GCE AS/A Level

0976/01

S17-0976-01

MATHEMATICS – C4
Pure Mathematics

FRIDAY, 16 JUNE 2017 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Express $\dfrac{8x^2+7x-25}{(x-1)^2(x+4)}$ in terms of partial fractions. [4]

(b) **Use your result to part *(a)*** to express $\dfrac{9x^2+5x-24}{(x-1)^2(x+4)}$ in terms of partial fractions. [3]

2. The curve *C* has equation

$$y^6 - 3x^4 - 9x^2y + 48 = 0.$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6xy+4x^3}{2y^5-3x^2}$. [3]

(b) Find the gradient of the tangent to *C* at each of the points where *C* crosses the x-axis. [3]

3. *(a)* Show that the equation

$$5\cos^2\theta + 7\sin 2\theta = 3\sin^2\theta$$

may be rewritten in the form

$$a\tan^2\theta + b\tan\theta + c = 0,$$

where a, b, c are non-zero constants whose values are to be found.
Hence, find all values of θ in the range $0° \leqslant \theta \leqslant 180°$ satisfying the equation

$$5\cos^2\theta + 7\sin 2\theta = 3\sin^2\theta.$$ [6]

(b) (i) Express $\sqrt{5}\cos\phi + \sqrt{11}\sin\phi$ in the form $R\cos(\phi - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Use your result to part (i) to find the least value of

$$\frac{1}{\sqrt{5}\cos\phi + \sqrt{11}\sin\phi + 6}.$$

Write down a value for ϕ for which this least value occurs. [6]

4. The region R is bounded by the curve $y = \cos x + \sec x$, the x-axis and the lines $x = \dfrac{\pi}{6}$, $x = \dfrac{\pi}{3}$. Find the volume of the solid generated when R is rotated through four right angles about the x-axis. Give your answer correct to two decimal places. [7]

5. *(a)* Expand $(1+4x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which your expansion is valid. [3]

(b) Use your answer to part *(a)* to expand $(1+4y+8y^2)^{-\frac{1}{2}}$ in ascending powers of y up to and including the term in y^2. [3]

6. The curve *C* has the parametric equations $x = at^2$, $y = bt^3$, where a, b are positive constants. The point *P* lies on *C* and has parameter p.

(a) Show that the equation of the tangent to *C* at the point *P* is

$$2ay = 3bpx - abp^3.$$

[5]

(b) The tangent to *C* at the point *P* intersects *C* again at the point with coordinates $(4a, 8b)$. Show that p satisfies the equation

$$p^3 - 12p + 16 = 0.$$

Hence find the value of p. [5]

7. *(a)* Find $\int \frac{\ln x}{x^4}\,dx$. [4]

(b) Use the substitution $u = x^2 + 1$ to evaluate

$$\int_0^1 x^3(x^2 + 1)^4\,dx.$$

[5]

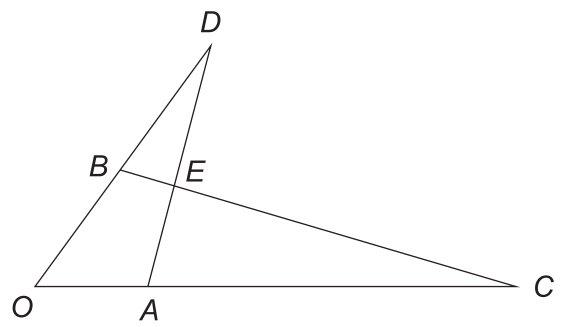
8. The size N of the population of a small island may be modelled as a continuous variable. At time t years, the rate of increase of N is assumed to be directly proportional to the value of $\sqrt{N}$.

(a) Write down a differential equation satisfied by N. [1]

(b) When t = 5, the size of the population was 256. When t = 7, the size of the population was 400. Find an expression for N in terms of t. [6]

TURN OVER

9. In the diagram below, the points O, A, B, C and D are as follows.
A lies on OC and $OC = 5OA$. B lies on OD and $OD = 2OB$.



Taking O as origin, the position vectors of A and B are denoted by $\mathbf{a}$ and $\mathbf{b}$ respectively.

(a) Write down the vector **AD** in terms of $\mathbf{a}$ and $\mathbf{b}$.
Hence show that the vector equation of the line AD may be expressed in the form

$$\mathbf{r} = (1 - \lambda)\mathbf{a} + 2\lambda\mathbf{b}.$$ [3]

(b) Find a similar expression for the vector equation of the line BC. [2]

(c) The lines AD and BC intersect at the point E. Find the position vector of E in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

10. Complete the following proof by contradiction to show that $\sqrt{7}$ is irrational.

Assume that $\sqrt{7}$ is rational. Then $\sqrt{7}$ may be written in the form $\frac{a}{b}$,

where a, b are integers having no factors in common.

$\therefore a^2 = 7b^2$.
$\therefore a^2$ has a factor 7.
$\therefore a$ has a factor 7 so that $a = 7k$, where k is an integer. [3]

END OF PAPER