geq 4$	B1 B1	B1 for implying that the pdf must be positive or zero (or cannot be negative) B1 for Correct statement leading to correct conclusion. ALTERNATIVE B1 for "If $b < 4$, $f(r)$is negative." B1 for stating that is not possible.
4 (b) (i)	$\int_1^4 kr(4-r)dr = 1$ $\int_1^4 (4kr - kr^2)dr = 1$ $k\left[\frac{4r^2}{2} - \frac{r^3}{3}\right]_1^4 = 1$ $k\left[\left(\frac{64}{2} - \frac{64}{3}\right) - \left(\frac{4}{2} - \frac{1}{3}\right)\right] = 1$ $k = \frac{1}{9}$ *ag	M1 A1 m1 A1	M1 Attempt at integration at least one power of r increasing by 1. Limits and = 1 not required here. A1 Correct integration. m1 substitution of correct limits and =1. Convincing

4 (b) (ii)	$F(r) = \frac{1}{9}\int_{1}^{r} t(4-t)dt$	M1	M1 Attempt at integrating $f(t)$ at least one power of t increasing by 1. Limits not required here.
	$= \frac{1}{9}\left[\frac{4t^2}{2} - \frac{t^3}{3}\right]_1^r$	A1	A1 Correct integration.
	$= \frac{1}{9}\left[2r^2 - \frac{r^3}{3} - \left(2 - \frac{1}{3}\right)\right]$ $= \frac{1}{9}\left(2r^2 - \frac{r^3}{3} - \frac{5}{3}\right)$	m1	m1 substituting correct limits Condone upper limit = x for m1 only
	$= \frac{1}{27}(6r^2 - r^3 - 5)$	A1	oe Mark final expression for $1 \le r \le 4$
(iii)	$P(2 \le R \le 3) = F(3) - F(2)$	M1	oe
	$= \frac{22}{27} - \frac{11}{27}$ $= \frac{11}{27}$	A1	FT their $F(r)$ for equivalent difficulty and provided probability is valid.
		Total [12]	

<table>
<tr>
<td>5</td>
<td>Let the random variable X be the number of 6s thrown from 3 dice.
If the dice are unbiased then $X \sim B(3, \frac{1}{6})$

H_0: The data can be modelled by the Binomial distribution $B(3, \frac{1}{6})$.
H_1: The data cannot be modelled by the Binomial distribution $B(3, \frac{1}{6})$.</td>
<td>B1

B1</td>
<td>si (implied by at least 3 correct expected frequencies)

or equivalent</td>
</tr>
<tr>
<td></td>
<td>
<table>
<tr><td>Number of sixes</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Observed</td><td>625</td><td>384</td><td>81</td><td>10</td></tr>
<tr><td>Expected</td><td>636.574</td><td>381.944</td><td>76.389</td><td>5.093</td></tr>
</table>

Use of χ^2 stat = $\sum \frac{(O-E)^2}{E}$ or $\sum \frac{O^2}{E} - N$

$= \frac{(625-636.574)^2}{636.574} + \frac{(384-381.944)^2}{381.944} + \frac{(81-76.389)^2}{76.389} + \frac{(10-5.093)^2}{5.093}$

$= 5.23$</td>
<td>M1
A1

M1

m1

A1</td>
<td>At least one correct.
All correct.

Must see at least 2 terms added

$\frac{625^2}{636.574} + \frac{384^2}{381.944} + \frac{81^2}{76.389} + \frac{10^2}{5.093} - 1100$

Accept anything which rounds to 5.2</td>
</tr>
<tr>
<td></td>
<td>DF = 3
5% CV = 7.815

Since $5.23 < 7.815$ we cannot reject H_0.
There is insufficient evidence at the 5% level to conclude that the set of dice are not fair.</td>
<td>B1
B1

B1
E1

Total [11]</td>
<td>
Accept other test levels.
1% CV = 11.345
10% CV = 6.251

FT their χ^2
Only award E1 if all five previous B1 awarded
E0 for categorical statements</td>
</tr>
</table>

6 (a)	H_0: Social media usage is independent of age. H_1: Social media usage is not independent of age	B1	
(b)	$\frac{1266\times352}{1953}$ $=228.18$ *ag	B1	oe
(c)	$s=\frac{(412-342.27)^2}{342.27}$ $s=14.2(0595699\ldots)$	M1 A1	
(d)	$(4-1)\times(2-1)=3$ degrees of freedom.	B1	
	5% CV = 7.815	B1	
	Add χ^2 contributions	M1	M1A1 if statement along the lines of "one contribution is > 7.815"
	$29.34+14.21+0.06+62.94+54.07+26.18$ $+0.11+115.99$ $=302.90$	A1	
	Since $302.91>7.815$ we can reject H_0.	B1	FT provided $\chi^2>$ 7.815
	There is (strong) evidence to suggest that social media usage is not independent of age.	E1	Only award E1 if previous three B1 awarded and part (a) correct
(e)	Valid explanation. e.g. The *p* value would not lead to rejecting H_0, which is the incorrect conclusion.	E1	
		Total [11]	

7 (a)	$b = \dfrac{96.60984}{88.42142}$	M1	
	$b = 1.09(26\ldots)$	A1	Accept 1.1
	$a = \dfrac{2738.656}{30} - 1.09(26\ldots) \times \dfrac{2850.836}{30}$	M1	FT their 'b' for M1
	$a =$ -12.5(39…)	A1	FT their 'b', following A0. Answer correct to 3sf
	$y = -12.5 + 1.09x$	A1	A1 FT 'their' gradient and intercept provided at least one M1 awarded.
(b)	Africa because 70 is out of the data set for Asia, The data points for Africa are closer to a straight line than those for the Arab World.	E1 E1	
		Total [7]	

GCE AS MARKING SCHEME

SUMMER 2022

AS (NEW)
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1

INTRODUCTION

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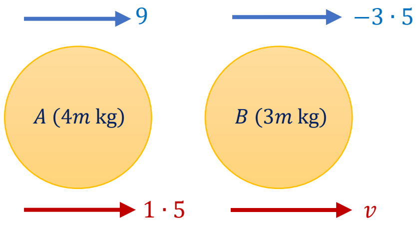
WJEC GCE AS FURTHER MATHEMATICS

UNIT 3 FURTHER MECHANICS A

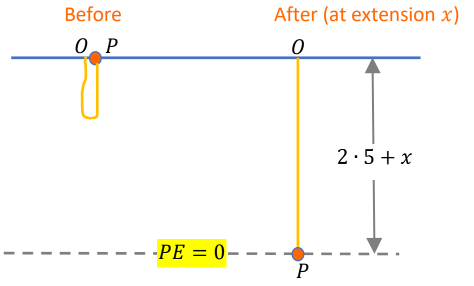
SUMMER 2022 MARK SCHEME

Q1	Solution	Mark	Notes
(a)	Angular velocity $\omega = \frac{v}{r}$ $\omega = \frac{8}{2}$ $\omega = 4 \quad (\text{rad s}^{-1})$	M1 A1 **[2]**	Used cao
(b)	N2L towards centre O Tension in the string $T = 1 \cdot 2a$ $T = 1 \cdot 2 \times \frac{8^2}{2}$ **or** $T = 1 \cdot 2 \times 4^2 \times 2$ $T = 38 \cdot 4 \quad (\text{N})$ or $\frac{192}{5}$	M1 A1 **[2]**	Used with $a = \begin{cases} \frac{v^2}{r} \\ \omega^2 r \end{cases}$ FT their ω from (a)
	Total for Question 1	4	

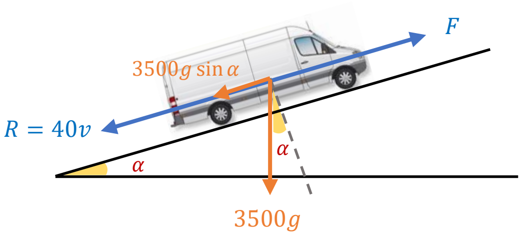
Q2	Solution	Mark	Notes
(a)	Using $\text{KE} = \frac{1}{2}mv^2$ with $m = 60, v = 7\cdot 8$	M1	Used
	$\text{KE} = \frac{1}{2}(60)(7\cdot 8)^2$		
	$\text{KE} = 1825\cdot 2$ (J) or $\frac{9126}{5}$	A1	cao
		[2]	
(b)	Using expression for PE **or** KE	M1	
	At start (platform),		
	$\text{PE} = 60g(10) \quad (= 600g = 5880 \text{ J})$	A1	
	At end (water),		
	$\text{KE} = \frac{1}{2}(60)v^2 \quad (= 30v^2)$	A1	
	Conservation of energy	M1	Used, all terms, allow sign errors
	$1825\cdot 2 + 5880 = 30v^2$ $(7705\cdot 2 \quad = 30v^2)$	A1	All correct, oe FT KE from (a)
	$v^2 = 256\cdot 84$ or $\frac{6421}{25}$		
	$v = 16\cdot 0262 \ldots \approx 16 \text{ (ms}^{-1})$	A1	Convincing, cso
		[6]	
(c)	Work-energy principle	M1	Used, all terms, allow sign errors
	$1825\cdot 2 + 5880 = \frac{1}{2}(60)(13)^2 + E_{lost}$ $(\quad 7705\cdot 2 \quad = \quad 5070 \quad + E \quad)$	A1	All correct, oe FT KE from (a) FT PE from (b)
	$E_{lost} = 2635\cdot 2$ (J) or $\frac{13176}{5}$	A1	FT their KE and PE
		[3]	
	<u>Alternative Solution</u>		
	Taking a difference in KE	(M1)	At least one v^2 correct
	$E_{lost} = \frac{1}{2}(60)\left(\frac{6421}{25}\right) - \frac{1}{2}(60)(13)^2$	(A1)	All correct, oe Accept $\frac{1}{2}(60)(16)^2 = 7680$
	$E_{lost} = 2635\cdot 2$ (J) or $\frac{13176}{5}$	(A1)	$E_{lost} = 2610$ (J) for $v = 16$
		([3])	
	Total for Question 2	**11**	

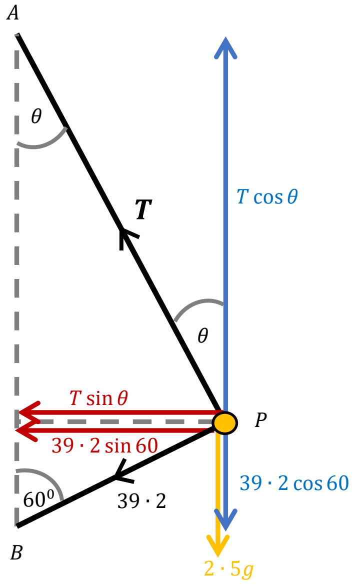
Q3	Solution	Mark	Notes
(a)			
	Conservation of momentum	M1	Attempted. Allow 1 sign error
	$(9)(4m) + (-3\cdot5)(3m) = (1\cdot5)(4m) + (v)(3m)$	A1	All correct
	$25\cdot5 = 6 + 3v$		
	$v = 6\cdot5 \quad (\mathrm{ms}^{-1})$	A1	Convincing
		[3]	
(b)	Restitution	M1	Attempted. Allow 1 sign error
	$6\cdot5 - 1\cdot5 = \ -e(-3\cdot5 - 9)$	A1	All correct, oe
	$5 = 12\cdot5e$		
	$e = \frac{2}{5}$	A1	cao
		[3]	
(c)	Change in momentum $= 36$	M1	
	$(4m)(9 - 1\cdot5) = 36 \qquad (30m = 36)$	A1	Correct equation, oe $(3m)(6\cdot5 - -3\cdot5) = 36$
	$m = 1\cdot2$	A1	cao
		[3]	
(d)	Valid reason, eg. Radii are equal Velocities are parallel to line of centres	E1	
		[1]	
	Total for Question 3	**10**	

Q4	Solution	Mark	Notes
(a)	$(9\mathbf{i}+6\mathbf{j}-12\mathbf{k})+(6\mathbf{i}-7\mathbf{j}+3\mathbf{k})+\mathbf{F}_3=0$	M1	
	$\mathbf{F}_3=-15\mathbf{i}+\mathbf{j}+9\mathbf{k}$ (N)	A1	
		[2]	
(b)	(i) $\mathbf{AB}=\mathbf{r}_B-\mathbf{r}_A=(8\mathbf{i}-5\mathbf{j}-\mathbf{k})-(2\mathbf{i}-9\mathbf{j}+7\mathbf{k})$	M1	or $\mathbf{BA}$
	$=6\mathbf{i}+4\mathbf{j}-8\mathbf{k}$	A1	oe, cao
	$\mathbf{F}_1=\frac{3}{2}\mathbf{AB}$ **or** $\mathbf{AB}=\frac{2}{3}\mathbf{F}_1$ ($\therefore$ parallel)	A1	Convincing
	(ii) Work done by $\mathbf{F}_1=\mathbf{F}_1.\mathbf{AB}$		
	$=(9\mathbf{i}+6\mathbf{j}-12\mathbf{k}).(6\mathbf{i}+4\mathbf{j}-8\mathbf{k})$	M1	Used. FT $\mathbf{AB}$
	$=(9)(6)+(6)(4)+(-12)(-8)$		
	$=174$ (J)	A1	FT their $\mathbf{AB}$
	(iii) Work done = change in KE		
	$174=\frac{1}{2}(0\cdot5)v^2-0$	M1	FT their '174'
	$v=26\cdot38(18\ldots)$ (ms^{-1})	A1	$v=\sqrt{696}=2\sqrt{174}$ FT their '174'
		[7]	
Total for Question 4		**9**	

Q5	Solution	Mark	Notes
(a)			
	Using expression for PE $= mgh$ **or** EE $= \frac{\lambda x^2}{2l}$	M1	
	Loss in PE $= 2g(2\cdot 5 + x)$ $\quad (= 5g + 2gx)$	A1	
	Gain in EE $= \frac{\lambda x^2}{2(2\cdot 5)} = \frac{30gx^2}{2(2\cdot 5)}$ $\quad (= 6gx^2)$	A1	
	Gain in KE $= \frac{1}{2}(2)v^2$ $\quad (= v^2)$	B1	
	Conservation of energy $v^2 + 6gx^2 = 5g + 2gx$	M1	Used with PE,KE and EE All terms, allow sign errors M0: PE $= 5g$ alone
	$v^2 = g(5 + 2x - 6x^2)$	A1	Convincing
		[6]	
(b)	At maximum extension, $v = 0$ $0 = g(5 + 2x - 6x^2)$	M1	Used
	$6x^2 - 2x - 5 = 0$		
	Attempting to solve $x = \frac{2\pm\sqrt{124}}{12}$	m1	$x = \frac{1\pm\sqrt{31}}{6}$ from calculator
	$x = 1\cdot 09(4627 \ldots)$ $\quad$ (or $x = -0\cdot 76(1294 \ldots)$)	A1	cao $x = -0\cdot 76$ ... clearly discarded
		[3]	
(c)	(i) When P attains its maximum speed, $a = 0$ so that Tension in $OP = 2g$	M1	Hooke's Law used with $T = 2g$
	$\frac{30gx}{2\cdot 5} = 2g$	A1	
	$x = \frac{1}{6}$ (m)	A1	
	Alternative Solution to (i)		
	(i) Differentiating to find for maximum v^2(or v) $\frac{\mathrm{d}(v^2)}{\mathrm{d}x} = 0$	(M1)	Condone the following incorrect notation $\frac{\mathrm{d}v}{\mathrm{d}x} = g(2 - 12x)$
	$g(2 - 12x) = 0$	(A1)	oe
	$x = \frac{1}{6}$	(A1)	

	(ii) Sub. $x = \frac{1}{6}$ into $v^2 = g(5 + 2x - 6x^2)$ Maximum speed is $7 \cdot 11(57103 \ldots)\ (\mathrm{ms}^{-1})$	M1 A1 **[5]**	FT their $x \geq 0$ $v = \sqrt{\frac{31g}{6}} = \sqrt{\frac{1519}{30}}$. FT their $x \neq 0$ for $v^2 > 0$
Total for Question 5		14	

Q6	Solution	Mark	Notes
(a)			$3500g\left(\frac{3}{49}\right) = 2100$
	$F = \frac{P}{25}$	B1	
	Using N2L up slope $F - R - mg\sin\alpha = ma$	M1 A1	All forces, dim. correct M1: Allow $mg\cos\alpha$ or sign errors, but not both
	$\frac{P}{25} - 40(25) - 3500g\left(\frac{3}{49}\right) = 3500(-0\cdot 2)$	A1	Correct equation FT their F
	$P = 60\,000$ (W) or 60 (kW)	A1	cao
		[5]	
(b)	$F = \frac{40 \times 1000}{20}$ $(= 2000)$	B1	si
	Using N2L with $a = 0$	M1	All forces, dim. correct M1: Allow $mg\cos\alpha$ or sign errors, but not both
	$F - R - mg\sin\alpha = 0$	A1	
	$2000 - 40(20) - 3500g\sin\alpha = 0$	A1	Correct equation FT their F
	$\sin\alpha = \frac{12}{343} = 0\cdot 03498\ldots$		
	$\alpha = 2^0$	A1	cao
		[5]	
	Total for Question 6	**10**	

Q7	Solution	Mark	Notes
(a)			$\sin\theta = 0\cdot6$ $\cos\theta = 0\cdot8$
	Resolving vertically,	M1	All forces, dim. correct
	$T\cos\theta = (39\cdot2)\cos 60 \ +2\cdot5g$	A1 A1	-1 each error
	$T(0\cdot8) = (39\cdot2)(0\cdot5) + (2\cdot5)(9\cdot8)$		
	$T = 55\cdot125 \quad (\mathrm{N})$	A1	cao
		[4]	
(b)	Using N2L towards C,	M1	All forces, dim. correct
	$T\sin\theta + (39\cdot2)\sin 60 = 2\cdot5a$	A1	Correct equation
	$(55\cdot125)(0\cdot6) + (39\cdot2)\left(\frac{\sqrt{3}}{2}\right) = (2\cdot5)\,\omega^2 r$	m1	$a = \begin{cases} \frac{v^2}{r} \\ \omega^2 r \end{cases}$
	$(55\cdot125)(0\cdot6) + (39\cdot2)\left(\frac{\sqrt{3}}{2}\right) = (2\cdot5)\,\omega^2(0\cdot9)$	B1	$r = 1\cdot5\sin\theta$ $r = 1\cdot5 \times 0\cdot6 = 0\cdot9$
	$\omega^2 = 29\cdot78808 \ldots$		
	$\omega = 5\cdot45\,(78463\ldots) \quad (\mathrm{rad}\ s^{-1})$	A1	cao
		[5]	
(c)	$v = \omega r$		
	$v = 5\cdot45\ldots \times 0\cdot9$ $v = 4\cdot91206\ldots$	M1	FT ω and $r \neq 1\cdot5$
	$KE = \frac{1}{2}(2\cdot5)(4\cdot91206\ldots)^2$	m1	FT v
	$KE = 30\cdot16\,(0438\ldots) \quad (\mathrm{J})$	A1	cao
		[3]	
	Total for Question 7	**12**	

GCE A LEVEL MARKING SCHEME

SUMMER 2022

A LEVEL (NEW)
FURTHER MATHEMATICS
UNIT 4 FURTHER PURE MATHEMATICS B
1305U40-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 4 FURTHER PURE MATHEMATICS B

SUMMER 2022 MARK SCHEME

1. a)	Differentiating $f'(x) = 3\cosh^2 x \sinh x - 3\sinh x$ oe	M1 A1	If identities used, must be a valid attempt at differentiation
	At a stationary point, $f'(x) = 0$, $\therefore\ 3\cosh^2 x \sinh x - 3\sinh x = 0$ $3\sinh x\,(\cosh^2 x - 1) = 0$	m1	
	THEN $3\sinh x = 0$ $x = 0$ or $\cosh^2 x - 1 = 0$ $\cosh x = 1$ or $\cosh x = -1$ $x = 0$ no solutions	A1	Award for any solution of hyperbolic equation
	$\therefore$ The only stationary point is at $x = 0$.	A1	Must be seen to discard equations with no solutions and show all remaining equations lead to $x = 0$
	OR As $\cosh^2 x - 1 = \sinh^2 x$ $3\sinh x\,(\cosh^2 x - 1) = 3\sinh^3 x = 0$ $\therefore \sinh^3 x = 0$ $\sinh x = 0$ $x = 0$	(A1)	Correct use of identity
	$\therefore$ The only stationary point is at $x = 0$.	(A1)	

b)	METHOD 1 Find gradient or value of f either side of $x = 0$ e.g. $f'(-1) = -4.869 \ldots < 0$ and $f'(1) = 4.869 \ldots > 0$ e.g. $\sinh x < 0$ and therefore $\sinh^3 x < 0$ for $x < 0$ and $\sinh x > 0$ and therefore $\sinh^3 x > 0$ for $x > 0$ Therefore, the stationary point at $x = 0$ is a minimum METHOD 2 Differentiating and substituting $x = 0$ $f''(x) = 3\cosh^3 x + 6\cosh x \sinh^2 x - 3\cosh x$ oe $f''(0) = 3 + 0 - 3 = 0$ Finding the gradient either side of $x = 0$ AND Stating the stationary point at $x = 0$ is a minimum	M1 A1 (M1) (A1)	Accept graphical method cao cao
c)	When $x = 0$, $f(x) = -2$ Therefore, largest range is $[-2, \infty)$	B1	Allow 'Range $f(x) \geq -2$'

2.	Let $z^4 = 9 - 3\sqrt{3}\mathrm{i}$		
	$\lvert z^4\rvert = \sqrt{9^2 + \left(3\sqrt{3}\right)^2} = \sqrt{108}$ or $6\sqrt{3}$	B1	si
	Finding the radius of the circle	M1	FT their $\lvert z^4\rvert$
	e.g. Radius of circle = $\sqrt[8]{108}$ or $108^{\frac{1}{8}}$ $= 1.795 \ldots$	A1	
	Circle: $x^2 + y^2 = 3.22$ or 1.795^2	A1	FT their radius Allow 1.8^2 Allow $\lvert z\rvert = 108^{1/8}$
3. a)	Substituting $\sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$ $4 \times \dfrac{2t}{1+t^2} + 5 \times \dfrac{1-t^2}{1+t^2} = 3$	M1	
	$4 \times 2t + 5(1-t^2) = 3(1+t^2)$ oe $8t + 5 - 5t^2 = 3 + 3t^2$	A1	Removal of fractions
	$4t^2 - 4t - 1 = 0$	A1	convincing
b)	Solving $4t^2 - 4t - 1 = 0$	M1	M0A0 no working
	$t = \frac{1\pm\sqrt{2}}{2}$ (–0.207106... or 1.207106...)	A1	
	Attempting to solve for θ $\tan\frac{\theta}{2} = \frac{1-\sqrt{2}}{2}$ or $\tan\frac{\theta}{2} = \frac{1+\sqrt{2}}{2}$	M1	FT their t
	$\frac{\theta}{2} = -11.7 \ldots (+180n)$ or	A1	$\frac{\theta}{2} = -0.2 \ldots (+\pi n)$
	$\frac{\theta}{2} = 50.36 \ldots (+180n)$	(A1)	$\frac{\theta}{2} = 0.87 \ldots (+\pi n)$
	Then, the general solution, $\theta = (-23.4(018 \ldots) + 360n)°$ oe or	A1	$\theta = (-0.408\ldots + 2\pi n)^{\mathrm{c}}$
	$\theta = (100.7(214 \ldots) + 360n)°$ oe	A1	$\theta = (1.758\ldots + 2\pi n)^{\mathrm{c}}$
			M0 M0 for -23.4… and 100.7… without working
4.	Volume= $\pi\int_1^3 \sin^2 y \,\mathrm{d}y$	B1	Correct notation required
	$\pi\displaystyle\int_1^3 \frac{1-\cos 2y}{2}\,\mathrm{d}y$	M1	Integrable form with no more than 1 slip
	$\pi\left[\frac{1}{2}y - \frac{1}{4}\sin 2y\right]_1^3$	A1	oe cao
	$\pi\left[\left(\frac{3}{2} - \frac{1}{4}\sin 6\right) - \left(\frac{1}{2} - \frac{1}{4}\sin 2\right)\right]$	m1	Attempt to substitute in correct limits
	Volume = 4.08	A1	cao

<table>
<tr><td>5. a)</td><td>$$\left(\begin{array}{ccc|c}1 & 2 & 0 & 3\\ 2 & -5 & 3 & 8\\ 0 & 6 & -2 & 0\end{array}\right)$$
$$\left(\begin{array}{ccc|c}1 & 2 & 0 & 3\\ 0 & -9 & 3 & 2\\ 0 & 6 & -2 & 0\end{array}\right)$$
$$\left(\begin{array}{ccc|c}1 & 2 & 0 & 3\\ 0 & -9 & 3 & 2\\ 0 & 0 & 0 & \frac{4}{3}\end{array}\right)$$
Valid statement.
Eg. As $0x + 0y + 0z \neq \frac{4}{3}$ there are no solutions.</td><td>M1
A1
A1
E1</td><td>Attempt at row reduction
1 row a multiple of another row
oe
If M0,
SC1 det A = 0
SC1 No **unique** solutions</td></tr>
<tr><td>b)</td><td>A correct statement involving 3 planes with no incorrect statements
e.g. 3 planes do not meet at a single point</td><td>B1</td><td>FT their (a)</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>6.</td><td>$\cos 2\theta - \cos 4\theta = -2\sin\frac{2\theta + 4\theta}{2}\sin\frac{2\theta - 4\theta}{2}$
$-2\sin 3\theta \sin(-\theta) = \sin 3\theta$
$2\sin 3\theta \sin\theta - \sin 3\theta = 0$
$\sin 3\theta\,(2\sin\theta - 1) = 0$
$\sin 3\theta = 0 \qquad \sin\theta = \frac{1}{2}$
$3\theta = 0, \pi, 2\pi, 3\pi$
$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi \qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$</td><td>M1
A1
A1
A1
A1A1</td><td>M0 no working
FT one slip for A1A1A1
Both solutions
A1 each set of solutions
If A1A1, penalise -1 for use of degrees</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>7. a)</td><td>$4x^2 + 10x - 24 = 4\left[x^2 + \frac{5}{2}x - 6\right]$
$= 4\left[\left(x + \frac{5}{4}\right)^2 - \frac{121}{16}\right]$
$= 4\left(x + \frac{5}{4}\right)^2 - \frac{121}{4}$
Therefore, $a = 4 \quad b = \frac{5}{4} \quad c = -\frac{121}{4}$</td><td>M1
m1
A1</td><td>$4x^2 + 10x - 24$
$= 4\left[x^2 + \frac{5}{2}x\right] - 24$
oe</td></tr>
</table>

b)	METHOD 1: $\int_3^5 \frac{6}{\sqrt{4x^2+10x-24}}\,\mathrm{d}x$		
	$= \int_3^5 \frac{6}{\sqrt{4\left(x+\frac{5}{4}\right)^2-\frac{121}{4}}}\,\mathrm{d}x$	M1	M0 no working FT (a) for equivalent difficulty
	$= \int_3^5 \frac{6}{2\sqrt{\left(x+\frac{5}{4}\right)^2-\frac{121}{16}}}\,\mathrm{d}x$	m1	Extracting a factor of $\sqrt{4}$ from denominator
	$= \left[3\cosh^{-1}\left(\frac{x+\frac{5}{4}}{\sqrt{\frac{121}{16}}}\right)\right]_3^5$	A1	oe
	$= \left[3\cosh^{-1}\left(\frac{4x+5}{11}\right)\right]_3^5$		
	$= \left[3\cosh^{-1}\left(\frac{25}{11}\right)-3\cosh^{-1}\left(\frac{17}{11}\right)\right]$	m1	
	$= 1.379$	A1	cao Must be 3d.p.
	METHOD 2: $\int_3^5 \frac{6}{\sqrt{4x^2+10x-24}}\,\mathrm{d}x$		
	$= \int_3^5 \frac{6}{\sqrt{4\left(x+\frac{5}{4}\right)^2-\frac{121}{4}}}\,\mathrm{d}x$	(M1)	M0 no working FT (a) for equivalent difficulty
	$= \int_3^5 \frac{6}{2\sqrt{\left(x+\frac{5}{4}\right)^2-\frac{121}{16}}}\,\mathrm{d}x$	(m1)	Extracting a factor of $\sqrt{4}$ from denominator
	$= \left[3\ln\left\{x+\frac{5}{4}+\sqrt{\left(x+\frac{5}{4}\right)^2-\frac{121}{16}}\right\}\right]_3^5$	(A1)	
	$= 3\ln\left[\frac{25}{4}+\sqrt{\frac{504}{16}}\right]-3\ln\left[\frac{17}{4}+\sqrt{\frac{168}{16}}\right]$	(m1)	
	$= 3\ln\left[\frac{25+\sqrt{504}}{17+\sqrt{168}}\right] = 3\ln\left[\frac{25+6\sqrt{14}}{17+2\sqrt{42}}\right]$		
	$= 1.379$	(A1)	cao Must be 3d.p.

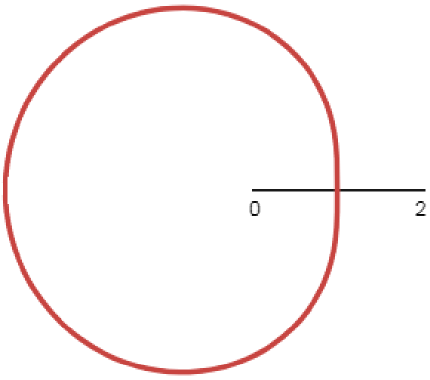
8.	$x = \sinh y$ $x = \dfrac{\mathrm{e}^y - \mathrm{e}^{-y}}{2}$	B1	
	$2x\mathrm{e}^y = (\mathrm{e}^y)^2 - 1$ $\therefore\ (\mathrm{e}^y)^2 - 2x\mathrm{e}^y - 1 = 0$	B1	
	Using quadratic formula,	M1	
	$\mathrm{e}^y = \dfrac{2x \pm \sqrt{4x^2+4}}{2} \qquad \left(= x \pm \sqrt{x^2+1}\right)$	A1	Allow omission of ±
	$y = \ln\left(x \pm \sqrt{x^2+1}\right)$	A1	
	As $x - \sqrt{x^2+1} < 0$, $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$	B1	Justification may be seen earlier
9. a) i)	$\left(\cos\dfrac{\theta}{3} + \mathrm{i}\sin\dfrac{\theta}{3}\right)^3$		
	$\cos^3\dfrac{\theta}{3} + 3\cos^2\dfrac{\theta}{3}\left(\mathrm{i}\sin\dfrac{\theta}{3}\right) + 3\cos\dfrac{\theta}{3}\left(\mathrm{i}\sin\dfrac{\theta}{3}\right)^2 + \left(\mathrm{i}\sin\dfrac{\theta}{3}\right)^3$	M1	Unsimplified Allow cis notation
	$= \cos^3\dfrac{\theta}{3} + 3\mathrm{i}\cos^2\dfrac{\theta}{3}\sin\dfrac{\theta}{3} - 3\cos\dfrac{\theta}{3}\sin^2\dfrac{\theta}{3} - \mathrm{i}\sin^3\dfrac{\theta}{3}$	A1	
ii)	$\left(\cos\dfrac{\theta}{3} + \mathrm{i}\sin\dfrac{\theta}{3}\right)^3 = \cos\theta + \mathrm{i}\sin\theta$	B1	si
	$\therefore \cos\theta = \cos^3\dfrac{\theta}{3} - 3\cos\dfrac{\theta}{3}\sin^2\dfrac{\theta}{3}$	M1	FT (i) for sign error only
	$= \cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}\left(1 - \cos^2\frac{\theta}{3}\right)$	A1	
	$= 4\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}$	A1	cao convincing
b)	METHOD 1: $\dfrac{\cos\theta}{\cos\dfrac{\theta}{3}} = \dfrac{4\cos^3\dfrac{\theta}{3} - 3\cos\dfrac{\theta}{3}}{\cos\dfrac{\theta}{3}} = 1$	M1	Substitution
	$4\cos^3\dfrac{\theta}{3} - 4\cos\dfrac{\theta}{3} = 0$ $4\cos\dfrac{\theta}{3}\left(\cos^2\dfrac{\theta}{3} - 1\right) = 0$	A1	Removing fraction
	$\cos\frac{\theta}{3} = 0$ (not a possible solution in this equation) or $\cos\frac{\theta}{3} = \pm 1$	A1	All three (including ±1)
	When $\cos\frac{\theta}{3} = 1,\ \ \frac{\theta}{3} = 2n\pi$ $\therefore\ \theta = 6n\pi$	M1	Use of general solution of $\cos\theta$
	When $\cos\frac{\theta}{3} = -1,\ \ \frac{\theta}{3} = \pi + 2n\pi$ $\therefore\ \theta = 3\pi + 6n\pi$	A1	Either θ
	General solution: $\theta = 3n\pi$	A1	

	METHOD 2: $\frac{\cos\theta}{\cos\frac{\theta}{3}} = 1$ $\cos\theta - \cos\frac{\theta}{3} = 0$ Then, $-2\sin\frac{\theta+\frac{\theta}{3}}{2}\sin\frac{\theta-\frac{\theta}{3}}{2} = 0$ Therefore, $\sin\frac{2\theta}{3} = 0$ or $\sin\frac{\theta}{3} = 0$ $\frac{2\theta}{3} = n\pi$ or $\frac{\theta}{3} = n\pi$ $\theta = \frac{3}{2}n\pi$ or $\theta = 3n\pi$ Odd multiple of $\frac{3}{2}n\pi$ are not a solution because $\cos\theta = 0$ $\theta = 3n\pi$	(B1) (M1) (A1) (A1) (M1) (A1)	Both

10. a)	$\det A = 4(\lambda \times \lambda) - (8 \times 8)$ $\det A = 4\lambda^2 - 64$ Singular when det **A** = 0 METHOD 1: $4\lambda^2 - 64 = 0$ $\lambda^2 = 16$ $\lambda = \pm 4$ so there are two values where **A** is singular METHOD 2: $4\lambda^2 - 64 = 0$ Discriminant = $0^2 - (4 \times 4 \times -64) = 1024$ As 1024 > 0 there are two roots of the equation so there are two values where **A** is singular	M1 A1 M1 A1 (M1) (A1)	oe Must reference >0
b) i)	Cofactor matrix: $\begin{pmatrix} 9 & -8 & -12 \\ -24 & 12 & 32 \\ -16 & 8 & 12 \end{pmatrix}$ Adjugate matrix $= \begin{pmatrix} 9 & -24 & -16 \\ -8 & 12 & 8 \\ -12 & 32 & 12 \end{pmatrix}$	B3	All correct B2 for 7 or 8 correct B1 for 5 or 6 correct
ii)	$\det A = (4 \times 3^2) - 64 = -28$ $\therefore A^{-1} = \frac{1}{-28}\begin{pmatrix} 9 & -24 & -16 \\ -8 & 12 & 8 \\ -12 & 32 & 12 \end{pmatrix}$	B1 B1	FT their (a) FT their adjugate Mark final answer

11. a) i)	$y = \mathrm{e}^{3x}\sin^{-1}x$		
	Use of product rule while differentiating	M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{3x}.\frac{1}{\sqrt{1-x^2}} + 3\mathrm{e}^{3x}\sin^{-1}x$	A2	A1 each term ISW
ii)	METHOD 1:		
	$y = \ln(\cosh(2x^2+7x))^2 = 2\ln(\cosh(2x^2+7x))$	M1	Log rule AND chain rule
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2 \times \sinh(2x^2+7x) \times (4x+7)}{\cosh(2x^2+7x)}$	A1 A1 A1	$\sinh(2x^2+7x)$ $4x+7$ oe Fully correct ISW
	METHOD 2: $y = \ln(\cosh(2x^2+7x))^2$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cosh(2x^2+7x) \times \sinh(2x^2+7x) \times (4x+7)}{(\cosh(2x^2+7x))^2}$	(M1) (A1) (A1)	Chain rule $\sinh(2x^2+7x)$ $4x+7$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2 \times \sinh(2x^2+7x) \times (4x+7)}{\cosh(2x^2+7x)}$	(A1)	oe Fully correct ISW
b)	METHOD 1:		
	$1 = \frac{1}{\sqrt{1+(y^2)^2}} \times \left(2y\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	M1 A1 A1	Must see chain rule Differentiate sinh^{-1} $2y\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\sqrt{1+y^4} = 2y\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{1+y^4}}{2y}$	A1	
	METHOD 2: $y^2 = \sinh x$		
	$2y\frac{\mathrm{d}y}{\mathrm{d}x} = \cosh x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cosh x}{2y}$	(M1) (A1) (A1) (A1)	$2y\frac{\mathrm{d}y}{\mathrm{d}x}$ Cosh
	METHOD 3: $y = \pm\sqrt{\sinh x}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm\frac{1}{2}\sinh^{-\frac{1}{2}}x\cosh x$	(M1) (A1) (A1) (A1)	$\frac{1}{2}\sinh^{-\frac{1}{2}}x$ Cosh ±
	THEN:		
	When $x = 1$, $y = \pm 1.084$,	B1	Both
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm 0.7117$	A1	cao Both
	$y - 1.084 = 0.7117(x-1)$	B1	FT their y and dy/dx
	$y + 1.084 = -0.7117(x-1)$	B1	FT their y and dy/dx

12.	Solve auxiliary $3t^2 + 5t - 2 = 0$ $(3t - 1)(t + 2) = 0$	M1	M0A0 no working
	$t = \frac{1}{3}$ or $t = -2$	A1	Both values
	Complementary function: $y = A\mathrm{e}^{\frac{1}{3}x} + B\mathrm{e}^{-2x}$		
	Use particular integral of the form $Cx^2 + Dx + E$	M1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2Cx + D$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2C$	A1	Both $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$
	Therefore, $6C + 5(2Cx + D) - 2(Cx^2 + Dx + E) = 8 + 6x - 2x^2$	A1	Substitution
	$-2C = -2 \quad \rightarrow \quad C = 1$ $10C - 2D = 6 \quad \rightarrow \quad D = 2$ $6C + 5D - 2E = 8 \quad \rightarrow \quad E = 4$	A1	All values
	General Solution: $y = A\mathrm{e}^{\frac{1}{3}x} + B\mathrm{e}^{-2x} + x^2 + 2x + 4$	M1	FT C,D,E for M1A1M1A1 Sub and differentiate
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}A\mathrm{e}^{\frac{1}{3}x} - 2B\mathrm{e}^{-2x} + 2x + 2$	A1	
	When $x = 0$, $y = A + B + 4 = 6$	M1	Substitution
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}A - 2B + 2 = 5$	A1	Both y and $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Solving, $A = 3$ and $B = -1$	B1	cao
	Therefore, $y = 3\mathrm{e}^{\frac{1}{3}x} - \mathrm{e}^{-2x} + x^2 + 2x + 4$	B1	cao

13. a)		G1 G1	For **shape**, to include reflection in the initial line. Fully correct
b) i)	$y = r\sin\theta$ $y = (2 - \cos\theta)\sin\theta$ $y = 2\sin\theta - \sin\theta\cos\theta$ THEN $\left(y = 2\sin\theta - \frac{1}{2}\sin 2\theta\right)$	M1	
	$\frac{dy}{d\theta} = 2\cos\theta - \cos 2\theta$	A1	
	When parallel to initial line, $2\cos\theta - \cos 2\theta = 0$	m1	
	$2\cos\theta - (2\cos^2\theta - 1) = 0$ $2\cos^2\theta - 2\cos\theta - 1 = 0$	A1	convincing
	OR		
	$\frac{dy}{d\theta} = 2\cos\theta - (\cos^2\theta - \sin^2\theta)$	(A1)	
	When parallel to initial line, $2\cos\theta - (\cos^2\theta - \sin^2\theta) = 0$	(m1)	
	$2\cos\theta - \cos^2\theta + (1 - \cos^2\theta) = 0$ $2\cos^2\theta - 2\cos\theta - 1 = 0$	(A1)	convincing
ii)	Solving $\cos\theta = \frac{2 \pm \sqrt{4+8}}{4}$	M1	
	$\cos\theta = 1.366$ therefore no solutions or	A1	
	$\cos\theta = -0.366$	A1	
	$\therefore\ \theta = 1.9455 \text{ or } 4.3377$	A1	Both values
	$r = 2.366$	B1	FT their θ

14.	$\frac{6x^2+2x+16}{x^3-x^2+3x-3}=\frac{6x^2+2x+16}{(x-1)(x^2+3)}$	M1 A1	Linear × Quadratic
	$\frac{6x^2+2x+16}{(x-1)(x^2+3)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+3}$	M1	FT their factorising if linear × quadratic of equivalent difficulty
	$6x^2+2x+16=A(x^2+3)+(Bx+C)(x-1)$	A1	
	When $x=1$, $24=4A$ $\rightarrow A=6$ When $x=0$, $16=3A-C$ $\rightarrow C=2$ Compare coefficients of x^2: $6=A+B$ $\therefore B=0$	A2	A2 all 3 values A1 any 2 values If M0, SC1 for A = 6, B = 0, C = 2.
	$\int_2^4 \frac{6x^2+2x+16}{x^3-x^2+3x-3}\,\mathrm{d}x$ $=\int_2^4\left(\frac{6}{x-1}+\frac{2}{x^2+3}\right)\mathrm{d}x$	M1	FT their A, B, C provided $a\neq 0$ and $c\neq 0$
	$=\left[6\ln(x-1)+\frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right]_2^4$	A2	A1 each term
	$=7.93362-0.98966=6.944$	A1	cao Answer only 0 marks

GCE A LEVEL MARKING SCHEME

SUMMER 2022

A LEVEL (NEW)
FURTHER MATHEMATICS
UNIT 5 FURTHER STATISTICS B
1305U50-1

INTRODUCTION

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WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 5 FURTHER STATISTICS B

SUMMER 2022 MARK SCHEME

Qu. No.	Solution	Mark	Notes
1	$\bar{x} = 15.37$	B1	
	Standard error = $\sqrt{\frac{0.9}{10}}$	B1	SE^2 = $\frac{0.9}{10}$
	Use of $\bar{x} \pm z \times \text{SE}$	M1	FT their $\bar{x}$ and SE $\neq \sqrt{0.9}$
	$=15.37 \pm 1.6449 \times \sqrt{\frac{0.9}{10}}$	A1	1.645 or better
	$[14.88, 15.86]$	A1	cao
		Total [5]	

Qu. No.	Solution	Mark	Notes
2 (a)(i)	$\mathrm{P}(X > 14) = 0.5793$	M1A1	M1 for correct method (calculator or standardizing)
(ii)	$\mathrm{P}(X > 14 \text{ for two out of three}) = 0.5793^2 \times 0.4207 \times 3$	M1	Ft for M1A1 "their (i)" and "1-(i)"
	$= 0.4235$	A1	awrt 0.423 or 0.424
		(4)	
(b)	Let $T = X_1 + X_2 + X_3 + \cdots + X_8$		
	$\mathrm{E}(T) = 120$	B1	
	$\mathrm{Var}(T) = 8\mathrm{Var}(X)$	M1	
	$\mathrm{Var}(T) = 200$	A1	
	$\mathrm{P}(T > 160) = 0.00234 \text{ (3sf)}$	A1	cao 0.00233 from tables
		(4)	
(c)	Let $A = X_1 + X_2 + X_3$ Let $B = X_1 + X_2 + X_3 + X_4 + X_5$ $A \sim \mathrm{N}(45, 75)$ and $B \sim \mathrm{N}(75, 125)$	B1	si
	Consider $U = B - 2A$ $\mathrm{E}(U) = -15$	M1 A1	M1A0 for $\mathrm{E}(U) = 105$ from $U = 2B - A$
	$\mathrm{Var}(U) = \mathrm{Var}(B) + 2^2\mathrm{Var}(A)$ $= 425$	M1 A1	M1A1 for $\mathrm{Var}(U) =$ 575 from $U = 2B - A$
	$\mathrm{P}(U > 0)$	m1	Dependent on 1st M1 and $U = B - 2A$ m0A0 if $U = 2B - A$
	$= 0.2334$	A1	cao 0.23270 from tables.
		(7)	
		Total [15]	

Qu. No.	Solution	Mark	Notes
3 (a)	Valid reason e.g. No knowledge of underlying distribution Data are ordinal. Interval scale assumption may not be valid.	E1 (1)	
(b)	H_0: Students from the north and the south of the county are similarly stressed. H_1: Students from the north and the south of the county are NOT similarly stressed.	B1	$H_0: \eta_N = \eta_S$ $H_1: \eta_N \neq \eta_S$
	Upper critical value = 22 Lower critical value = 5 × 5 – 22 = 3	B1	For either CV
	Use of the formula $U = \sum\sum z_{ij}$	M1	Attempt to use
	U = 4 + 0 + 4 + 3 + 4 = 15 OR U = 1 + 5 + 1 + 2 + 1 = 10	A1	
	Since 15 < 22 OR 10 > 3 and there is insufficient evidence to reject H_0.	B1	FT their CV and U
	There is not enough evidence to say that students from the North and from the South have different stress levels.	E1 (6)	cso
(c)	Valid improvement. e.g. Bigger sample size. e.g. Use a control test to see if students from the North and South are generally more stressed.	E1 (1)	
		Total [8]	

Qu. No.	Solution	Mark	Notes
4 (a)	$\hat{p} = \frac{940}{2000} = 0.47$	B1	
	$\text{ESE} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $= \sqrt{\frac{0.47 \times 0.53}{2000}}$	M1	FT their $\hat{p}$ for M1A1
	$= 0.01116 \ldots$	A1	
	95% confidence limits are $\hat{p} \pm z \times \text{ESE}$	M1	FT their $\hat{p}$ and ESE for M1A1
	$0.47 \pm 1.96 \times 0.01116 \ldots$	A1	
	Giving $[0.448, 0.492]$	A1	cao
		(6)	
(b)	Two valid reasons. Eg. We have used an approximate value for p (in calculating the standard error). The binomial distribution has been approximated by the normal distribution. No continuity correction has been used.	E2 (2)	E1 for one reason
(c)			Full FT their $\hat{p}$
	$2.5758 \ldots \times \sqrt{\frac{0.53 \times 0.47}{n}} \leq 0.02$	M1	Attempt at equation or inequality with $2.5758, n$ and 0.02 oe
	$\frac{2.5758 \ldots \times \sqrt{0.53 \times 0.47}}{0.02} \leq \sqrt{n}$	A1	Correct equation or inequality. 4132.4295… from $Z_{0.995} = 2.576$
	$n \geq 4131.88 \ldots\ldots$	A1	2133 from $Z_{0.995} = 2.576$
	Therefore, an additional (4132 – 2000 =) 2132 people.	A1	
		(4)	
		Total [12]	

Qu. No.	Solution	Mark	Notes
5 (a)	$\bar{x} = \frac{2163}{50}$		
	$= 43.26$	B1	
	$s^2 = \frac{1}{49} \times \left(98508 - \frac{2163^2}{50}\right)$	M1	
	$s^2 = 100.7473(469\ldots)$ or $s = 10.0(3729779\ldots)$	A1	si
	H_0: $\mu = 38$ H_1: $\mu > 38$	B1	
	Under H_0, $\bar{X} \sim N\left(38, \frac{100.747\ldots}{50}\right)$		*Alternative method* M1 for Test statistic = $\frac{43.26-38}{10.03729779\ldots/\sqrt{50}}$ if standardising. A1 *p*-value from tables = 0.00010
	p-value = $P(\bar{X} > 43.26 \mid H_0$ is true)	M1	
	p-value = 0.000105	A1	
	Since *p* << 0.05 there is strong evidence to reject H_0.	m1	
	There is strong evidence to reject the laboratory's claim that the average time taken for test results to be returned is 38 hours.	A1	FT from their *p*-value and corresponding conclusion above
	Valid Headline implying failure on the part of the laboratory. e.g. Lab lets down patients. e.g. Laboratory takes longer than claimed to process test results.	B1	
		(9)	
(b)	Valid explanation. e.g. Because n is large, the central limit theorem allows us to use the normal distribution. e.g. Because n is large, the CLT allows us to assume that the distribution of the sample mean is normal.	E1	
		(1)	
(c)	Valid explanation. e.g. Random sampling eliminates the bias that may occur from taking a batch from, say, the same day. e.g. 50 consecutive results might all come from a time when the process is having a good, or bad, run. Randomisation avoids this.	E1	
		(1)	
(d)(i)	A t-test because the sample size is small.	E1	
(ii)	The assumption would be that the time taken for results to be returned is normally distributed.	E1	
		(2)	
		Total [13]	

<table>
<tr><th>Qu. No.</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>6</td><td>H_0: median = 4.2
H_1: median ≠ 4.2</td><td>B1</td><td>Both
$H_0: \eta = 4.2$
$H_1: \eta \neq 4.2$
Condone mean Differences</td></tr>
<tr><td></td><td><table>
<tr><td>Diff
$x - 4.2$</td><td>+0.8</td><td>-1.0</td><td>+0.7</td><td>-0.2</td><td>-0.9</td><td>0</td><td>+1.9</td><td>+0.1</td><td>+0.6</td><td>+1.7</td></tr>
<tr><td>Rank</td><td>5</td><td>7</td><td>4</td><td>2</td><td>6</td><td>.</td><td>9</td><td>1</td><td>3</td><td>8</td></tr>
</table></td><td>M1
A1</td><td></td></tr>
<tr><td></td><td>$W^+ = 5 + 4 + 9 + 1 + 3 + 8$ OR $W^- = 7 + 2 + 6$</td><td>M1</td><td>Attempt at summing ranks</td></tr>
<tr><td></td><td>= 30 = 15</td><td>A1</td><td>For A1, FT their differences provided M1M1 awarded.
M1A0M1A0 if rank of 1 assigned to diff = 0
FT n = 10 if M1A0M1A0 awarded</td></tr>
<tr><td></td><td>Upper CV = 43 OR Lower CV = 2</td><td>B1</td><td>FT their TS and CV</td></tr>
<tr><td></td><td>Because 30 < 43 OR 15 > 2, there is insufficient evidence to reject H_0.</td><td>B1</td><td>cso</td></tr>
<tr><td></td><td>The test suggests that the zoologist should abandon his studies on this population.</td><td>E1</td><td></td></tr>
<tr><td></td><td></td><td>Total [8]</td><td></td></tr>
</table>

Qu. No.	Solution	Mark	Notes
7(a)	$(X+Y) \sim \mathrm{N}(180, 2\sigma^2)$	B1	si
	$\mathrm{P}(180-\sigma < X+Y < 180+\sigma)$ $= \mathrm{P}\left(\frac{180-\sigma-180}{\sqrt{2\sigma^2}} < Z < \frac{180+\sigma-180}{\sqrt{2\sigma^2}}\right)$	M1	
	$= \mathrm{P}\left(\frac{-1}{\sqrt{2}} < Z < \frac{1}{\sqrt{2}}\right)$	M1	SC2 for only doing one side leading to 0.7602 or 0.76115 from tables
	$= 0.52 \ldots$	A1	
		(4)	
(b)	$\mathrm{E}(T_1) = \mathrm{E}(45 + \frac{1}{4}(3X - Y))$		
	$\mathrm{E}(T_1) = 45 + \frac{3}{4}\mathrm{E}(X) - \frac{1}{4}\mathrm{E}(180 - X)$	M1	M1 for either first line.
	$\mathrm{E}(T_1) = 45 + \frac{3}{4}\alpha - 45 + \frac{1}{4}\alpha$	M1	
	$\mathrm{E}(T_1) = \alpha$	A1	Convincing
	T_1 is an unbiased estimator for α		
	$\mathrm{Var}(T_1) = \mathrm{Var}(45 + \frac{1}{4}(3X - Y))$		
	$\mathrm{Var}(T_1) = \frac{9}{16}\mathrm{Var}(X) + \frac{1}{16}\mathrm{Var}(Y)$	M1	
	$\mathrm{Var}(T_1) = \frac{5}{8}\sigma^2$	A1	
	$\mathrm{Var}(T_1) < \sigma^2$	E1	FT their $\mathrm{Var}(T_1) = k\sigma^2$ where $k < 1$
	$\therefore T_1$ is a better estimator than X	(6)	
(c)(i)	$\mathrm{E}(T_2) = \mathrm{E}(\lambda X + (1-\lambda)(180° - Y))$		
	$\mathrm{E}(T_2) = \lambda\alpha + (1-\lambda)(180° - \beta)$	M1	
	$\mathrm{E}(T_2) = \lambda\alpha + (1-\lambda)(\alpha)$		
	$\mathrm{E}(T_2) = \lambda\alpha + \alpha - \lambda\alpha = \alpha$	A1	Convincing
(ii)	$\mathrm{Var}(T_2) = \mathrm{Var}(\lambda X + (1-\lambda)(180° - Y))$		
	$\mathrm{Var}(T_2) = \lambda^2\mathrm{Var}(X) + (1-\lambda)^2\mathrm{Var}(180° - Y)$	M1	
	$\mathrm{Var}(T_2) = \lambda^2\sigma^2 + (1-\lambda)^2\sigma^2$	A1	oe ISW

Qu. No.	Solution	Mark	Notes
(iii)	$\frac{\mathrm{d}}{\mathrm{d}\lambda}\mathrm{Var}(T_2) = 2\lambda\sigma^2 - 2(1-\lambda)\sigma^2$	M1A1	M1 for attempt to differentiate with at least one decrease in power. For A1, FT $\mathrm{Var}(T_2)$ for equivalent difficulty only.
			Use of $\frac{\mathrm{d}}{\mathrm{d}\lambda}\mathrm{Var}(T_2) = 0$
	$\frac{\mathrm{d}}{\mathrm{d}\lambda}\mathrm{Var}(T_2) = 0$ gives the best estimator	M1	
	$2\lambda\sigma^2 = 2(1-\lambda)\sigma^2$		
	$2\lambda = 2 - 2\lambda$		cao
	$\lambda = \frac{1}{2}$	A1	Accept alternative justification
	$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\mathrm{Var}(T_2) = 4\sigma^2$ > 0, therefore minimum	E1	
		(9)	
		Total [19]	

GCE A LEVEL MARKING SCHEME

SUMMER 2022

A LEVEL (NEW)
FURTHER MATHEMATICS
UNIT 6 FURTHER MECHANICS B
1305U60-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

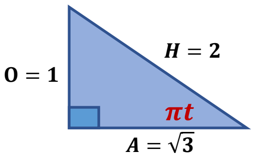
WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

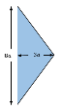
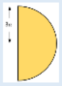
WJEC GCE A LEVEL FURTHER MATHEMATICS

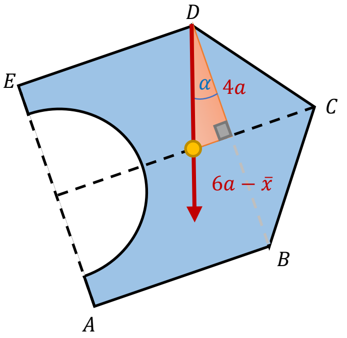
UNIT 6 FURTHER MECHANICS B

SUMMER 2022 MARK SCHEME

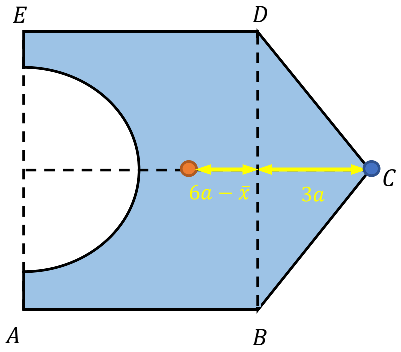
Q1	Solution	Mark	Notes
(a)	$a = v\frac{\mathrm{d}v}{\mathrm{d}x}$	M1	Used
	$\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{96}{(4x+9)^2}$	B1	
	$a = \frac{24}{4x+9} \times -24(4x+9)^{-2} \times 4$	A1	cao, isw
	$a = -\frac{2304}{(4x+9)^3}$	**[3]**	
(b)	(i) $-\frac{4}{3} = -\frac{2304}{(4x+9)^3}$	M1	FT their a from part (a)
	$4x + 9 = \sqrt[3]{1728}$	m1	Only FT $ax + b = \sqrt[3]{c}$ from the form $-\frac{4}{3} = \frac{k}{(4x+9)^3}$
	$x = \frac{3}{4}$	A1	cao
	(ii) $v = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{24}{4x+9}$		
	$\int (4x+9)\mathrm{d}x = 24\int \mathrm{d}t$	M1	Separation of variables
	$2x^2 + 9x = 24t\ (+C)$	A1	All correct
	When $t = 0, x = -2$ $(\Rightarrow\ C = -10)$	m1	Use of initial conditions
	$t = \frac{1}{24}(2x^2 + 9x + 10)$ or $t = \frac{1}{12}x^2 + \frac{3}{8}x + \frac{5}{12}$	A1	Correct expression only $(t =)$
	Substitute x from (i) into expression for t above	M1	Sub. their x into their t expression involving x and t
	$T = \frac{1}{24}\left(2\left(\frac{3}{4}\right)^2 + 9\left(\frac{3}{4}\right) + 10\right)$		
	$T = \frac{143}{192} = 0 \cdot 74(4791 \ldots)$	A1 **[9]**	FT their x if used in the correct expression only
	Total for Question 1	**12**	

Q2	Solution	Mark	Notes
(a)	(i) $x = \sin(\pi t) + \sqrt{3}\cos(\pi t)$.		
	$\frac{\mathrm{d}x}{\mathrm{d}t} = v = \pi\cos(\pi t) - \sqrt{3}\,\pi\sin(\pi t)$	B1	$\dot{x}, v = \cdots$
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\pi^2\sin(\pi t) - \sqrt{3}\,\pi^2\cos(\pi t)$	M1	$\ddot{x}, \dot{v}, a = \cdots$
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\pi^2 x$	A1	Convincing
	$\therefore$ motion is SHM (with $\omega = \pi$)		
	Value of x at the centre of motion $= 0$	B1	
	(ii) Period $= \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$ (s)	B1	Convincing
	Amplitude, $a =$ value of x when $v = 0$ $\pi\cos(\pi t) - \sqrt{3}\,\pi\sin(\pi t) = 0$	M1	FT their v
	$\tan(\pi t) = \frac{1}{\sqrt{3}}$ $\left(= \frac{\sqrt{3}}{3}\right)$		
	$\sin(\pi t) = \frac{1}{2}$ or $\cos(\pi t) = \frac{\sqrt{3}}{2}$ **OR** $x\vert_{t=\frac{1}{6}}$	m1	Either trig. ratio **OR** sub. $t = \frac{1}{6}$ into x
	$a = \left(\frac{1}{2}\right) + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$		
	$a = 2$ (m)	A1	cao 
		[8]	
(b)	Q has same period as $P \Rightarrow \omega = \pi$ amplitude is a		Condone repeated use of a
	$v^2 = \omega^2(a^2 - x^2)$, $\omega = \pi$, $x = \pm 2\sqrt{3}$, $v = \pm 2\pi$	M1	FT their $\omega = k\pi$
	$(2\pi)^2 = \pi^2\left(a^2 - \left(2\sqrt{3}\right)^2\right)$,	A1	Correct equation
	$a = 4$ (m)	A1	cao
		[3]	
(c)	$x = \pm 4\sin(\pi t)$	M1	Allow $\pm a\cos(\pi t)$, a from part (b)
	$\sin(\pi t) + \sqrt{3}\cos(\pi t) = \pm 4\sin(\pi t)$	m1	RHS $= \pm a\cos(\pi t)$
	$\tan(\pi t) = \frac{\sqrt{3}}{3}$ or $\tan(\pi t) = -\frac{\sqrt{3}}{5}$	A1	
	$t = \frac{1}{6} = 0 \cdot 16(66\ldots)$ or $t = 0 \cdot 89(385\ldots)$	A1	cao
		[4]	
	Total for Question 2	**15**	

<table>
<tr><th>Q3</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>(a)</td><td>$(\bar{y} =) \quad 4a$</td><td>B1
[1]</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th>Shape</th><th>Area/mass</th><th>Distance from AE</th></tr>
<tr><td></td><td>$8a \times 6a$
$(= 48a^2)$</td><td>$3a$</td></tr>
<tr><td> </td><td>$\frac{8a \times 3a}{2}$
$(12a^2)$</td><td>$6a + \frac{1}{3}(3a)\ (= 7a)$</td></tr>
<tr><td> </td><td>$\frac{\pi(3a)^2}{2}$
$\left(= \frac{9\pi a^2}{2}\right)$</td><td>$\frac{4(3a)}{3\pi}\left(= \frac{4a}{\pi}\right)$</td></tr>
<tr><td>*Lamina*</td><td>$a^2\left(60 - \frac{9\pi}{2}\right)$</td><td>$\bar{x}$</td></tr>
</table>

Moments about AE
$a^2\left(60 - \frac{9\pi}{2}\right)\bar{x} = (48a^2)(3a) + (12a^2)(7a) - \left(\frac{9\pi a^2}{2}\right)\left(\frac{4a}{\pi}\right)$
$\left(\frac{120 - 9\pi}{2}\right)\bar{x} = 144a + 84a - 18a$
$\bar{x} = \frac{140}{40 - 3\pi}a$</td><td>B3
B1
M1
A1
A1
[7]</td><td>Candidates may legitimately include a ρ term for mass per unit area
B3 6
B2 any 4 or 5,
B1 any 2 or 3 correct
Allow $-\frac{\pi(3a)^2}{2}$ or $-\frac{4(3a)}{3\pi}$
Masses and moments consistent
All terms, allow one sign error
FT Correct for their table, provided semicircle is subtracted in lamina area and moment
$\bar{x} = \frac{420}{120 - 9\pi}a$
Convincing</td></tr>
<tr><td>(c)</td><td>(i)

If hanging in equilibrium, vertical passes through centre of mass.
$\alpha = \tan^{-1}\left(\frac{6a - \bar{x}}{4a}\right)$ OR $\alpha = \tan^{-1}\left(\frac{4a}{6a - \bar{x}}\right)$
$\alpha = 90 - 70 \cdot 44(07 \ldots)^{\circ}$
$\alpha = 19 \cdot 55(92 \ldots)^{\circ}$</td><td>M1
A1
A1</td><td>Correct triangle identified
Condone missing a's
Note that
$6a - \bar{x} = \left(\frac{100 - 18\pi}{40 - 3\pi}\right)a$
$= (1 \cdot 4211 \ldots)a$
cso, accept answers rounding to
$\theta = 19^{\circ}$ or 20°</td></tr>
</table>

(ii)

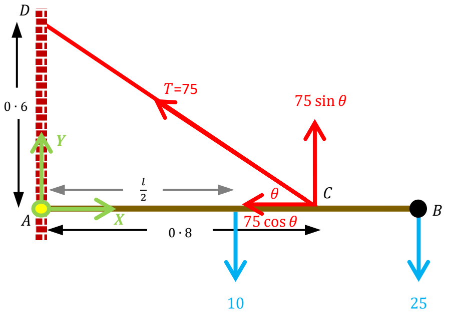
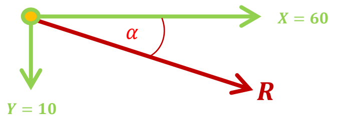


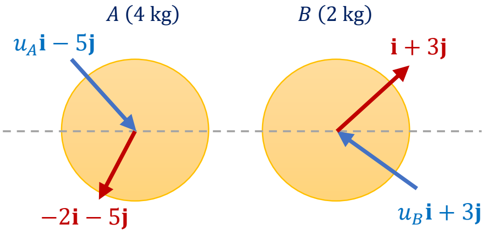
Moments about BD	M1	Condone missing a's
$M\times\left(6-\frac{140}{40-3\pi}\right)a=kM\times 3a$	A1	$M\times(6-\bar{x})a=kM\times 3a$
$k=\frac{1}{3}\left(6-\frac{140}{40-3\pi}\right)$		
$k=\frac{1}{3}(1\cdot 42\ldots)$		
$k=0\cdot 47(37\ldots)$ $\quad=\frac{1}{3}\left(\frac{100-18\pi}{40-3\pi}\right)$	A1 **[6]**	cso, accept answers rounding to $k=0\cdot 47$

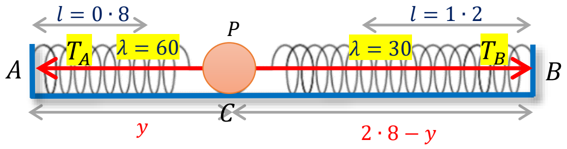
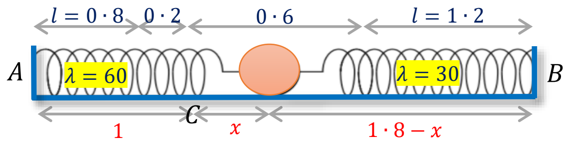
Alternative Solution

Shape	*Area/mass*	*Distance from AE*	*Distance from BD*
Lamina	M	$\bar{x}$	$6a-\bar{x}$
Particle	kM	$9a$	$3a$
New Lamina	$(k+1)M$	$6a$	0

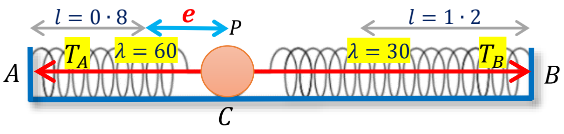
Moments about AE	M1	Condone missing a's
$(k+1)M\times 6a=M\times\bar{x}+kM\times 9a$	A1	
$k=\frac{1}{3}\left(6-\frac{140}{40-3\pi}\right)$		
$k=\frac{1}{3}(1\cdot 42\ldots)$		
$k=0\cdot 47(37\ldots)$ $\quad=\frac{1}{3}\left(\frac{100-18\pi}{40-3\pi}\right)$	A1	cso, accept answers rounding to $k=0\cdot 47$
Total for Question 3	14	

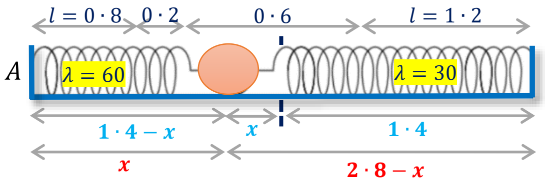
Q4	Solution	Mark	Notes
(a)			length of rod $AB = l$ $\sin\theta = 0\cdot6$ $\cos\theta = 0\cdot8$
	Moments about A	M1	Dim. correct equation with 3 terms
	$75\sin\theta\times0\cdot8 = 10\times\frac{l}{2}+25\times l$	A1 A1	-1 each error
	$l = 1\cdot2\quad(\mathrm{m})$	A1	cao
		[4]	
(b)	Resolve vertically $Y+75\sin\theta = 10+25$ $Y = -10\quad(\mathrm{N})$ Y pointing downwards $(75\sin\theta = Y+10+25)$ $Y = 10$	M1 A1	Dim. correct equation, no extra/missing forces
	Resolve horizontally $X = 75\cos\theta$ $X = 60\quad(\mathrm{N})$	M1 A1	Dim. correct equation, no extra forces
	$R = \sqrt{60^2+10^2}$	m1	Provided both M's awarded, FT their X and Y
	$R = 10\sqrt{37} = 60\cdot82(76\ldots)\ (\mathrm{N})$	A1	cao
			
	$\tan\alpha = \frac{10}{60}$	m1	Provided both M's awarded, FT their X and Y
	$\alpha = 9\cdot46(23\ldots)°$ **below the horizontal**	A1	cao
		[8]	
	Total for Question 4	**12**	

Q5	Solution	Mark	Notes
(a)			Before collision After collision $e=\frac{2}{5}$
	Con. of momentum (along line of centres)	M1	Attempted. Allow 1 sign error. $4(u_A\mathbf{i}-5\mathbf{j})+2(u_B\mathbf{i}+3\mathbf{j})=4(-2\mathbf{i}-5\mathbf{j})+2(\mathbf{i}+3\mathbf{j})$
	$4u_A+2u_B=4(-2)+2(1)$ $(2u_A+u_B=-3)$ $4u_A\mathbf{i}+2u_B\mathbf{i}=-6\mathbf{i}$	A1	All correct, oe Condone **i**'s, i.e.
	Restitution (along line of centres)	M1	Attempted. Allow 1 sign error.
	$(1)-(-2)=-\frac{2}{5}(u_B-u_A)$ $(2u_A-2u_B=15)$ $4u_A\mathbf{i}+2u_B\mathbf{i}=-6\mathbf{i}$	A1	All correct, condone **i**'s, $\frac{2}{5}=-\frac{1--2}{u_B-u_A}=\frac{1--2}{u_A-u_B}$
	Solving equations $u_A=\frac{3}{2}$ $u_B=-6$	m1	One variable eliminated
	Velocities before collision Sphere $A=\frac{3}{2}\mathbf{i}-5\mathbf{j}$ (ms^{-1})	A1	cao
	Sphere $B=-6\mathbf{i}+3\mathbf{j}$ (ms^{-1})	A1 **[7]**	cao
(b)	Wall is parallel to vector **i** since impulse only has a **j** component	B1 **[1]**	Parallel to vector **i** since … • No **i** component • No momentum in **i** direction • Perpendicular to wall
(c)	Impulse, $\mathbf{I}=$ change in momentum $32\mathbf{j}=4\mathbf{v}-4(-2\mathbf{i}-5\mathbf{j})$	M1	Used, $32\mathbf{j}=-4\mathbf{v}+4(-2\mathbf{i}-5\mathbf{j})$ $32=4v-4(-5)$ Condone **j**'s on the above
	$\mathbf{v}=-2\mathbf{i}+3\mathbf{j}$	A1	
	speed $=\sqrt{2^2+3^2}$ $=\sqrt{13}$ (ms^{-1}) or $=3\cdot60(55\ldots)$	B1 **[3]**	FT their $\sqrt{13}$ derived from $\mathbf{v}=-2\mathbf{i}+a\mathbf{j},\ a\neq0$
(d)	Loss in KE $=\frac{1}{2}(4)(2^2+5^2)-\frac{1}{2}(4)\left(\sqrt{13}^2\right)$ OR Loss in KE $=\frac{1}{2}(4)(5^2)-\frac{1}{2}(4)(3^2)$	M1	Difference in KE, any order At least one v^2 correct
	Loss in KE $=32$ (J)	A1 **[2]**	FT provided loss (in KE) >0
	Total for Question 5	**13**	

Q6	Solution	Mark	Notes
			$AB = 2\cdot 8$ m
(a)	Let $AC = y$		Use of Hooke's Law
	$T_A = \frac{60(y-0\cdot8)}{0\cdot8}$ $(= 75y - 60)$	M1	$\frac{60\,dist}{0\cdot8}$ or $\frac{30\,dist}{1\cdot2}$ Any algebraic extension/distance
	$T_B = \frac{30(2\cdot8-1\cdot2-y)}{1\cdot2}$ $(= 40 - 25y)$	A1	T_B or T_A correct
	In equilibrium, $T_A = T_B$	m1	
	$\frac{60(y-0\cdot8)}{0\cdot8} = \frac{30(2\cdot8-1\cdot2-y)}{1\cdot2}$		
	$75y - 60 = 40 - 25y$		
	$y = 1$ (m)	A1	Convincing
		[4]	
(b)			$AB = 2\cdot 8$ m
	(i) Let x denote the displacement of P from C		
	$T_A = \frac{60(0\cdot2+x)}{0\cdot8}$ $(= 15 + 75x)$	B1	either term, oe
	$T_B = \frac{30(0\cdot6-x)}{1\cdot2}$ $(= 15 - 25x)$		
	Apply N2L to P,	M1	Dim. correct. T_B, T_A opposing
	$T_B - T_A = 4\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$	A1	
	$\frac{30(0\cdot6-x)}{1\cdot2} - \frac{60(0\cdot2+x)}{0\cdot8} = 4\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$		
	$-100x = 4\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$		
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -25x$	A1	Allow for any defined x, e.g. $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -25(x-1)$
	$\therefore$ SHM with $\omega = 5$ (with centre at C)	B1	Must come from $\ddot{x} - \omega^2 x$
	Period $= \frac{2\pi}{\omega} = \frac{2\pi}{5}$	B1	FT ω

	(ii) Amplitude, $a = 1\cdot4 - 1 = 0\cdot4\ \ (\mathrm{m})$	B1	
	Using $x = \pm a\cos\omega t$ with $a = 0\cdot4,\ \ \omega = 5$	M1	Allow $x = \pm a\sin(\omega t)$ FT a and ω
	$-0\cdot2 = 0\cdot4\cos 5t$	A1	FT for $-0\cdot2 = a\cos\omega t$
	$t = \frac{2\pi}{15} = 0\cdot418(879\ldots)\qquad (s)$	A1	cao
		[10]	
	Total for Question 6	**14**	

Q6	Alternative Solution	Mark	Notes
			$AB = 2\cdot8$ m
(a)	Let e = extension in AP		Use of Hooke's Law
	$T_A = \frac{60}{0\cdot8}e \qquad (= 75e)$	M1	$\frac{60\ dist}{0\cdot8}$ or $\frac{30\ dist}{1\cdot2}$ Any algebraic distance/extension
	$T_B = \frac{30(0\cdot8-e)}{1\cdot2} \qquad (= 20 - 25e)$	A1	T_B or T_A correct
	In equilibrium, $T_A = T_B$	m1	
	$\frac{60}{0\cdot8}e = \frac{30(0\cdot8-e)}{1\cdot2}$		
	$75e = 20 - 25e \qquad \Rightarrow \qquad e = 0\cdot2$		
	$AC = 0\cdot8 + 0\cdot2 = 1\quad (\mathrm{m})$	A1	Convincing
		[4]	

Q6	Alternative Solution	Mark	Notes
(b)			$AB = 2\cdot8\text{ m}$
	(i) Let x denote the displacement of P from • the midpoint of AB • A		
	$T_A = \frac{60(1{\cdot}4-0{\cdot}8-x)}{0{\cdot}8}$ $T_A = \frac{60(x-0{\cdot}8)}{0{\cdot}8}$ $T_B = \frac{30(1{\cdot}4-1{\cdot}2+x)}{1{\cdot}2}$ $T_B = \frac{30(2{\cdot}8-1{\cdot}2-x)}{1{\cdot}2}$	B1	$T_A = 45 - 75x$ or $75x - 60$ either term, oe $T_B = 5 + 25x$ or $40 - 25x$
	Apply N2L to P,	M1	Dim. correct. T_B, T_A opposing
	$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \begin{cases} T_A - T_B \\ T_B - T_A \end{cases}$	A1	
	$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \begin{cases} \frac{60(1{\cdot}4-0{\cdot}8-x)}{0{\cdot}8} - \frac{30(1{\cdot}4-1{\cdot}2+x)}{1{\cdot}2} \\ \frac{30(2{\cdot}8-1{\cdot}2-x)}{1{\cdot}2} - \frac{60(x-0{\cdot}8)}{0{\cdot}8} \end{cases}$		
	$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \begin{cases} 40 - 100x \\ 100 - 100x \end{cases}$		
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \begin{cases} -25(x - 0\cdot4) \\ -25(x - 1) \end{cases}$	A1	
	∴ SHM with $\omega = 5$ (with centre at $x = 0\cdot4$, i.e. C) (with centre at $x = 1$, i.e. C)	B1	
	Period $= \frac{2\pi}{\omega} = \frac{2\pi}{5}$	B1	FT ω
	(ii) Amplitude, $a = 1\cdot4 - 1 = 0\cdot4\ \text{(m)}$	B1	
	Using $x - 0\cdot4 = \pm a\cos\omega t$ with $a = 0\cdot4$, $\omega = 5$	M1	Allow $x = \pm a\sin(\omega t)$ FT a and ω
	$0\cdot6 - 0\cdot4 = -0\cdot4\cos 5t$	A1	FT RHS with $x = 1\cdot4 - 0\cdot8$
	$t = \frac{2\pi}{15} = 0\cdot418(879\ldots)\quad\text{(s)}$	A1	cao
	OR		
	Using $x - 1 = \pm a\cos\omega t$ with $a = 0\cdot4\quad \omega = 5$	(M1)	
	$0\cdot8 = 1 + 0\cdot4\cos 5t$	(A1)	
	$-0\cdot2 = 0\cdot4\cos 5t$		
	$t = \frac{2\pi}{15} = 0\cdot418(879\ldots)\quad\text{(s)}$	(A1)	
		[10]	
	Total for Question 6	14	