GCE AS/A Level

0977/01

S17-0977-01

MATHEMATICS – FP1
Further Pure Mathematics

FRIDAY, 19 MAY 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}.$$

(a) Evaluate the determinant of $\mathbf{M}$. [2]

(b) (i) Find the adjugate matrix of $\mathbf{M}$.

(ii) Deduce the inverse matrix $\mathbf{M}^{-1}$. [3]

(c) Hence solve the system of equations

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \\ 17 \end{bmatrix}.$$

[2]

2. Consider the series

$$S_n = 1^2 + 4^2 + 7^2 + \ldots + (3n-2)^2.$$

Obtain an expression for S_n, giving your answer in the form $an^3 + bn^2 + cn$, where a, b, c are rational numbers. [6]

3. The complex number z is given by $z = \dfrac{(1+2\mathrm{i})(-3+\mathrm{i})}{(1+3\mathrm{i})}$.

Determine the modulus and the argument of z. [8]

4. The transformation T in the plane consists of a reflection in the x-axis, followed by a translation in which the point (x, y) is transformed to the point $(x-2, y+1)$, followed by an anticlockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$ [5]

(b) Show that T has no fixed points. [3]

5. Consider the following equations.

$$x + 3y - z = 1,$$
$$2x - y + 2z = 3,$$
$$3x - 5y + 5z = \lambda.$$

(a) Find the value of λ for which the equations are consistent. [4]

(b) For this value of λ, find the general solution of the equations. [3]

6. Use mathematical induction to prove that $9^n - 1$ is divisible by 8 for all positive integers n. [7]

7. The function f is defined on the domain $\left(0, \frac{\pi}{2}\right)$ by

$$f(x) = (\tan x)^{\tan x}.$$

(a) Show that

$$f'(x) = g(x)(1 + 1\text{n}(\tan x)),$$

where $g(x)$ is to be determined. [4]

(b) Find the x-coordinate of the stationary point on the graph of f, giving your answer correct to two decimal places. [3]

8. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$wz = 1.$$

(a) Obtain expressions for x and y in terms of u and v. [4]

(b) Given that the point P moves along the line $x + y = 1$,

(i) show that the locus of Q is a circle,

(ii) determine the radius and the coordinates of the centre C of the circle. [6]

(c) Given that P and Q have the same coordinates, find the two possible positions of P and Q. [3]

9. The roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$ are denoted by α, β, γ.

(a) (i) Show that

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = -\frac{7}{16}.$$

(ii) What does this result tell you about the nature of the roots of this cubic equation? [5]

(b) Determine the cubic equation whose roots are $\frac{\alpha}{\beta\gamma}, \frac{\beta}{\gamma\alpha}, \frac{\gamma}{\alpha\beta}$. [7]

END OF PAPER