

974/01

**MATHEMATICS C2**

**Pure Mathematics**

P.M. TUESDAY, 10 January 2006

(1½ hours)

**NEW SPECIFICATION**

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with six ordinates to find an approximate value for the integral

$$\int_0^1 \frac{1}{2+x^3} dx.$$

Show your working and give your answer correct to four significant figures. [4]

2. Find all values of  $\theta$  in the interval  $0^\circ \leq \theta \leq 360^\circ$  satisfying

(a)  $4 \cos^2 \theta - \cos \theta = 2 \sin^2 \theta,$  [6]

(b)  $\tan \theta = -\sqrt{3},$  [2]

(c)  $\sin 2\theta = \frac{1}{2}.$  [3]

3. The triangle  $ABC$  is such that  $AB = 12$  cm,  $BC = 10$  cm and  $\widehat{CAB} = 45^\circ$ .

(a) Find the possible values of  $\widehat{BCA}$  and  $\widehat{ABC}.$  [4]

(b) Find the possible values of the area of the triangle  $ABC.$  [2]

4. (a) A geometric series has first term  $a$  and common ratio  $r$ . Write down the  $n$ th term of the series and prove that the sum of the first  $n$  terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}. \quad [4]$$

- (b) The fourth term of a geometric series is 2 and the seventh term is 54.

(i) Find the common ratio of the series.

(ii) Find the sum of the first ten terms of the series, giving your answer correct to one decimal place.

(iii) Find the least value of  $n$  for the  $n$ th term to exceed 125 000. [10]

5. The sum of the first two terms of an arithmetic series is 3. The eighth term of the arithmetic series is 47.

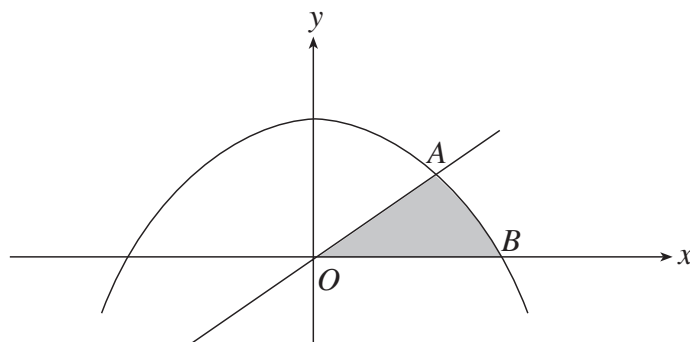
Find

(a) the first term and the common difference of the series, [4]

(b) the sum of the first twenty terms of the series. [2]

6. Integrate  $5x^{\frac{1}{3}} + 3x^{-3}$  with respect to  $x$ . [2]

7.



The diagram shows the curve  $y = 4 - x^2$  and the line  $y = 3x$  intersecting at the point  $A$ . The curve  $y = 4 - x^2$  intersects the  $x$ -axis at  $B$ .

- (a) Find the coordinates of  $A$  and  $B$ , showing your working. [5]
- (b) Evaluate the area of the shaded region. [7]
8. (a) Find the centre and radius of the circle  $C$  given by

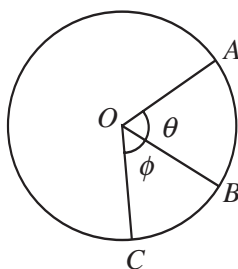
$$x^2 + y^2 - 8x + 4y + 11 = 0. \quad [3]$$

- (b) Given that the circle

$$x^2 + y^2 = a^2 \quad (a > 0)$$

touches  $C$  externally, find the value of  $a$ , giving your answer correct to two decimal places. [4]

9.



The diagram shows a circle of centre  $O$  and radius 4 cm. The points  $A$ ,  $B$  and  $C$  lie on the circle as shown, the angles  $\theta$  and  $\phi$  being measured in radians. The sum of the sector areas  $AOB$  and  $BOC$  is  $15.2 \text{ cm}^2$ .

- (a) Show that  $\theta + \phi = 1.9$ . [2]
- (b) Given that the arc length  $AB$  is 3.2 cm greater than the arc length  $BC$ , find the values of  $\theta$  and  $\phi$ . [4]
10. (a) Given that  $x > 0, y > 0$ , show that

$$\log_a(xy) = \log_a x + \log_a y. \quad [3]$$

- (b) Given that  $\int_1^3 \log_{10} x \, dx$  has an approximate value of 0.5628, find an approximate value for  $\int_1^3 \log_{10}(10x) \, dx$ . Give your answer correct to four decimal places. [4]